AS MATHEMATICS 7356/2

Paper 2

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

206A7356/2/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:
- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	1.1b	B1	$\frac{1}{\sqrt[5]{e^2}}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Ticks correct box	1.1b	B1	$-1 < y < 0$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3	Differentiates once, at least one term correct	1.1a	M1	$\frac{dy}{dx} = 12x^3 - \frac{2}{x^2} - \frac{1}{4}$
	Differentiates twice, both powers of x correct at least one coefficient correct (ACF) Condone one extra term from error differentiating $-\frac{x}{4}$ twice	1.1a	M1	$\frac{d^2y}{dx^2} = 36x^2 + \frac{4}{x^3}$
	Obtains completely correct expression with no errors	1.1b	A1	
	Total		**3**	

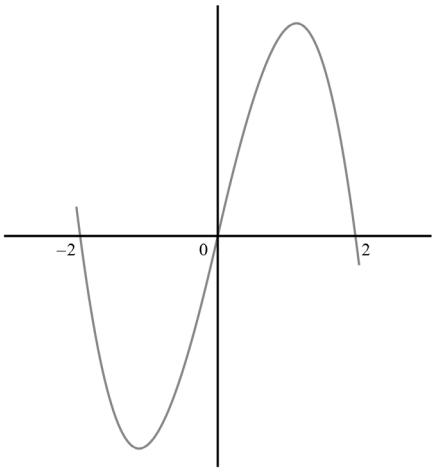
Q	Marking Instructions	AO	Marks	Typical Solution
4	Uses substitution $\cos^2 x = 1 - \sin^2 x$ in any form	1.2	B1	$9\sin^2 x - 6\sin x + (1 - \sin^2 x) = 0$ $8\sin^2 x - 6\sin x + 1 = 0$
	Solves 'their' quadratic to obtain two values for $\sin x$	1.1a	M1	$(4\sin x - 1)(2\sin x - 1)$ $\sin x = ¼$
	Finds two correct solutions for x	1.1b	A1	$\sin x = ½$ 14°, 30°, 150°, 166°
	Finds all four solutions for x and no extras (condone 14.5, 165.5 AWRT)	1.1b	A1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Chooses x^3 term (PI)	1.1b	B1	$(35)(2^4)(-3)^3 x^3$ $-15120x^3$ Joseph has the right number but the wrong sign
	Uses correct coefficient formula, allow use of either ${}^7C_3$ or ${}^7C_4$ (OE)	1.1a	M1	
	Obtains -15120 or 22680 as the value of the coefficient (ignore any power of x if included as part of the coefficient)	1.1b	A1F	
	Obtains -15120 and explains how the error could have occurred or what the error is	2.4	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Completes a square correctly once OE (PI by one correct coordinate of the centre)	1.1a	M1	$x^2 + 10x + 25 + y^2 - 4y + 4 - 100 = 0$ $(x + 5)^2 + (y - 2)^2 = 100$ Centre = (–5, 2)
	Obtains correct centre	1.1b	A1	
	Subtotal		**2**	
6(b)	Finds correct gradient from 'their' centre from (a) to the point (1, 10)	1.1b	B1F	Gradient from (–5, 2) to (1, 10) is $\frac{8}{6}$ So perpendicular gradient is $-\frac{3}{4}$ $y - 10 = -\frac{3}{4}(x - 1)$ $4y - 40 = -3x + 3$ $3x + 4y - 43 = 0$
	Uses perpendicular gradient property for 'their' gradient	1.1a	M1	
	Finds 'their' equation of the line based on 'their' perpendicular gradient. Finding c = 10.75 (OE) is sufficient for M1	1.1a	M1	
	Rearranges "their" equation of the line into form $ax + by + c = 0$ (FT 'their' centre of the circle only)	1.1b	A1F	
	Subtotal		**4**	
	Question Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Multiplies by ratio of populations in 1989 and 2019	3.1b	M1	6 million × $\frac{6 \text{ million}}{3.6 \text{ million}}$ = 10 million
	Obtains correct estimated population	1.1b	A1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical solution
8(a)	Uses rules of indices to express $2^{(2x+3)}$ in terms of y or 16^x in terms of y, must see an intermediary step, either $(2^4)^x$ or $(4^2)^x$, not just $16^x = (2^{2x})^2$	1.1a	M1	$2^{(2x+3)} = 2^{2x} \times 2^3 = 8y$ $16^x = (2^4)^x = (2^{2x})^2 = y^2$ $y^2 - 8y - 9 = 0$
	Completes rigorous solution expressing both terms in terms of y Condone working backwards from quadratic in y to both terms in x. Be convinced	2.1	R1	
	Subtotal		**2**	
