Question	Scheme	Marks	AOs
11 (a)	$\sqrt{\dfrac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$	B1	3.1a
	$(1+4x)^{0.5}=1+0.5\times(4x)+\dfrac{0.5\times-0.5}{2}\times(4x)^2$ $(1-x)^{-0.5}=1+(-0.5)(-x)+\dfrac{(-0.5)\times(-1.5)}{2}(-x)^2$ $(1+4x)^{0.5}=1+2x-2x^2$ and $(1-x)^{-0.5}=1+0.5x+0.375x^2$ oe	M1 M1 A1	1.1b 1.1b 1.1b
	$(1+4x)^{0.5}\times(1-x)^{-0.5}=\left(1+2x-2x^2......\right)\times\left(1+\dfrac{1}{2}x+\dfrac{3}{8}x^2...\right)$ $=1+\dfrac{1}{2}x+\dfrac{3}{8}x^2+2x+x^2-2x^2+...$ $=A+Bx+Cx^2$	dM1	2.1
	$=1+\dfrac{5}{2}x-\dfrac{5}{8}x^2......$ *	A1*	1.1b
		(6)	
(b)	Expression is valid $\|x\|<\dfrac{1}{4}$ Should not use $x=\dfrac{1}{2}$ as $\dfrac{1}{2}>\dfrac{1}{4}$	B1	2.3
		(1)	
(c)	Substitutes $x=\dfrac{1}{11}$ into $\sqrt{\dfrac{1+4x}{1-x}}\approx1+\dfrac{5}{2}x-\dfrac{5}{8}x^2$	M1	1.1b
	$\sqrt{\dfrac{3}{2}}=\dfrac{1183}{968}$	A1	1.1b
	(so $\sqrt{6}$ is) $\dfrac{1183}{484}$ or $\dfrac{2904}{1183}$	A1	2.1
		(3)	
		(10 marks)	

(a)

B1: Scored for key step in setting up the process so that it can be attempted using binomial expansions

This could be achieved by $\sqrt{\dfrac{1+4x}{1-x}}=(1+4x)^{0.5}\times(1-x)^{-0.5}$ See end for other alternatives

It may be implied by later work.

M1: Award for an attempt at the binomial expansion $(1+4x)^{0.5}=1+0.5\times(4x)+\dfrac{(0.5)\times(-0.5)}{2}\times(4x)^2$

There must be three (or more terms). Allow a missing bracket on the $(4x)^2$ and a sign slip so the correct application may be implied by $1+2x\pm0.5x^2$

M1: Award for an attempt at the binomial expansion $(1-x)^{-0.5}=1+(-0.5)(-x)+\dfrac{(-0.5)\times(-1.5)}{2}(-x)^2$

There must be three (or more terms). Allow a missing bracket on the $(-x)^2$ and a sign slips so the method may be awarded on $1\pm0.5x\pm0.375x^2$

A1: Both correct and simplified. This may be awarded for a correct final answer if a candidate does all their simplification at the end

dM1: In the main scheme it is for multiplying their two expansions to reach a quadratic. It is for the key step in adding 'six' terms to produce the quadratic expression. Higher power terms may be seen. Condone slips on

4 (a)	$\frac{1}{\sqrt{4-x}}=(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}\times(1\pm......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n=1+nax+\frac{n(n-1)}{2}a^2x^2+$	M1	1.1b
	$\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$	A1	1.1b
	$\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x=-14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert>4$	B1	2.4
		(1)	
(b)(ii)	States $x=-\frac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
			(6 marks)

(a)

M1: For the strategy of expanding $\frac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1\pm ax)^n=1\pm nax\pm\frac{n(n-1)}{2}a^2x^2+$

Condone sign slips and the "*a*" not being squared in term 3. Condone $a=\pm1$
Look for an attempt at the correct binomial coefficient for their *n*, being combined with the correct power of *ax*

A1: $\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-\frac{x}{4}\right)+\frac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\frac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1+\frac{x}{8}+\frac{3x^2}{128}$ Accept the terms with commas between.

