Question	Scheme	Marks	AOs
3	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\dfrac{12y+5}{1-3y}=2$ or e.g. $2=\log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3\dfrac{12y+5}{1-3y}=2 \Rightarrow \dfrac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=...$ or e.g. $\log_3(12y+5)=\log_3\left(3^2(1-3y)\right) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=...$	M1	2.1
	$y=\dfrac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

Notes

B1(M1 on EPEN): Applies at least one addition or subtraction law of logs correctly. Can also be awarded for using $2=\log_3 9$. This may be implied by e.g. $\log_3 ...=2 \Rightarrow ...=9$

M1: A rigorous argument with no incorrect working to remove the log or logs correctly and obtain a <u>correct</u> equation in any form **and** solve for *y*.

A1: Correct **exact** value. Allow equivalent fractions.

<u>Guidance on how to mark particular cases:</u>

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{\log_3(12y+5)}{\log_3(1-3y)}=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$$
$$\Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M0A0

$$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\frac{4}{39}$$

B1M1A1

Question	Scheme	Marks	AOs
9 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \rightarrow ...... \rightarrow ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \rightarrow a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

(a)

B1: States or uses $\log a - \log b = \log \frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make '*a*' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in *a* are on the same side of the equation and the term in *b* is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be correct.

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \rightarrow \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$.
They may state that *b* cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)
As a minimum accept that $b > 1$ as *a* cannot be negative.
Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

....

Alt (a)

Note that it is possible to attempt part (a) by substituting $a = \frac{b^2}{b-1}$ into both sides of the given identity.

Question	Scheme	Marks	AOs
1	$2^x \times 4^y = \dfrac{1}{2\sqrt{2}} \quad \left\{ = \dfrac{\sqrt{2}}{4} \right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \to 2^{x+2y}$ • $2^x \times 4^y \to 4^{\frac{1}{2}x+y}$ • $\dfrac{1}{2^x 2\sqrt{2}} \to 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \to x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \to x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$ • $y = \log\left(\dfrac{1}{2^x 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\dfrac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\dfrac{1}{2}x - \dfrac{3}{4}$ or $y = -\dfrac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\dfrac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = ...$	M1	2.1
	$y = \dfrac{-\log(2\sqrt{2}) - x\log 2}{\log 4} \quad \left\{ \Rightarrow y = -\dfrac{1}{2}x - \dfrac{3}{4} \right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow y = ...$	M1	2.1
	$y = \dfrac{\log\left(\dfrac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4} \quad \left\{ \Rightarrow y = -\dfrac{1}{2}x - \dfrac{3}{4} \right\}$	A1	1.1b
		(3)	
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\dfrac{3}{2} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\dfrac{1}{2}x - \dfrac{3}{4}$ or $y = -\dfrac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Way 5	$4^{\frac{1}{2}x} \times 4^{y} = 4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \frac{1}{2}x + y = -\frac{3}{4} \Rightarrow y = ...$	M1	2.1
	E.g. $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	

Notes for Question 1

	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain y written as a function of x.
A1:	$y = \dfrac{-\log(2\sqrt{2}) - x\log 2}{\log 4}$ or $y = \dfrac{-\ln(2\sqrt{2}) - x\ln 2}{\ln 4}$ or $y = \dfrac{\log\left(\frac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$ or $y = -\frac{1}{2}x - \frac{3}{4}$ or $y = -\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain y written as a function of x.
A1:	$y = -\frac{1}{2}x - \frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where y is written as a function of x
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^x \times 4^y = \dfrac{1}{2\sqrt{2}} \Rightarrow 4^y = \dfrac{1}{2^x 2\sqrt{2}} \Rightarrow \log_4(4^y) = \log_4\left(\dfrac{1}{2^x 2\sqrt{2}}\right)$ followed by M1 A1 for $y = \log_4\left(\dfrac{1}{2^x 2\sqrt{2}}\right)$ or $y = \log_4\left(\dfrac{2^{-x}}{2\sqrt{2}}\right)$ or $y = \log_4\left(\dfrac{\sqrt{2}}{4(2^x)}\right)$ or $y = -\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y = -\log_4(\sqrt{2}(2^{x+1}))$

Question	Scheme	Marks	AOs
2	$4^{3p-1}=5^{210}\Rightarrow(3p-1)\log 4=210\log 5$	M1	1.1b
	$\Rightarrow 3p=\dfrac{210\log 5}{\log 4}+1\Rightarrow p=...$	dM1	2.1
	$p=$ awrt 81.6	A1	1.1b
		(3)	
			(3 marks)
Notes:			

M1: Takes logs of both sides and uses the power law on **each** side.
Condone a missing bracket on lhs and slips.
Award for any base including ln but the logs must be the same base.

dM1: A full method leading to a value for *p*.
It is dependent upon the previous M mark and there must be an attempt to change the subject of the equation in the correct order.
Look for $(3p-1)\log 4=210\log 5\Rightarrow 3p=\dfrac{210\log 5}{\log 4}\pm 1\Rightarrow p=...$ condoning slips.
You may see numerical versions E.g. $(3p-1)\times 0.60=210\times 0.7\Rightarrow 1.8p-0.6=147\Rightarrow p=82$
Use of incorrect log laws would be dM0. E.g $(3p-1)\log 4=210\log 5\Rightarrow 3p=210\log\dfrac{5}{4}\pm 1$

A1: awrt 81.6 following a correct method. Bracketing errors can be recovered for full marks
A correct answer with no working scores 0 marks. The demand in the question is clear.

