

Mark Scheme (Results)

January 2009

GCE

GCE Mathematics (6666/01)

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6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks	
1 (a)	<p>$C: y^2 - 3y = x^3 + 8$</p> <p>$\left\{ \begin{array}{l} \frac{dy}{dx} \\ \frac{dy}{dx} \end{array} \right\} \times \left\{ \begin{array}{l} \frac{dy}{dx} \\ \frac{dy}{dx} \end{array} \right\} \times 2y \frac{dy}{dx} - 3 \frac{dy}{dx} = 3x^2$</p> <p>$(2y-3) \frac{dy}{dx} = 3x^2$</p> <p>$\frac{dy}{dx} = \frac{3x^2}{2y-3}$</p>	<p>Differentiates implicitly to include either $\pm ky \frac{dy}{dx}$ or $\pm 3 \frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx} = \right)$.)</p> <p>Correct equation.</p> <p>A correct (condoning sign error) attempt to combine or factorise their '$2y \frac{dy}{dx} - 3 \frac{dy}{dx}$'.</p> <p>Can be implied.</p> <p>$\frac{3x^2}{2y-3}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1 oe</p> <p>(4)</p>
(b)	<p>$y = 3 \Rightarrow 9 - 3(3) = x^3 + 8$</p> <p>$x^3 = -8 \Rightarrow \underline{x = -2}$</p> <p>$(-2, 3) \Rightarrow \frac{dy}{dx} = \frac{3(4)}{6-3} \Rightarrow \frac{dy}{dx} = 4$</p>	<p>Substitutes $y = 3$ into C.</p> <p>Only $\underline{x = -2}$</p> <p>$\frac{dy}{dx} = 4$ from correct working.</p> <p>Also can be ft using their 'x' value and $y = 3$ in the correct part (a) of $\frac{dy}{dx} = \frac{3x^2}{2y-3}$</p>	<p>M1</p> <p>A1</p> <p>A1 $\sqrt{\quad}$</p> <p>(3)</p>
	<p>1(b) final A1 $\sqrt{\quad}$. Note if the candidate inserts their x value and $y = 3$ into $\frac{dy}{dx} = \frac{3x^2}{2y-3}$, then an answer of $\frac{dy}{dx} =$ their x^2, may indicate a correct follow through.</p>		
			[7]

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<p>2 (a)</p> <p>Area(R) = $\int_0^2 \frac{3}{\sqrt{1+4x}} dx = \int_0^2 3(1+4x)^{-\frac{1}{2}} dx$</p> <p>$= \left[\frac{3(1+4x)^{\frac{1}{2}}}{\frac{1}{2} \cdot 4} \right]_0^2$</p> <p>$= \left[\frac{3}{2}(1+4x)^{\frac{1}{2}} \right]_0^2$</p> <p>$= \left(\frac{3}{2}\sqrt{9} \right) - \left(\frac{3}{2}(1) \right)$</p> <p>$= \frac{9}{2} - \frac{3}{2} = \underline{3} \text{ (units)}^2$</p> <p>(Answer of 3 with no working scores M0A0M0A0.)</p> <p>(b)</p> <p>Volume = $\pi \int_0^2 \left(\frac{3}{\sqrt{1+4x}} \right)^2 dx$</p> <p>$= (\pi) \int_0^2 \frac{9}{1+4x} dx$</p> <p>$= (\pi) \left[\frac{9}{4} \ln 1+4x \right]_0^2$</p> <p>$= (\pi) \left[\left(\frac{9}{4} \ln 9 \right) - \left(\frac{9}{4} \ln 1 \right) \right]$</p> <p>Note that $\ln 1$ can be implied as equal to 0.</p> <p>So Volume = $\frac{9}{4} \pi \ln 9$</p> <p>Note the answer must be a one term exact value. Note, also you can ignore subsequent working here.</p>	<p><i>Integrating</i> $3(1+4x)^{-\frac{1}{2}}$ to give $\pm k(1+4x)^{\frac{1}{2}}$.</p> <p><u>Correct integration.</u> Ignore limits.</p> <p>Substitutes limits of 2 and 0 into a changed function and subtracts the correct way round.</p> <p>Use of $V = \pi \int y^2 dx$. Can be implied. Ignore limits and dx.</p> <p>$\pm k \ln 1+4x$ $\frac{9}{4} \ln 1+4x$</p> <p>Substitutes limits of 2 and 0 and subtracts the correct way round.</p> <p>$\frac{9}{4} \pi \ln 9$ or $\frac{9}{2} \pi \ln 3$ or $\frac{18}{4} \pi \ln 3$</p> <p>Note that = $\frac{9}{4} \pi \ln 9 + c$ (oe.) would be awarded the final A0.</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p><u>3</u> A1</p> <p>(4)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>dM1</p> <p>A1 oe isw</p> <p>(5)</p> <p>[9]</p>