A1: $\frac{1}{\sqrt{4-x}}=\frac{1}{2}+\frac{1}{16}x+\frac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}}=4^{-\frac{1}{2}}+\left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2+..$

M1: For $4^{-\frac{1}{2}}+.....$ **M1:** As above but allow slips on the sign of *x* and the value of *n* **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.
Look for a suitable value and a suitable reason for both parts.

(b)(i)

Question	Scheme	Marks	AOs
1 (a)	$(1+8x)^{\frac{1}{2}} = 1+\frac{1}{2}\times 8x+\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(8x)^2+\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(8x)^3$	M1 A1	1.1b 1.1b
	$=1+4x-8x^2+32x^3+....$	A1	1.1b
		(3)	
(b)	Substitutes $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to give $\frac{\sqrt{5}}{2}$	M1	1.1b
	Explains that $x=\frac{1}{32}$ is substituted into $1+4x-8x^2+32x^3$ and you multiply the result by 2	A1ft	2.4
		(2)	
			(5 marks)
Notes:			

(a)

M1: Attempts the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4.

Award for the correct coefficient with the correct power of x. Do not accept ${}^{n}\mathrm{C}_{r}$ notation for coefficients.

For example look for term 3 in the form $\frac{\frac{1}{2}\times-\frac{1}{2}}{2!}\times(*x)^2$ or $\frac{\frac{1}{2}\times-\frac{1}{2}\times-\frac{3}{2}}{3!}\times(*x)^3$

A1: Correct (unsimplified) expression. May be implied by correct simplified expression

A1: $1+4x-8x^2+32x^3$

Award if there are extra terms (even if incorrect).

Award if the terms are listed $1,\ 4x, -8x^2, 32x^3$

(b)

M1: Score for substituting $x=\frac{1}{32}$ into $(1+8x)^{\frac{1}{2}}$ to obtain $\frac{\sqrt{5}}{2}$ or equivalent such as $\sqrt{\frac{5}{4}}$

Alternatively award for substituting $x=\frac{1}{32}$ into **both sides** and making a connection between the two sides by use of an = or $\approx$.

E.g. $\left(1+\frac{8}{32}\right)^{\frac{1}{2}}=1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ following through on their expansion

Also implied by $\frac{\sqrt{5}}{2}=\frac{1145}{1024}$ for a correct expansion

It is not enough to state substitute $x=\frac{1}{32}$ into " the expansion" or just the rhs "$1+4x-8x^2+32x^3$"

A1ft: Requires a full (and correct) **explanation** as to how the expansion can be used to estimate $\sqrt{5}$

E.g. Calculates $1+4\times\frac{1}{32}-8\times\left(\frac{1}{32}\right)^2+32\times\left(\frac{1}{32}\right)^3$ and multiplies by 2.

This can be scored from an incorrect binomial expansion or a binomial expansion with more terms.

The explanation could be mathematical. So $\frac{\sqrt{5}}{2}=\frac{1145}{1024}\rightarrow\sqrt{5}=\frac{1145}{512}$ is acceptable.

SC: For 1 mark, M1,A0 score for a statement such as "substitute $x=\frac{1}{32}$ into both sides of part (a) and make $\sqrt{5}$ the subject"

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow(4-x)^2-x-8=0$$

$$\Rightarrow16-8x+x^2-x-8\Rightarrow x^2-9x+8=0$$

Scores B1M1A1

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=0$$

$$\Rightarrow\frac{(4-x)^2}{(x+8)}=1\Rightarrow16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$$

Scores B1M0A0

(a) Alternative:
B1: Writes $x^2 - 9x + 8 = 0$ as $(4 - x)^2 - x - 8 = 0$ or equivalent
M1: Proceeds correctly to reach $\log(4-x)^2=\log(x+8)$
A1: Obtains $2\log(4-x)=\log(x+8)$ and makes a (minimal) conclusion e.g. hence proved, QED, #, square etc.