....

There are alternatives:

E.g. A starting point could be $4^{3p-1}=5^{210}\Rightarrow\dfrac{4^{3p}}{4}=5^{210}$
but the first M mark is still for using the power law correctly on each side

In such a method the dM1 mark is for using **all** log rules correctly and proceeding to a value for *p*.

....

Using base 4 or 5

E.g. $4^{3p-1}=5^{210}\Rightarrow(3p-1)=\log_4 5^{210}$
The M mark is not scored until $(3p-1)=210\log_4 5$

....

3(a)	$2\log(4-x)=\log(4-x)^2$	B1	1.2
	$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)$ $(4-x)^2=(x+8)$ **or** $2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2-\log(x+8)=0$ $\dfrac{(4-x)^2}{(x+8)}=1$	M1	1.1b
	$16-8x+x^2=x+8\Rightarrow x^2-9x+8=0$ *	A1*	2.1
		(3)	
	(a) Alternative - working backwards:		
	$x^2-9x+8=0\Rightarrow(4-x)^2-x-8=0$	B1	1.2
	$\Rightarrow(4-x)^2=x+8$ $\Rightarrow\log(4-x)^2=\log(x+8)$	M1	1.1b
	$\Rightarrow 2\log(4-x)=\log(x+8)$ * Hence proved.	A1	2.1
(b)	(i) (x =) 1, 8	B1	1.1b
	(ii) 8 is **not** a solution as log(4 – 8) cannot be found	B1	2.3
		(2)	
			(5 marks)

Notes:

(a)

B1: States or uses $2\log(4-x)=\log(4-x)^2$

M1: Correct attempt at eliminating the logs to form a quadratic equation in x.

Note that this may be implied by e.g. $\log\dfrac{(4-x)^2}{(x+8)}=0\Rightarrow(4-x)^2=x+8$

A1*: Proceeds to the given answer with at least one line where the $(4-x)^2$ has been multiplied out.
There must be no errors or omissions but condone invisible brackets around the arguments of the logs e.g. allow $\log 16-8x+x^2$ for $\log(16-8x+x^2)$ and $\log x+8$ for $\log(x+8)$

Note we will allow a start of $(4-x)^2=x+8$ with no previous work for full marks.

Some examples of how to mark (a) in particular cases:

$$2\log(4-x)=\log(x+8)\Rightarrow\log(4-x)^2=\log(x+8)\Rightarrow\frac{\log(4-x)^2}{\log(x+8)}=1$$

$(4-x)^2$

Question	Scheme	Marks	AOs
5	$15-2^{x+1}=3\times 2^{x}$	B1	1.1b
	$\Rightarrow 15-2\times 2^{x}=3\times 2^{x}\Rightarrow 2^{x}=3$ or e.g. $\Rightarrow \dfrac{15}{2^{x}}-2=3\Rightarrow 2^{x}=3$	M1	1.1b
	$2^{x}=3\Rightarrow x=...$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
		(4)	
	Alternative		
	$y=3\times 2^{x}\Rightarrow 2^{x}=\dfrac{y}{3}\Rightarrow y=15-2\times\dfrac{y}{3}$	B1	1.1b
	$3y+2y=45\Rightarrow y=9\Rightarrow 3\times 2^{x}=9\Rightarrow 2^{x}=3$	M1	1.1b
	$2^{x}=3\Rightarrow x=...$	dM1	1.1b
	$x=\log_2 3$	A1cso	1.1b
			(4 marks)

