Question	Scheme	Marks	AOs
1(a) **Way 1**	$\det \mathbf{M} = -4\times -4 - 4\sqrt{3}\times -4\sqrt{3} = \ldots \Rightarrow k = \sqrt{\det \mathbf{M}} = \ldots$	M1	3.1a
	$k = 8$	A1	1.1b
	$\Rightarrow \mathbf{Q} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \ldots$	M1	1.1b
	$\left(\cos\theta < 0, \sin\theta > 0 \Rightarrow \text{ Quadrant 2 so}\right) \quad \theta = 120°$	A1	1.1b
		(4)	
Way 2	$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix} = k\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} -4 & -4\sqrt{3} \\ 4\sqrt{3} & -4 \end{pmatrix}$	M1	3.1a
	Achieves both the equations $k\cos\theta = -4$ and $k\sin\theta = 4\sqrt{3}$	A1	1.1b
	$\frac{k\sin\theta}{k\cos\theta} = \frac{4\sqrt{3}}{-4} \Rightarrow \tan\theta = -\sqrt{3} \Rightarrow \theta = \ldots$	M1	1.1b
	$\theta = 120°$ and $k = 8$	A1	1.1b
		(4)	
(b)	Area of S' = area of $S \times k^2$ (The area of the square $S = 2a^2$)	M1	1.1b
	Area of $S' = 128a^2$	A1ft	2.2a
		(2)	
		(6 marks)	

Notes:

(a) Way 1

M1: A full method to find *k* such as attempting the square root of the determinant of **M**. It is immediately deducible so the method may be implied by $k = 8$.

A1: $k = 8$

M1: A full method to find a value of θ using their *k*, no need to justify quadrant. Only one equation needed for this mark. Allow if a radians answer is given. May be implied by a correct angle.

A1: Correct angle in degrees.

Way 2

M1: Multiplies the correct matrix representing transformation *Q* by the matrix representing transformation *P* and sets equal to matrix **M**. Allow for the matrices either way round as the transformations commute. No need to see the identity matrix, just multiplying through by *k* is sufficient.

A1: Both correct equations. Note that if a correct value of *k* is found, this A is scored under Way 1.

M1: Solves their simultaneous equations to find a value for θ (or *k*)

A1: $\theta = 120°$ and $k = 8$

(b)

M1: Complete method to find the area of S' : 'their k^2' × 'their $2a^2$'. Must be an attempt at the area of *S* but it need not be correct.

A1ft: Deduces the correct area for S', follow through their value of *k*

Question	Scheme		Marks	AOs
2	$\begin{pmatrix}4 & -2\\ 5 & 3\end{pmatrix}\begin{pmatrix}x\\ mx+c\end{pmatrix}=\begin{pmatrix}X\\ mX+c\end{pmatrix}$ leading to an equation in x, m, c and X		M1	3.1a
	$4x-2(mx+c)=X$ and $5x+3(mx+c)=mX+c$		A1	1.1b
	$5x+3(mx+c)=m\big(4x-2(mx+c)\big)+c$ leading to $5+3m=4m-2m^2$	$(3c=-2mc+c)$	M1	2.1
	$2m^2-m+5=0\Rightarrow b^2-4ac=(-1)^2-4(2)(5)=...$	Solves $3c=-2mc+c\Rightarrow m=...$	dM1	1.1b
	Correct expression for the discriminant = $\{-39\}$ < 0 therefore there are no invariant lines.	$m=-1$ and shows a contradiction in $5+3m=4m-2m^2$ therefore there are no invariant lines.	A1	2.4
	<u>Alternative</u> $\begin{pmatrix}4 & -2\\ 5 & 3\end{pmatrix}\begin{pmatrix}x\\ mx\end{pmatrix}=\begin{pmatrix}X\\ mX\end{pmatrix}$ leading to an equation in x, m and X		M1	3.1a
	$4x-2(mx)=X$ and $5x+3(mx)=mX$		A1	1.1b
	$5x+3(mx)=m\big(4x-2(mx)\big)$ leading to $5+3m=4m-2m^2$		M1	2.1
	$2m^2-m+5=0\Rightarrow b^2-4ac=(-1)^2-4(2)(5)=...$		dM1	1.1b
	Correct expression for the discriminant = $\{-39\}$ < 0 therefore there are no invariant lines that pass through the origin no invariant lines.		A1	2.4
			(5)	
				(5 marks)

Notes:

M1: Sets up a matrix equation in an attempt to find a fixed line and extract at least one equation.