(b)
B1: Writes down (x =) 1, 8
B1: Chooses 8 (no follow through here) and gives a reason why it should be rejected by referring to logs and which log it is.
They must refer to the 8 as the required value but allow e.g. $x \neq 8$ and there must be a reference to $\log(4 - x)$ or log of lhs or log(–4) or the 4 – 8. Some acceptable reasons are: log(–4) can't be found/worked out/is undefined, log(–4) gives math error, log(–4) = n/a, lhs is log(negative) so reject, you can't do the log of a negative number which would happen with 4 – 8
Do **not** allow "you can't have a negative log" unless this is clarified further and do **not** allow "you get a math error" in isolation
There must be no contradictory statements.

Note that this is an independent mark but must have $x = 8$ (i.e. may have solved to get $x = -1, 8$ for first B mark)

Question	Scheme	Marks	AOs
4	${}^7C_4a^3(2x)^4$	M1	1.1b

	$\frac{7!}{4!3!}a^3 \times 2^4 = 15120 \Rightarrow a = ...$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes:

M1: For an attempt at the correct coefficient of x^4.
The coefficient must have

- the correct binomial coefficient
- the correct power of a
- 2 or 2^4 (may be implied)

May be seen within a full or partial expansion.

Accept ${}^7C_4 a^3(2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{4}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{4}a^3 16x^4$ etc.

or ${}^7C_4 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{4}a^3 2^4$, $35a^3 2^4$, $560a^3$ etc.

or ${}^7C_3 a^3(2x)^4$, $\frac{7!}{4!3!}a^3(2x)^4$, $\binom{7}{3}a^3(2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{3}a^3 16x^4$ etc.

or ${}^7C_3 a^3 2^4$, $\frac{7!}{4!3!}a^3 2^4$, $\binom{7}{3}a^3 2^4$, $35a^3 2^4$, $560a^3$

You can condone missing brackets around the "2x" so allow e.g. $\frac{7!}{4!3!}a^3 2x^4$

An alternative is to attempt to expand $a^7\left(1+\frac{2x}{a}\right)^7$ to give $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$

Allow M1 for e.g. $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...\binom{7}{4}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...35\left(\frac{2x}{a}\right)^4...\right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7C_3$, $\binom{7}{3}$ etc. are equivalent to ${}^7C_4$, $\binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x)$… etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For $"560"a^3 = 15120 \Rightarrow a = ...$ Condone slips on copying the 15120 but their "560" must be an attempt at ${}^7C_4{\times}2$ or ${}^7C_4{\times}2^4$ and must be attempting the <u>cube root</u> of $\frac{15120}{"560"}$. **Depends on the first mark**.

A1: $a = 3$ and no other values i.e. ±3 scores A0

Note that this is fairly common:

$${}^7C_4 a^3 2x^4 = 70a^3x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0

Question	Scheme	Marks	AOs
5	$15 - 2^{x+1} = 3 \times 2^x$	B1	1.1b

Question Number	Scheme	Marks
6 (a)	$\dfrac{6}{\sqrt{(9+Ax^2)}} = 6(9+Ax^2)^{-\frac{1}{2}} = 6\times 9^{-\frac{1}{2}}\left(1+\dfrac{A}{9}x^2\right)^{-\frac{1}{2}}$ $9^{-\frac{1}{2}}$ or $\dfrac{1}{3}$	B1
	$= 2\times\underline{\left(1+\left(-\dfrac{1}{2}\right)\left(\dfrac{A}{9}x^2\right)+\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\dfrac{A}{9}x^2\right)^2+\ldots\right)}$	M1A1
	$= 2\times\left(1-\dfrac{A}{18}x^2+\dfrac{A^2}{216}x^4+\ldots\right)$	
	$= 2-\dfrac{A}{9}x^2+\dfrac{A^2}{108}x^4+\ldots$	
	Compare to $B-\dfrac{2}{3}x^2+Cx^4 \Rightarrow$ $B=2$	B1
	$A=6$	B1
	$C = '\dfrac{1}{108}'\times'A^2' = \dfrac{1}{3}$	dM1A1
		(7)
(b)	Coefficient of $x^6 = 2\times\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\times\left(\dfrac{A}{9}\right)^3 = -\dfrac{5}{27}$	M1A1
		(2)
		(9 marks)