Question Number	Scheme	Marks
<p>3 (a)</p>	$27x^2 + 32x + 16 \equiv A(3x+2)(1-x) + B(1-x) + C(3x+2)^2$ $x = -\frac{2}{3}, \quad 12 - \frac{64}{3} + 16 = \left(\frac{5}{3}\right)B \Rightarrow \frac{20}{3} = \left(\frac{5}{3}\right)B \Rightarrow B = 4$ $x = 1, \quad 27 + 32 + 16 = 25C \Rightarrow 75 = 25C \Rightarrow C = 3$ <p>Equate x^2: $27 = -3A + 9C \Rightarrow 27 = -3A + 27 \Rightarrow 0 = -3A \Rightarrow A = 0$</p> $x = 0, \quad 16 = 2A + B + 4C$ $\Rightarrow 16 = 2A + 4 + 12 \Rightarrow 0 = 2A \Rightarrow A = 0$	<p>Forming this identity M1</p> <p>Substitutes either $x = -\frac{2}{3}$ or $x = 1$ into their identity or equates 3 terms or substitutes in values to write down three simultaneous equations. M1</p> <p>Both $B = 4$ and $C = 3$ A1</p> <p>(Note the A1 is dependent on both method marks in this part.)</p> <p>Compares coefficients or substitutes in a third x-value or uses simultaneous equations to show $A = 0$. B1</p> <p>(4)</p>
<p>(b)</p>	$f(x) = \frac{4}{(3x+2)^2} + \frac{3}{(1-x)}$ $= 4(3x+2)^{-2} + 3(1-x)^{-1}$ $= 4\left[2\left(1 + \frac{3}{2}x\right)^{-2}\right] + 3(1-x)^{-1}$ $= 1\left(1 + \frac{3}{2}x\right)^{-2} + 3(1-x)^{-1}$ $= 1\left\{1 + (-2)\left(\frac{3x}{2}\right) + \frac{(-2)(-3)}{2!}\left(\frac{3x}{2}\right)^2 + \dots\right\}$ $+ 3\left\{1 + (-1)(-x) + \frac{(-1)(-2)}{2!}(-x)^2 + \dots\right\}$ $= \left\{1 - 3x + \frac{27}{4}x^2 + \dots\right\} + 3\left\{1 + x + x^2 + \dots\right\}$ $= 4 + 0x + \frac{39}{4}x^2$	<p>Moving powers to top on any one of the two expressions M1</p> <p>Either $1 \pm (-2)\left(\frac{3x}{2}\right)$ or $1 \pm (-1)(-x)$ from either first or second expansions respectively dM1;</p> <p>Ignoring 1 and 3, any one correct {.....} expansion. A1</p> <p>Both {.....} correct. A1</p> <p>$4 + (0x) + \frac{39}{4}x^2$ A1; A1</p> <p>(6)</p>