Question Number	Scheme	Marks
6.	$\log_4 \frac{a}{b} = 3$ or $\log_4 a + \log_4 b = \log_4 25$ or $\log_4 \frac{a}{\frac{25}{a}} = 3$ or $\log_4 \frac{\frac{25}{b}}{b} = 3$ (If this is preceded by wrong algebra (e.g. b = 25 –a) M1 can still be given if their b is used	M1
	$\log_4 64 = 3$ or $4^3 = 64$ (may be implied by the use of 64) or see $\log a = \frac{1}{2}(\log 25 + 3)$ become $a = 4^{\frac{1}{2}(\log 25+3)}$ or see $\log b = \frac{1}{2}(\log 25 - 3)$ become $b = 4^{\frac{1}{2}(\log 25-3)}$ (these latter two statements will be implied by correct answers)	B1
	Correct algebraic elimination of a variable to obtain expression in *a* or *b* without logs	dM1
	$a = 40$ **or** $b = \frac{5}{8}$	A1
	Substitutes to give second variable or solves again from start	dM1
	$a = 40$ **and** $b = \frac{5}{8}$ and no other answers.	A1
		[6]
		6 marks
	Notes	
	M1: Uses addition or subtraction law correctly for logs (N.B. $\log_4 a + \log_4 b = 25$ **is M0)** **B1**: See number 64 used (independent of M mark) or or see $\log a = \frac{1}{2}(\log 25 + 3)$ become $a = 4^{\frac{1}{2}(\log 25+3)}$ or see $\log b = \frac{1}{2}(\log 25 - 3)$ become $b = 4^{\frac{1}{2}(\log 25-3)}$ **dM1**: Dependent on first M mark. Eliminates *a* or *b* (with appropriate algebra) and eliminates logs **A1**: Either *a* or *b* correct **dM1**: Dependent on first M mark . Attempts to find second variable **A1**: Both *a* and *b* correct – allow *b* = 0.625 If *a* = -40 and *b* = -5/8 are **also** given as answers **lose the last A mark**. **NB** Log *a* + log *b* = 2.3219..will not yield exact answers If they round their answers to 40 and 0.625 after decimal work, do not give final A mark. NB: Some will change the base of the log and use loga – log b = 3log4	

Question Number	Scheme	Marks
3 (a)	$4^a = 20 \Rightarrow a\log 4 = \log 20 \Rightarrow a = ...$	M1
	$= \dfrac{\log 20}{\log 4} = \text{awrt } 2.16$	A1
		(2)
(b)	$3 + 2\log_2 b = \log_2 30b \Rightarrow 3 + \log_2 b^2 = \log_2 30b$	M1
	$\Rightarrow 3 = \log_2 30b - \log_2 b^2$	
	$\Rightarrow 3 = \log_2\left(\dfrac{30b}{b^2}\right)$	M1A1
	$\Rightarrow 2^3 = \dfrac{30b}{b^2} \Rightarrow b = ..$	dM1
	$b = 3.75$	A1
		(5) **(7 marks)**
Alt (b)	$3 + 2\log_2 b = \log_2(30b) \Rightarrow 3 + 2\log_2 b = \log_2 30 + \log_2 b$	2nd M1
	$\Rightarrow 3 + \log_2 b = \log_2 30$	
	$\Rightarrow \log_2 8 + \log_2 b = \log_2 30$	1st M1
	$\Rightarrow \log_2 8b = \log_2 30$	A1
	$\Rightarrow 8b = 30 \Rightarrow b = ..$	dM1
	$\Rightarrow b = 3.75$	A1
		(5)

(a)

M1 Takes logs of both sides leading to $a = ...$

Accept for this mark $\log_4 20, \dfrac{\log 20}{\log 4}$ or $a\log 4 = \log 20 \Rightarrow a = ..$

A1 awrt 2.16 . Just the answer with no incorrect working scores both marks.

Do not accept answers from trial and error.

(b) Note that this part is B1M1M1dM1A1 on e pen. We are scoring it M1M1A1dM1A1

M1 Score for a correct use of the power law for logs

Accept either $2\log_2 b = \log_2 b^2$, $2\log b = \log b^2$ or $3 = \log_2 8$

M1 Score for a use of the addition or subtraction law of logs.

Examples of this would be $\log(30b) = \log 30 + \log b$ and $\log(30b) - \log(b^2) = \log\left(\dfrac{30b}{b^2}\right)$

Do not accept attempts such as $\log(30b) - 2\log(b) = \log\left(\dfrac{30b}{2b}\right)$

A1 Achieving a correct intermediate line of the form $\log_2(..) = ..$ or $\log_2 ... = \log_2 ...$

Accept exact equivalents of $3 = \log_2\left(\dfrac{30b}{b^2}\right)$, $\log_2 8b = \log_2 30$ and $\log_2 8b^2 = \log_2 30b$, $b = 2^{\log_2(30) - 3}$

dM1 Dependent upon both previous M's it is for correctly undoing the logs and solving to get a value for b

A1 $b = 3.75$ or exact equivalent. Ignore any reference to $b = 0$

Question Number	Scheme	Marks
6.	(a) Use or state $2\log_4(2x+3)=\log_4(2x+3)^2$	M1
	Use or state $\log_4 4=1$ or $4^1=4$	M1
	Use or state $\log_4 x+\log_4(2x-1)=\log_4 x(2x-1)$ or $\log_4(2x+3)^2-\log_4 x=\log_4\frac{(2x+3)^2}{x}$ etc	M1
	$(2x+3)^2=4x(2x-1)$ or equivalent including correct rational equations	A1
	Then $4x^2+12x+9=8x^2-4x$ and so $4x^2-16x-9=0$ *	A1*
		[5]
	(b) $(2x+1)(2x-9)=0$ so $x=$ (or use other method e.g formula or completion of square)	M1
	$x=(-\frac{1}{2}$ or $)\ \frac{9}{2}$	A1
		[2]
		7 marks
	Notes	