Question Number	Scheme	Marks
5.	$\left(8+27x^3\right)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1+\dfrac{27x^3}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1+\dfrac{27x^3}{8}\right)^{\frac{1}{3}}$ $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$	<u>B1</u>
	$=\{2\}\left[1+\left(\tfrac{1}{3}\right)\left(kx^3\right)+\dfrac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2+\dots\right]$	M1 A1
	$=\{2\}\left[1+\left(\dfrac{1}{3}\right)\left(\dfrac{27x^3}{8}\right)+\dfrac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(\dfrac{27x^3}{8}\right)^2+\dots\right]$	
	$= 2\left[1+\dfrac{9}{8}x^3;-\dfrac{81}{64}x^6+\dots\right]$	
	$= 2+\dfrac{9}{4}x^3;-\dfrac{81}{32}x^6+\dots$	A1; A1
		[5]
Method 2	$\left\{\left(8+27x^3\right)^{\frac{1}{3}}\right\} = (8)^{\frac{1}{3}}+\left(\dfrac{1}{3}\right)(8)^{-\frac{2}{3}}(27x^3)+\dfrac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}(8)^{-\frac{5}{3}}(27x^3)^2$	
	$(8)^{\frac{1}{3}}$ or 2	B1
	Any two of three (un-simplified or simplified) terms correct	M1
	All three (un-simplified or simplified) terms correct.	A1
	$= 2+\dfrac{9}{4}x^3;-\dfrac{81}{32}x^6+\dots$	A1; A1
		[5] **5**

Notes

<u>Method 1:</u>

<u>B1</u>: $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ outside brackets then isw or $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$ as candidate's constant term in their binomial expansion.

M1: Expands $\left(\dots+kx^3\right)^{\frac{1}{3}}$ to give any 2 terms out of 3 terms correct for their k simplified or un-simplified

Eg: $1+\left(\tfrac{1}{3}\right)\left(kx^3\right)$ or $\left(\tfrac{1}{3}\right)\left(kx^3\right)+\dfrac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2$ or $1+\dots\dots..+\dfrac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2$ [Allow $\left(\tfrac{1}{3}-1\right)$ for $\left(-\tfrac{2}{3}\right)$]

where $k \neq 1$ are acceptable for M1. Allow omission of brackets. [k will usually be 27, 27/8 or 27/2…]

A1: A correct simplified or un-simplified $1+\left(\tfrac{1}{3}\right)\left(kx^3\right)+\dfrac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(kx^3\right)^2$ expansion with consistent $\left(kx^3\right)$ {or (kx) – for special case only}. **Note** that $k \neq 1$. The bracketing must be correct and now need all three terms correct for their k.

A1: $2+\dfrac{9}{4}x^3$ **- allow** $2+2.25x^3$ **or** $2+2\tfrac{1}{4}x^3$

A1: $-\dfrac{81}{32}x^6$ allow $-2.53125x^6$ or $-2\tfrac{17}{32}x^6$ (Ignore extra terms of higher power)

<u>Method 2:</u>

B1: $(8)^{\frac{1}{3}}$ or 2

M1: Any two of three (un-simplified or simplified) terms correct – condone missing brackets

A1: All three (un-simplified or simplified) terms correct. The bracketing must be correct but it is acceptable for them to recover this mark following "invisible" brackets.

A1A1: as above.