8(b)	Solves the equation in terms of 2^{2x} (PI by seeing either $2^{2x} = 9$ or $2^{2x} = -1$)	1.1a	M1	$y = 9$ or $y = -1$ $2^{2x} = 9$ or $2^{2x} = -1$
	States that $2^{2x} = -1$ has no (real) solutions (PI by negative numbers do not have a square root)	2.4	E1	$2^{2x} = -1$ has no solutions
	Uses logs to solve $2^{2x} = 9$ OE Or Square roots first then correctly takes logs Accept $\log_2 9$ or $\log_4 9$ seen as evidence for M1	1.1a	M1	$2x = \log_2 9$ $x = \frac{1}{2}\log_2 9$ $= \log_2 9^{\frac{1}{2}}$ $= \log_2 3$
	Completes rigorous solution. Must include justification for 9 becoming 3	2.1	R1	
	Subtotal		**4**	
	Question Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)(i)	Integrates to obtain terms in x^2 and x^4	1.1a	M1	$2x^2 - \frac{x^4}{4} + c$
	Obtains fully correct integral (ISW) Condone omission of +c	1.1b	A1	
	Subtotal		**2**	
9(a)(ii)	Obtains answer of 0 *nb* correct answer can be obtained directly from calculator, if working shown then CSO	1.1b	B1	0
	Subtotal		**1**	
9(b)	Shows curve with three zeros with correct orientation	1.1b	B1	 The area between -2 and 0 lies below the axis so its integral has a negative value
	Explains that the integral for the area below the axis has a negative value (PI) (Allow even if the graph is incorrectly drawn or omitted)	2.4	E1	
	Subtotal		**2**	
9(c)	Uses $2 \times \int_0^2 4x - x^3\, dx$ (OE) **Or** Obtains values of 4, and -4 or 4 from two separate integrals	1.1a	M1	$2 \times \int_0^2 4x - x^3\, dx$ $2\left[2x^2 - \frac{x^4}{4}\right]_0^2$ $= 8$
	Obtains correct area	1.1b	A1	
	Subtotal		**2**	
	Question Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	States $\frac{dy}{dx}$ = 0 at a turning point OE	2.4	E1	At a turning point $\frac{dy}{dx}$ = 0 $3x^2 - 12x + c = 0$ c = –15 Integrate to find y $y = x^3 - 6x^2 - 15x + k$ 1 = –1 – 6 + 15 + k k = –7 $\frac{dy}{dx}$ = 0 gives x = –1 and x = 5 $y = 5^3 - 6(5^2) - 15(5) - 7$ y = -107 (5, –107)
	Substitutes x = –1 into $\frac{dy}{dx}$ = 0	3.1a	M1	
	Obtains correct value for c	1.1b	A1	
	Obtains x = 5 at other turning point	1.1b	A1	
	Integrates to find y, at least one term correct and substitutes point (–1 , 1) into their integrated expression to find 'their' k	3.1a	M1	
	Obtains correct y coordinate	1.1b	A1	
	Subtotal		**6**	
10(b)	Obtains lower inequality condone inclusion of equality	1.1b	B1	x < –1 and x > 5
	Obtains upper inequality condone inclusion of equality	1.1b	B1	
	Subtotal		**2**	
	Question Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)(i)	Substitutes $y = 0$ to give a quadratic equation (PI)	3.1b	M1	$0 = -0.0125x^2 + 0.5x - 2.55$
	Solves quadratic equation	1.1b	M1	$x^2 - 40x + 204 = 0$ $(x - 6)(x - 34)$
	Selects lower root to obtain the correct value of a. Condone x=6 if clearly chosen.	3.2a	A1	$x = 6$ or 34 $a = 6$
	Subtotal		**3**	
11(a)(ii)	Subtracts their two values for a to find the correct distance travelled ie 28m FT provided both values for their a are positive and the smallest was chosen for a	3.2a	B1F	$34 - 6 = 28$ metres
	Subtotal		**1**	
11(b)	Differentiates, at least one term correct Or Uses the symmetry of the curve (PI)	3.1b	M1	$\frac{dy}{dx} = -0.025x + 0.5$ $-0.025x + 0.5 = 0$
	Sets $\frac{dy}{dx} = 0$ to find maximum Or Identifies that the maximum value will be halfway between 'their' solutions to (a)(i) (PI)	1.1a	M1	$x = 20$ Max height = 2.45 m
	Obtains correct value of $x = 20$ PI by correct value of y	1.1b	A1	
	Substitutes back into y to find the max height = 2.45m CAO must include units	3.2a	A1	
	Subtotal		**4**	