(a) M1**:** Uses power law for logs

M1: Connects 1 with 4 correctly

M1: Uses addition (or subtraction) law correctly

e.g. $\log_4 x+\log_4(2x-1)=\log_4 x(2x-1)$ or $\log_4(2x+3)^2-\log_4 x=\log_4\frac{(2x+3)^2}{x}$ or

$\log_4(2x+3)^2-\log_4 x-\log_4(2x-1)=\log_4\frac{(2x+3)^2}{x(2x-1)}$ or even $\log_4 x+\log_4 4=\log_4 4x$ or

$\log_4(2x-1)+\log_4 4=\log_4 4(2x-1)$ or $\log_4(2x-1)+\log_4 4+\log_4 x=\log_4 4x(2x-1)$ etc…

A1: Correct equation (unsimplified) after correct work. e.g. $\frac{(2x+3)^2}{x(2x-1)}=4$

A1: Obtains printed answer correctly (This is a given answer so needs previous A mark to have been awarded and needs correct expansion)

Special case :

$$\log_4(2x+3)^2=1+\log_4 x(2x-1)\quad so\quad \frac{\log_4(2x+3)^2}{\log_4 x(2x-1)}=1\quad so\quad \frac{4x^2+12x+9}{2x^2-x}=4$$

This can have M1, M1, M1, A0, A0 so 3/5 losing accuracy because of the error in the second step.

(b) Some candidates who did not achieve marks in part (a) begin the log work again and make more progress here. Mark the better work. So credit for (a) may be given in (b). Credit for (b) should not be given in (a)

M1: Uses solution of their quadratic or of printed quadratic(see notes). This must be in part (b)

A1: x = 4.5 and discards x = - 0.5 (any equivalent form) Giving $x=-\frac{1}{2}$, $\frac{9}{2}$ is A0 This must be in part (b)

Question Number	Scheme	Marks
14(i)	$\log_a x + \log_a 3 = \log_a 27 - 1$ so $\log_a \frac{3x}{27} = -1$ Or $\log_a x + \log_a 3 = \log_a 27 - \log_a a$ so $\log_a 3x = \log_a \frac{27}{a}$ Or $\log_a x + 1 = \log_a 27 - \log_a 3 = \log_a 9$ so $\log_a ax = \log_a 9$	M1 A1
	$\frac{3x}{27} = a^{-1}$	M1
	$x = 9a^{-1}$ or $\frac{9}{a}$	A1
		[4]
(ii)	$x^2 - 7x + 12 = 0$ and attempt to solve to give $x = \ldots$ or $\log_5 y = \ldots$ (implied by correct answers)	M1
	x (or $\log_5 y$) = 3 **and** 4	A1
	$y = 5^3$ **or** 5^4	dM1
	y = 125 **and** 625	A1
		[4]
		8 marks
	Notes	
(i)	**M1**: Uses sum or difference of logs correctly e.g. $\log x + \log 3 = \log 3x$ or $\log 27 - \log 3 = \log 9$ or $\log 27 - \log x = \log \frac{27}{x}$ etc. or writes 1 as $\log_a a$ **A1:** Uses **two** rules correctly to obtain correct log equation **M1:** Removes logs correctly to obtain an equation connecting x and a **A1:** Correct simplified answer Note that some candidates interpret $\log_a 27 - 1$ as $\log_a (27-1)$. This can score a maximum of 1 out of 4 if they have $\log x + \log 3 = \log 3x$ Note that $\log_a x + \log_a 3 = \log_a 27 - 1$ so $\frac{\log_a 3x}{\log_a 27} = -1 \Rightarrow \frac{3x}{27} = a^{-1}$ etc. scores **M1A0M0A0** Note that $\log_a x + \log_a 3 = \log_a 27 - 1$ so $\frac{\log_a x \log_a 3}{\log_a 27} = -1 \Rightarrow \frac{3x}{27} = a^{-1}$ etc. scores **no marks**	
(ii)	**M1:** Recognise and attempt to solve quadratic **A1:** Obtain both 3 and 4 (Both correct implies M1A1) **dM1:** Uses powers **correctly** to find a value for y **(Dependent on first method mark)** **A1:** Both values correct	

Question Number	Scheme	Marks
3 (i)	Either $4^{2x+1}=2^{2(2x+1)}$ and $8^{4x}=2^{3\times 4x}$ or $8^{4x}=4^{\frac{3}{2}\times 4x}$	M1
	$2(2x+1)=12x\Rightarrow x=\frac{1}{4}$	dM1A1
		(3)
(ii)(a)	$3\sqrt{18}-\sqrt{32}=9\sqrt{2}-4\sqrt{2}=5\sqrt{2}$	M1A1
		(2)
(b)	$\sqrt{n}=5\sqrt{2}\Rightarrow n=\left(5\sqrt{2}\right)^2=25\times 2=50$	M1A1
		(2)
		(7 marks)
Alt 3 (i)	Taking logs of both sides and proceeding to $(2x+1)\log 4=4x\log 8$	M1
	$\Rightarrow x=\dfrac{\log 4}{4\log 8-2\log 4}$	
	$\Rightarrow x=\dfrac{\log 4}{\log 256}=\dfrac{1}{4}$	dM1A1
		(3)