Special case (either method) uses x instead of x^3 throughout to obtain $= 2+\tfrac{9}{4}x;-\tfrac{81}{32}x^2+\dots$ **gets B1M1A1A0A0**

Question Number	Scheme	Marks
5 (a)	$\frac{1}{\sqrt{1-2x}}=(1-2x)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(-2x)^2$	M1A1
	$=1+x+\frac{3}{2}x^2+\dots$	
	$\frac{2+3x}{\sqrt{1-2x}}=(2+3x)(1+x+\frac{3}{2}x^2+\dots$	M1
	$=2+5x+6x^2$	A1*
		(4)
(b)	Sub $x=\frac{1}{20}$ into both sides of $\frac{2+3x}{\sqrt{1-2x}}=2+5x+6x^2$	M1
	$\frac{43\sqrt{10}}{60}=\frac{453}{200}$ oe $\frac{43}{20}=\frac{453}{200}\times\frac{3}{\sqrt{10}}$	dM1
	$\sqrt{10}=\frac{1359}{430}$ Accept $\frac{4300}{1359}$	A1
		(3)
		(7 marks)

Question Number	Scheme	Marks
2(a) Way 1	$4(x^2+6) = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$	M1
	Let $x=-2 \Rightarrow 40 = 5C \Rightarrow C = 8$	dM1
	Let $x=\frac{1}{2} \Rightarrow 25 = 6.25A \Rightarrow A = 4$ $\qquad A=4, C=8$	A1
	Compare constants / terms in *x* or substitute another value of x into identity and conclude that $B=0$ e.g. $24 = 4A+2B+C \Rightarrow B=0$ *	A1* **(4)**
Way 2 (a)	$4(x^2+6) = A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$	M1
	Compare x^2 : so $4 = A-2B$, x: so $0 = 4A-3B-2C$, constants: so $24 = 4A+2B+C$	dM1
	So $A=4, C=8$, and $B=0$*	A1, A1
Way 1 (b)	$\frac{4(x^2+6)}{(1-2x)(2+x)^2} = 4(1-2x)^{-1} + 8(2+x)^{-2} \quad = 4(1-2x)^{-1} + 8\times\frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$	B1 ft
	See...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ or ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	M1
	...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ and ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	A1
	$= 4(1+2x+4x^2+..)+2\left(1-x+\frac{3}{4}x^2\right) \qquad = 6+6x+\frac{35}{2}x^2$	dM1A1 **(5)**
Way 2 (b)	Or $\frac{4(x^2+6)}{(1-2x)(2+x)^2} = 4(x^2+6)\times(1-2x)^{-1}\times\frac{1}{2^2}\left(1+\frac{x}{2}\right)^{-2}$	B1
	See...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ or ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	M1
	...$\left(1+(-1)(-2x)+\frac{(-1)(-2)(-2x)^2}{2!}\right)$ and ...$\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{2}\right)^2+\right)$	A1
	$= 4(x^2+6)(1+2x+4x^2+..)\times\frac{1}{4}\left(1-x+\frac{3}{4}x^2\right) = 6+6x+\frac{35}{2}x^2$	dM1A1 **(5)** **(9 marks)**

Question Number	Scheme	Marks
1	$(3-2x)^{-4} = 3^{-4}\left(1-\frac{2}{3}x\right)^{-4}$ $\quad 3^{-4}$ or $\frac{1}{81}$	B1
	$=\frac{1}{81}\times\underline{\left(1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+\ldots\right)}$	$\underline{\text{M1A1}}$
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$	A1
		(4 marks)
	Alternative: $(3-2x)^{-4} = 3^{-4}+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2+\ldots$	B1 M1 A1
	$=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$	A1
		(4 marks)

B1 For taking out a factor of 3^{-4}

Evidence would be seeing either 3^{-4} or $\frac{1}{81}$ before the bracket.

M1 For the form of the binomial expansion with $n=-4$ and a term of (kx)

To score M1 it is sufficient to see just the second and third term with the correct coefficient multiplied by the correct power of x. Condone sign slips. Look for $\ldots+(-4)(kx)+\frac{(-4)(-5)}{2!}(kx)^2\ldots$

A1 Any (unsimplified) form of the binomial expansion. Ignore the factor before the bracket.