11(c)	Substitutes 11 + a into equation to find y.	3.1b	M1	Using x = 17, y = 2.3375 I have assumed the jet of water has no size 2.3375 > 2.3 so passes over the wall
	Explains a limitation of the model eg that the model assumes that jet has no size or a size less than 3.75cm **or** Wall has no width or has some width **or** No air resistance	3.5b	E1	
	Compares correct value with 2.3 to infer that jet passes over wall / fails to pass over the wall. Inference must be consistent with stated assumption. Condone a valid comparison if no assumption given	2.2b	R1	
	Subtotal		**3**	
	Question Total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Circles correct answer	2.2a	B1	A
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
13	Circles correct answer	1.1b	B1	72
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Explains clearly that the sample is likely to be biased (OE) Do not accept "the sample is not random" alone. Accept "this method does not guarantee to get an employee or opinions from each or other store(s)" (OE) Accept "people from same large store may have similar opinions" (OE)	2.4	E1	Likely to be a biased sample
	Subtotal		**1**	
14(b)	Expresses the idea that method B ensures that there is representation from every store (OE) Method B gives every employee the opportunity of being chosen (OE) Do not accept references to time	2.4	E1	It guarantees to get responses from each store
	Subtotal		**1**	
14(c)(i)	States Systematic (sampling) (CAO)	1.2	B1	Systematic sampling
	Subtotal		**1**	
14(c)(ii)	Explains clearly, in context, the idea that all samples of size 100 are not possible to be chosen using this method so the method is not random. Do not accept each person is not equally likely to be selected.	2.4	E1	Not all subsets of employees of size 100 are possible, so each sample of size 100 is not equally likely to be selected, so not random.
	Subtotal		**1**	
	Question Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	States (AWRT) 22.9 Condone (AWRT) 24.1	1.1b	B1	22.9
	Subtotal		**1**	
15(b)	Explains or shows how 145.8 is generated from the two separate means or Explains that we do not know that the sample sizes are the same for the two years.	2.4	E1	(171.2 + 120.4) ÷ 2 = 145.8 However, the sample size for 2016 is larger so the claim is incorrect
	Explains that the sample size for 2016 is larger than 2002	2.4	E1	
	Subtotal		**2**	
	Question Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)(i)	Finds P(X = 8) correctly AWRT 0.207	3.4	B1	0.207
	Subtotal		**1**	
16(a)(ii)	Finds P($X \leq 11$) – P($X \leq 6$). Must be two terms subtracted with at least one term correct Or Terms for P(X = 7,8,9,10,11) added, at most one error	1.1a	M1	P($X \leq 11$) – P($X \leq 6$) = 0.96020841 – 0.1501401 = 0.810
	Obtains correct probability (AWRT 0.81)	1.1b	A1	
	Subtotal		**2**	
16(b)	States one clear assumption in context linked to independence or the constant probability of success. Do not accept a fixed number of trials or she either fails or does not fail or the probability is 0.6.	3.5b	E1	Being able to solve one puzzle is independent of being able to solve any other puzzle
	Subtotal		**1**	
	Question Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
17(a)	Shows clearly where $\frac{5}{18}$ comes from. Need to see $\frac{100}{360}$ and simplification.	3.1b	B1	P(X = 1) = $\frac{100}{360}$ = $\frac{5}{18}$
	Subtotal		**1**	
17(b)	States at least one pair of (X , P(X = x)correctly	1.1a	M1	X | 2 | 3 | 5 P(X = x) | $\frac{5}{12}$ | $\frac{2}{9}$ | $\frac{1}{12}$
	Completes table correctly ACF	1.1b	A1	
	Subtotal		**2**	
	Question Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
18(a)	Finds the sum of two or more products each term in the products having denominators of 12 and 11	1.1a	M1	P(one blue) = $\frac{7}{12} \times \frac{5}{11} + \frac{5}{12} \times \frac{7}{11}$ = $\frac{35}{66}$
	Obtains $\frac{35}{66}$ OE (accept 0.53 or better)	1.1b	A1	
	Subtotal		**2**	
18(b)	Uses correct denominator (12 × 10) at least once	1.1b	B1	P(Red from Bag B) = $\frac{8}{12} \times \frac{6}{10} + \frac{4}{12} \times \frac{7}{10}$ = $\frac{19}{30}$
	Uses correct numerator in two or three termed expression (8×6)+(4×7) or (7×6)+(4×7)+(1×6) (OE)	1.1a	M1	
	Obtains $\frac{19}{30}$ OE (accept 0.63 or better)	1.1b	A1	
	Subtotal		**3**	
	Question Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
19	States both hypotheses correctly for a two-tailed test. Accept population proportion for p. Accept 15%, but not 'x =' or '$\overline{x}$ =' or 'μ ='	2.5	B1	X is 'Number of residents who buy Local News' H_0: $p = 0.15$ H_1: $p \neq 0.15$ Under H_0: $X \sim B(50, 0.15)$ $P(X \leq 3) = 0.04604657$ As $0.046 > 0.025$ Accept H_0 There is insufficient evidence to suggest that the proportion of residents buying 'Local News' has changed.
	States model used (PI by 0.046, 0.032, 0.014, 0.112) (AWRT)	1.1a	M1	
	Evaluates $P(X \leq 3) = 0.046$ (PI) (condone 0.014 for A1)	1.1b	A1	
	Compares 0.046 or 0.014 with 0.025. Must see clear comparison (inequality or diagram)	3.5a	M1	
	Infers H_0 is not rejected (PI)(CSO)(OE)	2.2b	A1	
	Concludes correctly in context CSO 'insufficient evidence' OE required. Only award for full complete correct solution.	3.2a	R1	
	Total		**6**	