(i)

M1 Writes both sides as powers of 2 or equivalent Eg $2^{2(2x+1)}=2^{3\times 4x}$

Alternatively writes both sides as powers of 4 or 8 or 64. Eg $8^{4x}=4^{\frac{3}{2}\times 4x}$

Note that expressions such as $2^{2+(2x+1)}=2^{3+4x}$ would be M0

Condone poor (or missing) brackets $2^{2\times 2x+1}=2^3$ but not incorrect index work eg $4^{2x+1}=8^{\frac{1}{2}(2x+1)}$

It is possible to use logs. most commonly with base 2 or 4. Using logs it is for reaching a linear form of the equation, again condoning poor bracketing .

$4^{2x+1}=8^{4x}\Rightarrow \log 4^{2x+1}=\log 8^{4x}\Rightarrow (2x+1)\log 4=4x\log 8$

dM1 Dependent upon the previous M. It is for equating the indices and proceeding to $x=..$

Condone sign/bracketing errors when manipulating the equation but not processing errors

If logs are used they must be evaluated without a calculator. Lengthy decimals would be evidence of this and would be dM0

$(2x+1)\log_2 4=4x\log_2 8\Rightarrow (2x+1)\times 2=4x\times 3\Rightarrow x=..$

$4^{2x+1}=8^{4x}\Rightarrow 2x+1=4x\log_4 8\Rightarrow 2x+1=\frac{3}{2}\times 4x\Rightarrow x=..$ is fine

A1 $x=\frac{1}{4}$ or equivalent

(ii)(a) Mark part (ii) as one complete question. Marks in (a) can be gained from (b)

M1 Writes either $\sqrt{18}=3\sqrt{2}$ or $3\sqrt{18}=9\sqrt{2}$ or $\sqrt{32}=4\sqrt{2}$

If the candidate writes $3\sqrt{18}-\sqrt{32}=k\sqrt{2}$ it can be scored for $\dfrac{3\sqrt{18}}{\sqrt{2}}=9$ or $\dfrac{\sqrt{32}}{\sqrt{2}}=4$

A1 $5\sqrt{2}$ or states $k=5$

The answer without working (the M1) would be 0 marks

(ii)(b)

M1 Moves from $\sqrt{n}=k\sqrt{2}$ to $n=2k^2$

Also accept for this mark $\sqrt{n}=\sqrt{50}$ or indeed $\sqrt{50}$ on its own

A1 $(n=)50$

Question Number	Scheme	Marks
13 (a)	$2\log_2 y = 5 - \log_2 x \Rightarrow \log_2 y^2 = 5 - \log_2 x$	M1
	$\Rightarrow \log_2 y^2 = \log_2 32 - \log_2 x \Rightarrow \log_2 y^2 = \log_2\left(\frac{32}{x}\right)$	
	$\Rightarrow y^2 = \frac{32}{x}$	M1A1
		(3)
(b)	$\log_x y = -3 \Rightarrow y = x^{-3}$	M1
	Sub $y = x^{-3}$ into $y^2 = \frac{32}{x} \Rightarrow x^{-6} = \frac{32}{x} \Rightarrow x^5 = \frac{1}{32} \Rightarrow x = \frac{1}{2}$	M1A1
	Sub $x = \frac{1}{2}$ into either eqn $\Rightarrow y = 8$	M1A1
		(5)
		(8 marks)
Alt (b)	Sub $y^2 = \frac{32}{x}$ into $\log_x y = -3 \Rightarrow \log_x \sqrt{\frac{32}{x}} = -3$	2nd M1
	$\Rightarrow \sqrt{\frac{32}{x}} = x^{-3}$	1st M1
	$\Rightarrow x^5 = \frac{1}{32} \Rightarrow x = \frac{1}{2}$	A1

(a)

M1 Uses one correct log law.
Eg Uses the index law and writes $2\log_2 y = \log_2 y^2$.
Alternatively writes 5 as $\log_2 32$. This may well come from $\log_2 ... = 5 \Rightarrow ... = 32$
Note that $2\log_2 y + \log_2 x = 2\log_2 xy$ is M0

M1 Uses two correct log laws
Award for $\log_2 y^2 = \log_2(32) - \log_2 x$
or $\log_2 x + \log_2 y^2 = 5 \Rightarrow \log_2 xy^2 = 5$

A1 Proceeds correctly to $y^2 = \frac{32}{x}$

(b)

M1 Undoes the log in the second equation $\log_x y = -3 \Rightarrow y = x^{-3}$
This may well appear later in the solution

M1 Combines both equations to form a single equation in one variable.