The bracketing must be correct but it is acceptable for them to recover from "missing" brackets for full marks.

Look for $1+(-4)\left(-\frac{2}{3}x\right)+\frac{(-4)(-5)}{2}\left(-\frac{2}{3}x\right)^2+$ or $1+\frac{8}{3}x+\frac{40}{9}x^2+$

A1 cao $=\frac{1}{81}+\frac{8}{243}x+\frac{40}{729}x^2+\ldots$. Ignore any further terms.

Alternative

B1 For seeing either 3^{-4} or $\frac{1}{81}$ as the first term

M1 It is sufficient to see the second and third term (unsimplified or simplified) condoning missing brackets.

ie. Look for $\ldots+(-4)(3)^{-5}(kx)+\frac{(-4)(-5)}{2}(3)^{-6}(kx)^2$

A1 Any (un simplified) form of the binomial expansion. $\ldots+(-4)(3)^{-5}(-2x)+\frac{(-4)(-5)}{2}(3)^{-6}(-2x)^2$

A1 Must now be simplified cao

Question Number	Scheme	Notes	Marks
3(a)	$(1+ax)^{-3} = 1+(-3)(ax)+\frac{(-3)(-4)}{2!}(ax)^2+\frac{(-3)(-4)(-5)}{3!}(ax)^3+...$ Uses the binomial expansion with $n=-3$ and $'x'=ax$. Minimum for M1 is $1+(-3)(ax)$ but can be scored for a correct 3rd or 4th term e.g. $\frac{(-3)(-4)}{2!}(ax)^2$ or $\frac{(-3)(-4)(-5)}{3!}(ax)^3$		M1
	$=1-3ax+6a^2x^2-10a^3x^3+...$ or $=1-3ax+6(ax)^2-10(ax)^3+...$	A1: Three of the four terms correct and simplified A1: All four terms correct and simplified and seen in part (a).	A1A1
			(3)
(b)	$\text{f}(x)=\frac{2+3x}{(1+ax)^3}=(2+3x)(1-3ax+6a^2x^2-10a^3x^3)$ Writes $\text{f}(x)$ as $(2+3x)(1-3ax+6a^2x^2-10a^3x^3)$ using their expansion from part (a). This may be implied by their expansion. Do not condone 'invisible' brackets around 2 + 3x or part(a) unless their presence is implied by later work and allow to recover in (b) from missing brackets in (a) e.g. ax^2 now becoming a^2x^2		M1
	NB $\text{f}(x)=2+(3-6a)x+(12a^2-9a)x^2+(18a^2-20a^3)x^3$		
	$12a^2-9a=3$	Multiplies out and sets their coefficient of x^2 (which comes from exactly 2 terms from their expansion – the two terms may have been combined earlier) = 3.	dM1
	$4a^2-3a-1=(4a+1)(a-1)\Rightarrow a=...$ Correct method of solving a 3TQ. If working is shown see general guidance for correct methods. If no working is shown then you may need to check their values if their quadratic is incorrect.		ddM1
	$a=-\frac{1}{4}$	Cao. Accept equivalent answers but must come from the **correct quadratic** and must be clearly identified.	A1
			(4)
(c)	$18\left(-\frac{1}{4}\right)^2-20\left(-\frac{1}{4}\right)^3$	Subs their $a=-\frac{1}{4}$ (positive or negative) into their coefficient of x^3 (which comes from exactly 2 terms from their expansion)	M1
	Coefficient of x^3 is $\frac{23}{16}$	Cao. Allow $\frac{23}{16}x^3$	A1
			(2)
			9 marks

