2. The functions f and g are defined by

$$f(x) = 7 - 2x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{3x}{5x-1} \qquad x \in \mathbb{R} \qquad x \neq \frac{1}{5}$$

(a) State the range of f

(1)

(b) Find gf(1.8)

(2)

(c) Find $g^{-1}(x)$

(2)

1. $$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

Answer ALL questions. Write your answers in the spaces provided.

1.

$$g(x) = \frac{2x+5}{x-3} \qquad x \geqslant 5$$

(a) Find gg(5).

(2)

(b) State the range of g.

(1)

(c) Find $g^{-1}(x)$, stating its domain.

(3)

Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

5. $$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write $f(x)$ in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

6.

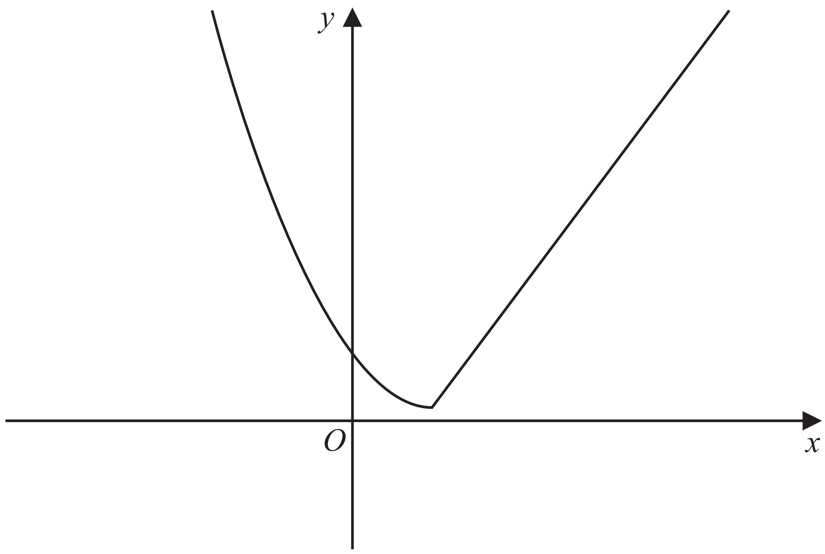

Figure 4

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of $\mathrm{gg}(0)$.

(2)

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

4. The function f is defined by

$$f(x) = \frac{3x - 7}{x - 2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$

(2)

(b) Show that $ff(x) = \dfrac{ax + b}{x - 3}$ where a and b are integers to be found.

(3)

7.

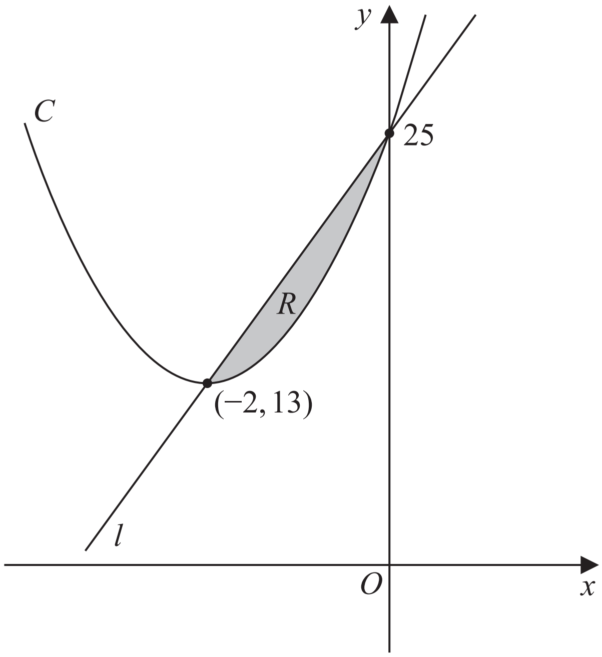

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define R.

(5)

4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3\ln x, \quad x > 0,\ x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

(Total for Question 4 is 5 marks)

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7. The function f is defined by

$$f : x \mapsto \frac{3x - 5}{x + 1}, \quad x \in \mathbb{R}, x \neq -1$$

(a) Find an expression for $f^{-1}(x)$

(3)

(b) Show that

$$ff(x) = \frac{x + a}{x - 1}, \quad x \in \mathbb{R}, x \neq -1, x \neq 1$$

where a is an integer to be determined.

(4)

The function g is defined by

$$g : x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, \ 0 \leqslant x \leqslant 5$$

(c) Find the value of fg(2)

(2)

(d) Find the range of g

(3)

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4. Given that

$$f(x) = \frac{4}{3x + 5}, \quad x > 0$$

$$g(x) = \frac{1}{x}, \qquad x > 0$$

(a) state the range of f,
(2)

(b) find $f^{-1}(x)$,
(3)

(c) find $fg(x)$.
(1)

(d) Show that the equation $fg(x) = gf(x)$ has no real solutions.
(4)

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3. The function g is defined by

$$g(x) = \frac{6x}{2x+3} \qquad x > 0$$

(a) Find the range of g.
(2)

(b) Find $g^{-1}(x)$ and state its domain.
(3)

(c) Find the function $gg(x)$, writing your answer as a single fraction in its simplest form.
(3)

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3.