A1 $x = \frac{1}{2}$ or $y = 8$. Condone a solution $y = \pm 8$ for this mark

M1 Substitutes their $x = \frac{1}{2}$ into an equation to find y.
Alternatively substitutes their $y = 8$ into an equation to find x

A1 $x = \frac{1}{2}$ and $y = 8$ only. Note $x = \frac{1}{2}$ and $y = \pm 8$ is A0

SC. If a candidate uses $y = \frac{k}{x}$ with $\log_x y = -3$ this can potentially score M1: Undoing logs, M0: as combining the equations has been made easier, A0: M1: If they substitute their x to find y and vice versa followed by A0: scoring 10010

Question Number	Scheme	Marks
2.(a)	$e^{3x-9}=8\Rightarrow 3x-9=\ln 8$	M1
	$\Rightarrow x=\dfrac{\ln 8+9}{3}, =\ln 2+3$	A1, A1
		(3)
(b)	$\ln(2y+5)=2+\ln(4-y)$	
	$\ln\left(\dfrac{2y+5}{4-y}\right)=2$	M1
	$\left(\dfrac{2y+5}{4-y}\right)=e^2$	M1
	$2y+5=e^2(4-y)\Rightarrow 2y+e^2y=4e^2-5\Rightarrow y=\dfrac{4e^2-5}{2+e^2}$	dM1, A1
		(4)
		7 marks

(a)

M1 Takes ln's of both sides and uses the power law. You may even accept candidates taking logs of both sides

A1 A correct unsimplified answer $\dfrac{\ln 8+9}{3}$ or equivalent such as $\dfrac{\ln 8e^9}{3}$, $3+\ln\left(\sqrt[3]{8}\right)$, $\dfrac{\log 8}{3\log e}+3$ or even 3.69

A1 cso $\ln 2+3$. Accept $\ln 2e^3$

....

Alt I (a)

$e^{3x-9}=8\Rightarrow\dfrac{e^{3x}}{e^9}=8\Rightarrow e^{3x}=8e^9\Rightarrow 3x=\ln\left(8e^9\right)$ for M1 (Condone slips on index work and lack of bracket)

Alt II (a)

$e^{x-3}=\sqrt[3]{8}\Rightarrow x-3=\ln\left(\sqrt[3]{8}\right)$ for M1 (Condone slips on the 9. Eg $e^{x-9}=2\Rightarrow x-9=\ln 2$)

....

(b)

M1 Uses a correct method to combine two terms to create a single ln term.

Eg. Score for $2+\ln(4-y)=\ln\left(e^2(4-y)\right)$ or $\ln(2y+5)-\ln(4-y)=\ln\left(\dfrac{2y+5}{4-y}\right)$

Condone slips on the signs and coefficients of the terms, but not on the e^2

M1 Scored for an attempt to undo the ln's to get an equation in y This must be awarded after an attempt to combine the ln terms. Award for $\ln(g(y))=2\Rightarrow g(y)=e^2$ and can be scored eg where $g(y)=2y+5-(4-y)$

It cannot be awarded for just $2y+5=e^2+4-y$ where the candidate attempts to undo term by term

dM1 Dependent upon **both** previous M's. It is for making y the subject. Expect to see both terms in y collected and factorised (may be implied) before reaching y =. Condone slips, for eg, on signs. y =2.615 scores this.

A1 $y=\dfrac{4e^2-5}{2+e^2}$ or equivalent such as $y=4-\dfrac{13}{2+e^2}$ ISW after you see the correct answer.

....

Special Case: $\ln(2y+5)-\ln(4-y)=2\Rightarrow\dfrac{\ln(2y+5)}{\ln(4-y)}=2\Rightarrow\dfrac{2y+5}{4-y}=e^2\Rightarrow$ Correct answer score M0 M1 M1 A0

Question	Scheme	Marks
9(a)	Subs $D=15$ and $t=4$ $\quad x=15\mathrm{e}^{-0.2\times4}=6.740\ (mg)$	M1A1
		(2)
(b)	$15\mathrm{e}^{-0.2\times7}+15\mathrm{e}^{-0.2\times2}=13.754(mg)$	M1A1*
		(2)
(c)	$15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times(T+5)}=7.5$	M1
	$15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times T}\mathrm{e}^{-1}=7.5$	
	$15\mathrm{e}^{-0.2\times T}(1+\mathrm{e}^{-1})=7.5\Rightarrow \mathrm{e}^{-0.2\times T}=\dfrac{7.5}{15(1+\mathrm{e}^{-1})}$	dM1
	$\mathrm{T}=-5\ln\left(\dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}\right)=5\ln\left(2+\dfrac{2}{\mathrm{e}}\right)$	A1, A1
		(4)
		(8 marks)

(a)

M1 Attempts to substitute both $D=15$ and $t=4$ in $x=D\mathrm{e}^{-0.2t}$

It can be implied by sight of $15\mathrm{e}^{-0.8}$, $15\mathrm{e}^{-0.2\times4}$ or awrt 6.7