Qu	Scheme	Marks
3(a)	$\dfrac{9+11x}{(1-x)(3+2x)} = \dfrac{A}{1-x} + \dfrac{B}{3+2x}$ and attempt to find A or B	M1
	$A = 4\ ,\ B = -3$	A1, A1 **(3)**
(b)	$(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$	B1
	$(3+2x)^{-1} = \dfrac{1}{3} \times \left(1 + (-1)\left(\dfrac{2}{3}x\right) + \dfrac{(-1)(-2)}{2}\left(\dfrac{2}{3}x\right)^2 + \dfrac{(-1)(-2)(-3)}{6}\left(\dfrac{2}{3}x\right)^3 \ldots\right)$	B1 M1
	Attempts $'4'\times(\ldots..) + '-3'\times(\ldots..)$	M1
	$= 3 + \dfrac{14}{3}x + \dfrac{32}{9}x^2 + \dfrac{116}{27}x^3 \ldots$	A1, A1 **(6)**
		(9 marks)

(a)

M1: For expression in markscheme or $9 + 11x = A(3+2x) + B(1-x)$ and use of substitution or comparison of coefficients in an attempt to find A or B (Condone slips on the terms)

A1: One correct value (this implies the M1)

A1: Both correct values (attached to the correct fraction).
You do not explicitly need to see the expression rewritten in PF form.

(b)

B1: Correct expansion $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots$ with or without working. Must be simplified

B1: For taking out a factor of 3^{-1} Evidence would be seeing either 3^{-1} or $\dfrac{1}{3}$ before the bracket or could be implied by the candidate multiplying their B by $\dfrac{1}{3}$.

M1: For the form of the binomial expansion with $n = -1$ and a term of $\left(\pm\dfrac{2}{3}x\right)$.

To score M1 it is sufficient to see just any two terms of the expansion. eg. $1 + \ldots. + \dfrac{(-1)(-2)}{2}\left(\pm\dfrac{2}{3}x\right)^2$

M1: Attempts to combine the two series expansions. Condone slips on signs but there must have been some attempt to combine terms (at least once) and to use both their coefficients

A1: Two terms correct which may be unsimplified.

A1: All four terms correct. (cao) Could be mixed number fraction form. ISW after a correct answer

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Alternative use of binomial in line 2 of scheme: ie. $3^{-1} + (-1)(3)^{-2}(2x)$

B1: For seeing either 3^{-1} or $\dfrac{1}{3}$ as the first term

M1: It is sufficient to see just the first two terms (unsimplified) then marks as before

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Way 2: Otherwise method: Use of $(9+11x)(1-x)^{-1}(3+2x)^{-1}$: B1 B1 M1 : as before
Then M1: Attempt to multiply three brackets and obtain 3 + .. A1: two terms correct A1: All four correct

Way 3: Use of $(9+11x)\left(3-\left(x+2x^2\right)\right)^{-1}$ or alternatives is less likely – send to review

Question Number	Scheme	Marks
4(a)	$27(3-5x)^{-2}=27\times\frac{1}{9}\left(1-\frac{5}{3}x\right)^{-2}$	B1, B1
	$=3\left(1+(-2)\left(-\frac{5}{3}x\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5}{3}x\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(-\frac{5}{3}x\right)^3+...\right)$	M1
	$=3+10x+25x^2+\frac{500}{9}x^3+...$	A1, A1
		(5)
(b)	$27(3+5x)^{-2}=27(3+5x)^{-2}$	
	$=3-10x+25x^2-\frac{500}{9}x^3+...$	B1ft
		(1)
(c)	$27(3-x)^{-2}=3+\frac{10}{5}x+\frac{25}{5^2}x^2+\frac{500}{9\times5^3}x^3$	M1
	$=3+2x+x^2+\frac{4}{9}x^3$	A1
		(2)
		(8 marks)
4(a) alt	$27(3-5x)^{-2}=27\left(\begin{array}{l}3^{-2}+(-2)\times3^{-3}\times(-5x)+\frac{(-2)(-3)}{2}\times3^{-4}\times(-5x)^2\\+\frac{(-2)(-3)(-4)}{3!}(3)^{-5}(-5x)^3\end{array}\right)$	B1 B1 M1
	$=27\left(\frac{1}{9}+\frac{10x}{27}+\frac{25x^2}{27}+\frac{500x^3}{243}+..\right)$ $=3+10x+25x^2+\frac{500}{9}x^3+...$	A1 A1