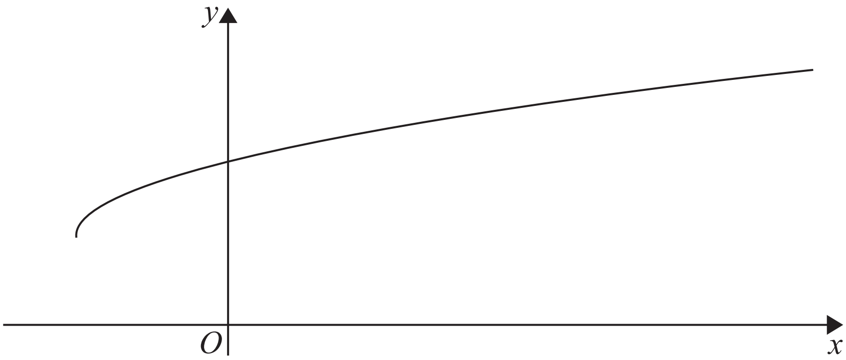

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = 3 + \sqrt{x+2}\,, \qquad x \geqslant -2$$

(a) State the range of g. **(1)**

(b) Find $\mathrm{g}^{-1}(x)$ and state its domain. **(3)**

(c) Find the exact value of x for which

$$\mathrm{g}(x) = x$$

(4)

(d) Hence state the value of a for which

$$\mathrm{g}(a) = \mathrm{g}^{-1}(a)$$

(1)

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1. The functions f and g are defined by

$$\text{f} : x \rightarrow 7x - 1, \qquad x \in \mathbb{R}$$

$$\text{g} : x \rightarrow \frac{4}{x-2}, \qquad x \neq 2,\ x \in \mathbb{R}$$

(a) Solve the equation $\text{fg}(x) = x$

(4)

(b) Hence, or otherwise, find the largest value of a such that $\text{g}(a) = \text{f}^{-1}(a)$

(1)

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7.

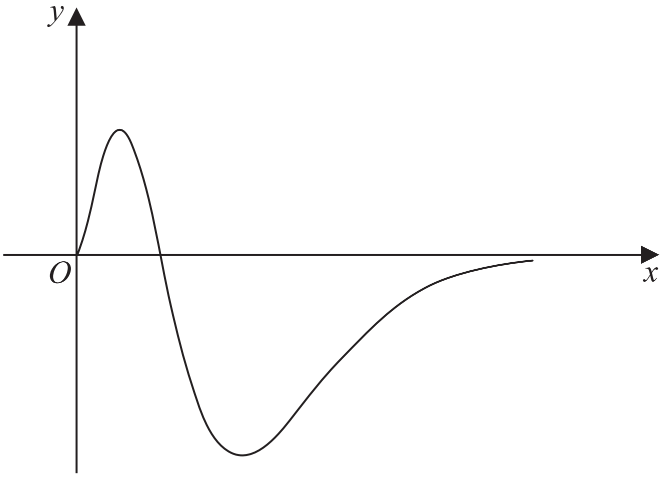

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geqslant 0$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g.

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

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6. The function f is defined by

$$f : x \rightarrow e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

(a) State the range of f.

(1)

(b) Find f^{-1} and state its domain.

(3)

The function g is defined by

$$g : x \rightarrow \ln(2x), \qquad x > 0$$

(c) Solve the equation

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form.

(4)

(d) Find fg(x), giving your answer in its simplest form.

(2)

(e) Find, in terms of the constant k, the solution of the equation

$$fg(x) = 2k^2$$

(2)

Leave blank

5.

$$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \quad x > 3$$

(a) Show that $g(x) = \dfrac{x+1}{x-2}, \quad x > 3$

(4)

(b) Find the range of g.

(2)

(c) Find the exact value of a for which $g(a) = g^{-1}(a)$.

(4)

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7. The function f has domain $-2 \leqslant x \leqslant 6$ and is linear from (–2, 10) to (2, 0) and from (2, 0) to (6, 4). A sketch of the graph of $y = \mathrm{f}(x)$ is shown in Figure 1.

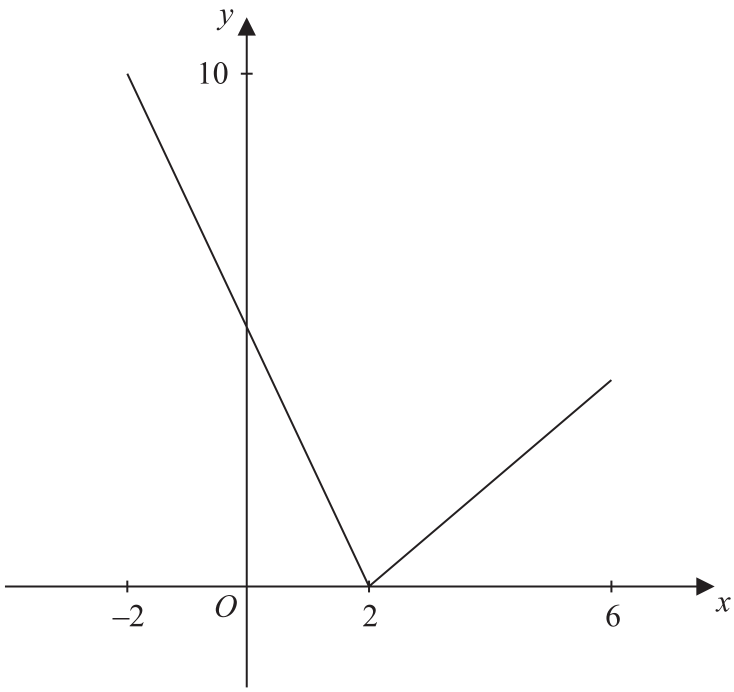

Figure 1

(a) Write down the range of f.
(1)

(b) Find ff(0).
(2)

The function g is defined by

$$\mathrm{g} : x \rightarrow \frac{4 + 3x}{5 - x}, \qquad x \in \mathbb{R}, \qquad x \neq 5$$

(c) Find $\mathrm{g}^{-1}(x)$
(3)

(d) Solve the equation $\mathrm{gf}(x) = 16$
(5)