Condone slips on the power. Eg you may see -0.02

A1 CAO 6.740 (mg) Note that 6.74 (mg) is A0

(b)

M1 Attempt to find the sum of two expressions with $D=15$ in both terms with t values of 2 and 7

Evidence would be $15\mathrm{e}^{-0.2\times7}+15\mathrm{e}^{-0.2\times2}$ or similar expressions such as $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times2}$

Award for the sight of the two numbers awrt **3.70** and awrt **10.05**, followed by their total awrt **13.75**

Alternatively finds the amount after 5 hours, $15\mathrm{e}^{-1}=$ awrt **5.52** adds the second dose = **15** to get a total of awrt **20.52** then multiplies this by $\mathrm{e}^{-0.4}$ to get awrt **13.75**.

Sight of 5.52+15=20.52 $\rightarrow$ 13.75 is fine.

A1* cso so both the expression $15\mathrm{e}^{-0.2\times7}+15\mathrm{e}^{-0.2\times2}$ and $13.754(mg)$ are required

Alternatively both the expression $\left(15\mathrm{e}^{-0.2\times5}+15\right)\times\mathrm{e}^{-0.2\times2}$ and $13.754(mg)$ are required.

Sight of just the numbers is not enough for the A1*

(c)

M1 Attempts to write down a correct equation involving T or t. Accept with or without correct bracketing

Eg. accept $15\mathrm{e}^{-0.2\times T}+15\mathrm{e}^{-0.2\times(T\pm5)}=7.5$ or similar equations $\left(15\mathrm{e}^{-1}+15\right)\mathrm{e}^{-0.2\times T}=7.5$

dM1 Attempts to solve their equation, dependent upon the previous mark, by proceeding to $\mathrm{e}^{-0.2\times T}=\ldots$

An attempt should involve an attempt at the index law $x^{m+n}=x^m\times x^n$ and taking out a factor of $\mathrm{e}^{-0.2\times T}$ Also score for candidates who make $\mathrm{e}^{+0.2\times T}$ the subject using the same criteria

A1 Any correct form of the answer, for example, $-5\ln\left(\dfrac{7.5}{15\left(1+\mathrm{e}^{-1}\right)}\right)$

A1 CSO $\mathrm{T}=5\ln\left(2+\dfrac{2}{\mathrm{e}}\right)$ Condone t appearing for T throughout this question.

Question Number	Scheme	Marks
2.(a)	$2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$	M1
	$\Rightarrow x=\dfrac{e^5-1}{2}$	A1
		(2)
(b)	$3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$	
	$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$ oe	A1
		(4) **6 marks**
Alt 1 2(b)	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$	
	$3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=...$	dM1
	$x=\dfrac{7}{(\ln 3+4)}$	A1
		(4)
Alt 2 2(b) Using logs	$3^x e^{4x}=e^7 \Rightarrow \log(3^x e^{4x})=\log e^7$	
	$\log 3^x+\log e^{4x}=\log e^7 \Rightarrow x\log 3+4x\log e=7\log e$	M1, M1
	$x(\log 3+4\log e)=7\log e \Rightarrow x=...$	dM1
	$x=\dfrac{7\log e}{(\log 3+4\log e)}$	A1
		(4)
Alt 3 2(b) Using $\log_3$	$3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$	
	$3^x=e^{7-4x} \Rightarrow x=(7-4x)\log_3 e$	M1,M1
	$x(1+4\log_3 e)=7\log_3 e \Rightarrow x=...$	dM1
	$x=\dfrac{7\log_3 e}{(1+4\log_3 e)}$	A1
		(4)
Alt 4 2(b) Using $3^x=e^{x\ln 3}$	$3^x e^{4x}=e^7 \Rightarrow e^{x\ln 3}e^{4x}=e^7$	
	$\Rightarrow e^{x\ln 3+4x}=e^7, \Rightarrow x\ln 3+4x=7$	M1,M1
	$x(\ln 3+4)=7 \Rightarrow x=... \qquad x=\dfrac{7}{(\ln 3+4)}$	dM1 A1
		(4)

Question Number	Scheme	Marks
6(a)	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	M1, M1
	So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$	A1
	Solve $5x^2-32x+35=0$ to give $x=\frac{7}{5}$ oe (Ignore the solution $x=5$)	M1A1
		(5)
(b)	Take $\log_e$'s to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x\ln 2 + (3x+1)\ln e = \ln 10$	M1
	$x(\ln 2 + 3\ln e) = \ln 10 - \ln e \Rightarrow x = ..$	dM1
	and uses $\ln e = 1$	M1
	$x = \frac{-1+\ln 10}{3+\ln 2}$	A1
		(5)
	Note that the 4th M mark may occur on line 2	**(10 marks)**

Notes for Question 6

(a)

M1 Uses addition law on lhs of equation. Accept slips on the signs. If one of the terms is taken over to the rhs it would be for the subtraction law.

