

2. Given that

$$f(x) = \ln x, \quad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x -axis.
In each case, state the equation of the asymptote.

(7)



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Question 2 continued



Question 2 continued



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Question 2 continued

Q2

(Total 7 marks)



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Question 3 continued

Lined writing area for the exam question.



P 4 3 0 1 6 A 0 9 3 2

4. $f(x) = 25x^2e^{2x} - 16, \quad x \in \mathbb{R}$

(a) Using calculus, find the exact coordinates of the turning points on the curve with equation $y = f(x)$.

(5)

(b) Show that the equation $f(x) = 0$ can be written as $x = \pm \frac{4}{5} e^{-x}$

(1)

The equation $f(x) = 0$ has a root α , where $\alpha = 0.5$ to 1 decimal place.

(c) Starting with $x_0 = 0.5$, use the iteration formula

$$x_{n+1} = \frac{4}{5} e^{-x_n}$$

to calculate the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places.

(3)

(d) Give an accurate estimate for α to 2 decimal places, and justify your answer.

(2)

Horizontal lines for writing answers.



5. Given that

$$x = \sec^2 3y, \quad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of y .

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x . Give your answer in its simplest form.

(4)



6. Find algebraically the exact solutions to the equations

(a) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1)$, $-1 < x < 2$ (5)

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\frac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers. (5)

Lined area for student answers.



7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

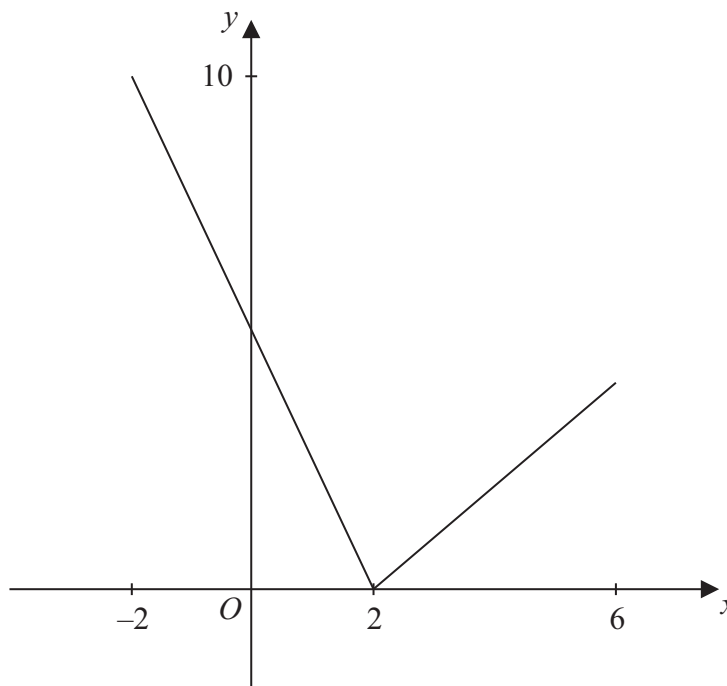


Figure 1

- (a) Write down the range of f . (1)
- (b) Find $ff(0)$. (2)

The function g is defined by

$$g : x \rightarrow \frac{4 + 3x}{5 - x}, \quad x \in \mathbb{R}, \quad x \neq 5$$

- (c) Find $g^{-1}(x)$ (3)
- (d) Solve the equation $gf(x) = 16$ (5)



8.

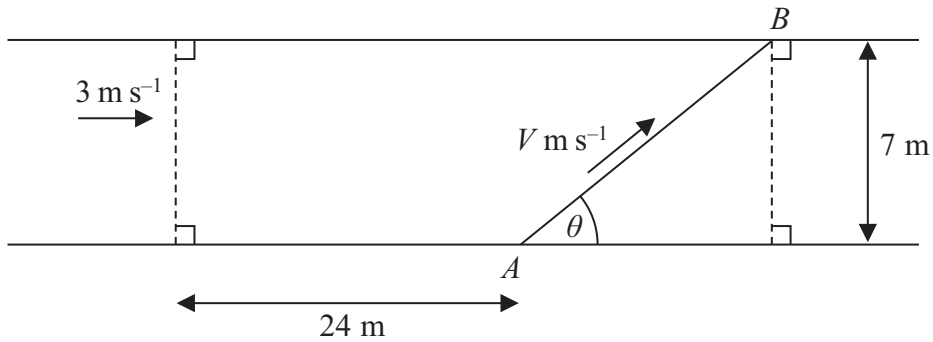


Figure 2

Kate crosses a road, of constant width 7 m, in order to take a photograph of a marathon runner, John, approaching at 3 m s^{-1} .

Kate is 24 m ahead of John when she starts to cross the road from the fixed point A .

John passes her as she reaches the other side of the road at a variable point B , as shown in Figure 2.

Kate's speed is $V \text{ m s}^{-1}$ and she moves in a straight line, which makes an angle θ , $0 < \theta < 150^\circ$, with the edge of the road, as shown in Figure 2.

You may assume that V is given by the formula

$$V = \frac{21}{24 \sin \theta + 7 \cos \theta}, \quad 0 < \theta < 150^\circ$$

- (a) Express $24 \sin \theta + 7 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where R and α are constants and where $R > 0$ and $0 < \alpha < 90^\circ$, giving the value of α to 2 decimal places.

(3)

Given that θ varies,

- (b) find the minimum value of V .

(2)

Given that Kate's speed has the value found in part (b),

- (c) find the distance AB .

(3)

Given instead that Kate's speed is 1.68 m s^{-1} ,

- (d) find the two possible values of the angle θ , given that $0 < \theta < 150^\circ$.

(6)