(a)

B1: Writes down $(3-5x)^{-2}$ or uses a power of -2

B1: Takes out a factor of 3^{-2} which can be implied by $\frac{1}{9}$ or $3\times(....)$ or a first term of 3

M1: Expands $(1+kx)^{-2}$, $k\neq\pm1$ with the structure for at least 2 terms correct (not including the "1"), from $\left(1+(-2)kx+\frac{(-2)(-3)}{2}(kx)^2+\frac{(-2)(-3)(-4)}{3!}(kx)^3+...\right)$ with or without the bracket around the kx

A1: Two of the four terms correct and simplified **but the method mark must have been awarded!**

A1: Fully correct simplified expansion $3+10x+25x^2+\frac{500}{9}x^3+...$ all on one line but isw once a correct expansion is seen.

Alternative for (a):

B1: Writes down $(3-5x)^{-2}$ or uses a power of -2

B1: For a first term of 3^{-2} in the bracket

Question Number	Scheme	Marks
7(a)	2^{-3} or $\frac{1}{2^3}$ or 0.125	B1
	$\frac{1}{(2-3x)^3}=(2-3x)^{-3}=\frac{1}{2^3}\left(1-\frac{3x}{2}\right)^{-3}$	
	$=\frac{1}{8}\left(1+(-3)\times\left(-\frac{3x}{2}\right)+\frac{-3\times-4}{2!}\times\left(-\frac{3x}{2}\right)^2+\ldots\right)$	M1A1
	$=\frac{1}{8}+\frac{9}{16}x\,;+\frac{27}{16}x^2+\ldots$	A1; A1
		(5)
(b)	$\frac{4+kx}{(2-3x)^3}=(4+kx)\left(\frac{1}{8}+\frac{9}{16}x+\frac{27}{16}x^2+\ldots\right)$	
	Compares x^2 terms $\frac{27}{4}+\frac{9k}{16}=\frac{81}{16}\Rightarrow k=\ldots$	M1
	$k=-3$	A1
		(2)
(c)	Compares x terms $\frac{9}{4}+\frac{1}{8}\times{}'-3'=A\Rightarrow A=\ldots$	M1
	$A=\frac{15}{8}$	A1
		(2)
		(9 marks)
7(a) ALT	2^{-3} or $\frac{1}{2^3}$	B1
	$(2-3x)^{-3}=2^{-3}+(-3)2^{-4}(-3x)+\frac{(-3)(-4)}{2}2^{-5}(-3x)^2$ M1: For 2^{-3} and the structure of at least one of the other terms correct A1: Fully correct	M1A1
	$=\frac{1}{8}+\frac{9}{16}x\,;+\frac{27}{16}x^2+\ldots$	A1; A1
		(5)

(a)

B1: For taking out a factor of 2^{-3} or $\frac{1}{2^3}$ or $\frac{1}{8}$ or 0.125

M1: Score for the form of the binomial expansion with index -3

Eg $=\left\{\cancel{\frac{1}{8}}\right\}\left[1+(-3)(**x)+\frac{(-3)(-4)}{2!}(**x)^2+\ldots\right]$ where $**\neq 1$ or -1

Requires 1 + … with the structure of at least one of the other terms correct as shown above.

A1: Correct un-simplified form $=\left\{\cancel{\frac{1}{8}}\right\}\left(1+(-3)\times\left(-\frac{3x}{2}\right)+\frac{-3\times-4}{2!}\times\left(-\frac{3x}{2}\right)^2+\ldots\right)$

Condone missing brackets around the $**x$ provided they are recovered later.