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(c)	<p>Actual = $f(0.2) = \frac{1.08 + 6.4 + 16}{(6.76)(0.8)}$ $= \frac{23.48}{5.408} = 4.341715976... = \frac{2935}{676}$</p> <p>Or</p> <p>Actual = $f(0.2) = \frac{4}{(3(0.2) + 2)^2} + \frac{3}{(1 - 0.2)}$ $= \frac{4}{6.76} + 3.75 = 4.341715976... = \frac{2935}{676}$</p> <p>Estimate = $f(0.2) = 4 + \frac{39}{4}(0.2)^2$ $= 4 + 0.39 = 4.39$</p> <p>%age error = $\frac{ 4.39 - 4.341715976... }{4.341715976...} \times 100$ $= 1.112095408... = 1.1\% (2sf)$</p>	<p>Attempt to find the actual value of $f(0.2)$ or seeing awrt 4.3 and believing it is candidate's actual $f(0.2)$.</p> <p>Candidates can also attempt to find the actual value by using $\frac{A}{(3x + 2)} + \frac{B}{(3x + 2)^2} + \frac{C}{(1 - x)}$ with their A, B and C.</p> <p>Attempt to find an estimate for $f(0.2)$ using their answer to (b)</p> <p>$\left \frac{\text{their estimate} - \text{actual}}{\text{actual}} \right \times 100$</p> <p>1.1%</p> <p>M1</p> <p>M1 $\sqrt{\quad}$</p> <p>M1</p> <p>A1 cao (4)</p> <p>[14]</p>

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<p>4 (a) $\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$</p> <p>As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\} = \frac{(-2 \times q) + (1 \times 2) + (-4 \times 2)}{\quad}$</p> <p>$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG</p> <p>(b) Lines meet where:</p> $\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$ <p>$\mathbf{i}: 11 - 2\lambda = -5 + q\mu$ (1) First two of $\mathbf{j}: 2 + \lambda = 11 + 2\mu$ (2) $\mathbf{k}: 17 - 4\lambda = p + 2\mu$ (3)</p> <p>(1) + 2(2) gives: $15 = 17 + \mu \Rightarrow \mu = -2$</p> <p>(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$</p> <p>(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$ $\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$</p> <p>(c) $\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5 \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2 \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$</p> <p>Intersect at $\mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or $\underline{(1, 7, -3)}$</p>	<p>Apply dot product calculation between two direction vectors, ie. $\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$</p> <p>Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$</p> <p>Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)</p> <p>Attempts to solve (1) and (2) to find one of either λ or μ Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$ Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$</p> <p>Attempt to substitute their λ and μ into their \mathbf{k} component to give an equation in p alone.</p> <p>Substitutes their value of λ or μ into the correct line l_1 or l_2.</p>	<p>M1</p> <p>A1 cso</p> <p>(2)</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>A1</p> <p>ddM1</p> <p>A1 cso</p> <p>(6)</p> <p>M1</p> <p>A1</p> <p>(2)</p>

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(d)	<p>Let $\vec{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection</p> $\vec{AX} = \vec{OX} - \vec{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ <p>Finding vector \vec{AX} by finding the difference between \vec{OX} and \vec{OA}. Can be ft using candidate's \vec{OX}.</p> $\vec{OB} = \vec{OA} + \vec{AB} = \vec{OA} + 2\vec{AX}$ $\vec{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2 \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix} \qquad \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2 \left(\text{their } \vec{AX} \right)$ <p>Hence, $\vec{OB} = \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $\vec{OB} = \underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ $\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $\underline{-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}}$ or $\underline{(-7, 11, -19)}$</p>	<p>M1 $\sqrt{\pm}$</p> <p>dM1 $\sqrt{}$</p> <p>A1</p> <p>(3)</p> <p>[13]</p>