M1 Uses power rule for logs write the $2\ln(x+1)$ term as $\ln(x+1)^2$. Condone invisible brackets

A1 Undoes the logs to obtain the 3TQ =0. $5x^2-32x+35=0$. Accept equivalences. The equals zero may be implied by a subsequent solution of the equation.

M1 Solves a quadratic by any allowable method.
The quadratic cannot be a version of $(4-2x)(9-3x)=0$ however.

A1 Deduces x = 1.4 or equivalent. Accept both x=1.4 and x=5. Candidates do not have to eliminate $x=5$.
You may ignore any other solution as long as it is not in the range $-1 < x < 2$. Extra solutions in the range scores A0.

Question Number	Scheme	Marks
7. (i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x=)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \frac{(9y+b)}{(2y-b)} = 2$ — Applies quotient law of logarithms	M1
	$\frac{(9y+b)}{(2y-b)} = 3^2$ — Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ — Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or: $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ — 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ — 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ — Multiplies across and makes y the subject	M1 A1cso **(4)** **[7]**

Notes

(i)

1st M1: Applies power law of logarithms correctly to one side of the equation

M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x+a = \sqrt{16a^6}$ isw so allow $x+a = \pm\ 4a^3$ for Method mark. Also allow $x+a = 4a^4$ or $x+a = \pm\ 4a^{5.5}$ or even $x+a = 16a^3$ as there is evidence of attempted square root.

May see the correct $x+a = 10^{(\log 4 + 3\log a)}$ *so* $x = -a + 10^{(\log 4 + 3\log a)}$ which gains M1A0 unless followed by the answer in the scheme.

A1: Do not allow $x = \pm\ 4a^3 - a$ for accuracy mark. You may see the factorised $a(2a+1)(2a-1)$ o.e.

(ii)

M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in y

M1: Uses $\log_3 3^2 = 2$

3rd M1: Obtains **correct** linear equation in y usually the one in the scheme and attempts $y =$

A1cso: $y = \frac{10}{9}b$ or correct equivalent after **completely correct** work.

Special case:

$\frac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3 \frac{(9y+b)}{(2y-b)} = 2$

Candidates may then write $\frac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.

Question Number	Scheme		Marks
8(i)	Two Ways of answering the question are given in part (i)		
Way 1	$\log_3\left(\frac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\frac{a-2}{3b+1}\right) = 1$	Applying the subtraction law of logarithms	M1
	$\frac{3b+1}{a-2} = 3^{-1}\left\{=\frac{1}{3}\right\}$ or $\left(\frac{a-2}{3b+1}\right) = 3$	Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3 = a-2 \Rightarrow\}\ b = \frac{1}{9}a - \frac{5}{9}$	$b = \frac{1}{9}a - \frac{5}{9}$ or $b = \frac{a-5}{9}$	A1 oe
			[3]
	In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs		
(i) **Way 2**	Either $\log_3(3b+1) - \log_3(a-2) = -\log_3 3$ or $\log_3(3b+1) + \log_3 3 = \log_3(a-2)$		2nd M1
	$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\frac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$		1st M1
	$\{3b+1 = \frac{a-2}{3}\}\ b = \frac{1}{9}a - \frac{5}{9}$		A1
			[3]
	Five Ways of answering the question are given in part (ii)		
(ii) **Way 1**	$32(2^{2x}) - 7(2^x) = 0$	Deals with power 5 correctly giving ×32	M1
See also common approach below in notes	So, $2^x = \frac{7}{32}$	$2^x = \frac{7}{32}$ or $y = \frac{7}{32}$ or awrt 0.219	A1 oe
	$x\log 2 = \log\left(\frac{7}{32}\right)$ or $x = \frac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x = \log_2\left(\frac{7}{32}\right)$	A valid method for solving $2^x = \frac{7}{32}$ Or $2^x = k$ to achieve $x = ...$	dM1
	$x = -2.192645...$	awrt -2.19	A1
			[4]
	Begins with $2^{2x+5} = 7(2^x)$ (for Way 2 and Way 3) (see notes below)		
(ii) **Way 2**	$(2x+5)\log 2 = \log 7 + x\log 2$	Correct application of **either** the power law **or** addition law of logarithms	M1
		Correct result after applying the power **and** addition laws of logarithms.	A1
	$2x\log 2 + 5\log 2 = \log 7 + x\log 2$ $\Rightarrow\ x = \frac{\log 7 - 5\log 2}{\log 2}$	Multiplies out, collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$	awrt -2.19	A1
			[4]
(ii) **Way 3**	$2x + 5 = \log_2 7 + x$	Evidence of $\log_2$ and either $2^{2x+5} \to 2x+5$ or $7(2^x) \to \log_2 7 + \log_2(2^x)$	M1
		$2x + 5 = \log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow\ x = \log_2 7 - 5$	Collects x terms to achieve $x = ...$	dM1
	$x = -2.192645...$	awrt -2.19	A1
			[4]