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5	<p>(a) Similar triangles $\Rightarrow \frac{r}{h} = \frac{16}{24} \Rightarrow r = \frac{2h}{3}$</p> <p>$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{2h}{3}\right)^2 h = \frac{4\pi h^3}{27}$ AG</p> <p>(b) From the question, $\frac{dV}{dt} = 8$</p> <p>$\frac{dV}{dh} = \frac{12\pi h^2}{27} = \frac{4\pi h^2}{9}$</p> <p>$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = 8 \times \frac{9}{4\pi h^2} = \frac{18}{\pi h^2}$</p> <p>When $h = 12$, $\frac{dh}{dt} = \frac{18}{144\pi} = \frac{1}{8\pi}$</p> <p>Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\frac{18}{144\pi}$.</p>	<p>Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe. M1</p> <p>Substitutes $r = \frac{2h}{3}$ into the formula for the volume of water V. A1</p> <p>(2)</p> <p>$\frac{dV}{dt} = 8$ B1</p> <p>$\frac{dV}{dh} = \frac{12\pi h^2}{27}$ or $\frac{4\pi h^2}{9}$ B1</p> <p>Candidate's $\frac{dV}{dt} \div \frac{dV}{dh}$; M1;</p> <p>$8 \div \left(\frac{12\pi h^2}{27}\right)$ or $8 \times \frac{9}{4\pi h^2}$ or $\frac{18}{\pi h^2}$ oe A1</p> <p>$\frac{18}{144\pi}$ or $\frac{1}{8\pi}$ A1 oe isw</p> <p>(5)</p> <p>[7]</p>

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<p>6 (a) $\int \tan^2 x \, dx$</p> <p>[NB: <u>$\sec^2 A = 1 + \tan^2 A$</u> gives <u>$\tan^2 A = \sec^2 A - 1$</u>]</p> <p>$= \int \sec^2 x - 1 \, dx$</p> <p>$= \underline{\tan x - x} (+ c)$</p> <p>(b) $\int \frac{1}{x^3} \ln x \, dx$</p> <p>$\left\{ \begin{array}{l} u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} \Rightarrow v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{array} \right\}$</p> <p>$= -\frac{1}{2x^2} \ln x - \int -\frac{1}{2x^2} \cdot \frac{1}{x} \, dx$</p> <p>$= -\frac{1}{2x^2} \ln x + \frac{1}{2} \int \frac{1}{x^3} \, dx$</p> <p>$= \underline{-\frac{1}{2x^2} \ln x + \frac{1}{2} \left(-\frac{1}{2x^2} \right)} (+ c)$</p>	<p>The correct <u>underlined identity</u>.</p> <p>Correct integration with/without + c</p> <p>Use of ‘integration by parts’ formula in the correct direction. Correct direction means that $u = \ln x$. Correct expression.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>An attempt to multiply through $\frac{k}{x^n}, n \in \mathbb{Z}, n \neq 2$ by $\frac{1}{x}$ and an attempt to ...</p> <p>... “integrate”(process the result);</p> </div> <p><u>correct solution</u> with/without + c</p>	<p>M1 oe</p> <p>A1</p> <p>(2)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 oe</p> <p>(4)</p>

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(c)	$\int \frac{e^{3x}}{1+e^x} dx$ $\left\{ u = 1 + e^x \Rightarrow \frac{du}{dx} = e^x, \frac{dx}{du} = \frac{1}{e^x}, \frac{dx}{du} = \frac{1}{u-1} \right\}$ $= \int \frac{e^{2x} \cdot e^x}{1+e^x} dx = \int \frac{(u-1)^2 \cdot e^x}{u} \cdot \frac{1}{e^x} du$ <p>or $= \int \frac{(u-1)^3}{u} \cdot \frac{1}{(u-1)} du$</p> $= \int \frac{(u-1)^2}{u} du$ $= \int \frac{u^2 - 2u + 1}{u} du$ $= \int u - 2 + \frac{1}{u} du$ $= \frac{u^2}{2} - 2u + \ln u (+c)$ $= \frac{(1+e^x)^2}{2} - 2(1+e^x) + \ln(1+e^x) + c$ $= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$ $= \frac{1}{2} + e^x + \frac{1}{2}e^{2x} - 2 - 2e^x + \ln(1+e^x) + c$ $= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) - \frac{3}{2} + c$ $= \frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k \quad \mathbf{AG}$	<p>Differentiating to find any one of the <u>three underlined</u></p> <p>Attempt to substitute for $e^{2x} = f(u)$, their $\frac{dx}{du} = \frac{1}{e^x}$ and $u = 1 + e^x$</p> <p>or $e^{3x} = f(u)$, their $\frac{dx}{du} = \frac{1}{u-1}$ and $u = 1 + e^x$.</p> $\int \frac{(u-1)^2}{u} du$ <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>An attempt to multiply out their numerator to give at least three terms and divide through each term by u</p> </div> <p>Correct integration with/without $+c$</p> <p>Substitutes $u = 1 + e^x$ back into their integrated expression with at least two terms.</p> $\frac{1}{2}e^{2x} - e^x + \ln(1+e^x) + k$ <p>must use a $+c$ and "$-\frac{3}{2}$" combined.</p> <p>B1</p> <p>M1*</p> <p>A1</p> <p>dM1*</p> <p>A1</p> <p>dM1*</p> <p>A1 cso</p> <p>(7)</p> <p>[13]</p>

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<p>7</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>At A, $x = -1 + 8 = 7$ & $y = (-1)^2 = 1 \Rightarrow A(7,1)$</p> <p>$x = t^3 - 8t$, $y = t^2$,</p> <p>$\frac{dx}{dt} = 3t^2 - 8$, $\frac{dy}{dt} = 2t$</p> <p>$\therefore \frac{dy}{dx} = \frac{2t}{3t^2 - 8}$</p> <p>At A, $m(\mathbf{T}) = \frac{2(-1)}{3(-1)^2 - 8} = \frac{-2}{3-8} = \frac{-2}{-5} = \frac{2}{5}$</p> <p>$\mathbf{T}: y - (\text{their } 1) = m_T(x - (\text{their } 7))$</p> <p>or $1 = \frac{2}{5}(7) + c \Rightarrow c = 1 - \frac{14}{5} = -\frac{9}{5}$</p> <p>Hence $\mathbf{T}: y = \frac{2}{5}x - \frac{9}{5}$</p> <p>gives $\mathbf{T}: \underline{2x - 5y - 9 = 0}$ AG</p> <p>$2(t^3 - 8t) - 5t^2 - 9 = 0$</p> <p>$2t^3 - 5t^2 - 16t - 9 = 0$</p> <p>$(t+1)\{(2t^2 - 7t - 9) = 0\}$</p> <p>$(t+1)\{(t+1)(2t-9) = 0\}$</p> <p>$\{t = -1 \text{ (at A)}\} t = \frac{9}{2} \text{ at B}$</p> <p>$x = (\frac{9}{2})^2 - 8(\frac{9}{2}) = \frac{729}{8} - 36 = \frac{441}{8} = 55.125$ or awrt 55.1</p> <p>$y = (\frac{9}{2})^2 = \frac{81}{4} = 20.25$ or awrt 20.3</p> <p>Hence $B(\frac{441}{8}, \frac{81}{4})$</p>	<p>B1</p> <p>(1)</p> <p>M1</p> <p>A1</p> <p>Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$</p> <p>Correct $\frac{dy}{dx}$</p> <p>Substitutes for t to give any of the four underlined oe:</p> <p>Finding an equation of a tangent with their point and their tangent gradient or finds c and uses $y = (\text{their gradient})x + "c"$.</p> <p>dM1</p> <p>A1 cso</p> <p>(5)</p> <p>M1</p> <p>Substitution of both $x = t^3 - 8t$ and $y = t^2$ into \mathbf{T}</p> <p>A realisation that $(t+1)$ is a factor.</p> <p>dM1</p> <p>A1</p> <p>$t = \frac{9}{2}$</p> <p>Candidate uses their value of t to find either the x or y coordinate</p> <p>ddM1</p> <p>One of either x or y correct.</p> <p>Both x and y correct.</p> <p>A1</p> <p>awrt</p> <p>(6)</p>
	<p>[12]</p>	