A1: Correct equations.

M1: Eliminates X from the simultaneous equations and equates the coefficients of x leading to a quadratic equation in terms of m.

dM1: Dependent on the previous method, finds the value of the discriminant, this can be seen in an attempt to solve the quadratic using the formula.

Alternatively solves $3c=-2mc+c$ and finds a value for m

Note: If the quadratic equation in m is solved on a calculator and complex roots given this is M0 as they are not showing why there are no real roots.

A1: Correct expression for the discriminant, states < 0 and draws the required conclusion. Alternatively, correct value for m, shows a contradiction in $5+3m=4m-2m^2$ and draws the required conclusion.

Alternative

M1: Sets up a matrix equation in an attempt to find a fixed line and extract at least one equation.

A1: Correct equations.

Question	Scheme	Marks	AOs
3(a)	$n = 1 \Rightarrow \mathbf{M}^1 = \begin{pmatrix} 3^1 & \frac{a}{2}(3^1 - 1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ {So the result is true for $n = 1$}	B1	2.2a
	Assume true for $n = k$ Or assume $\mathbf{M}^n$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & \frac{a}{2}(3^k - 1) \\ 0 & 1 \end{pmatrix}$	M1	2.4
	A correct method to find an expression for $n = k + 1$ $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3^k & \frac{a}{2}(3^k - 1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3^k & \frac{a}{2}(3^k - 1) \\ 0 & 1 \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 3(3^k) & a(3^k) + \frac{a}{2}(3^k - 1) \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 3(3^k) & 3 \times \frac{a}{2}(3^k - 1) + a \\ 0 & 1 \end{pmatrix}$	A1	1.1b
	$\begin{pmatrix} 3^{k+1} & \frac{a}{2}[2(3^k) + (3^k - 1)] \\ 0 & 1 \end{pmatrix} =$ $\begin{pmatrix} 3^{k+1} & \frac{a}{2}[3(3^k) - 1] \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3^{k+1} & \frac{a}{2}[3^{k+1} - 1] \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 3(3^k) & 3 \times \frac{a}{2}(3^k - 1) + a \\ 0 & 1 \end{pmatrix} =$ $\begin{pmatrix} 3^{k+1} & \frac{a}{2}(3(3^k - 1) + 2) \\ 0 & 1 \end{pmatrix}$ $= \begin{pmatrix} 3^{k+1} & \frac{a}{2}(3^{k+1} - 1) \\ 0 & 1 \end{pmatrix}$	A1	2.1
	If true for $n = k$ **then** true for $n = k + 1$ and as it is **true** for $n = 1$ the statement is **true for** all (positive integers) ***n***	A1	2.4
		(6)	
(b)(i)	$det(\mathbf{M}^n) = 3^n$ or $det(\mathbf{M}) = 3$	B1	1.1b
	Uses $5 \times det(\mathbf{M}^n) = 1215 \Rightarrow p^n = q \Rightarrow n = \ldots$ $5 \times 3^n = 1215 \Rightarrow 3^n = 243 \Rightarrow n = \ldots$	M1	3.1a
	$n = 5$	A1	1.1b
(ii)	$\begin{pmatrix} 3^n & \frac{a}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 123 \\ -2 \end{pmatrix} \Rightarrow 2(3^n) - 2\frac{a}{2}(3^n - 1) = 123$ $\Rightarrow a = \ldots$	M1	1.1b

	$\begin{pmatrix} 243 & \frac{a}{2}(243-1) \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 123 \\ -2 \end{pmatrix} \Rightarrow 2(243) - 2\frac{a}{2}(243-1) = 123 \Rightarrow a = \ldots$ $\frac{1}{243}\begin{pmatrix} 1 & -\frac{a}{2}(243-1) \\ 0 & a \end{pmatrix}\begin{pmatrix} 123 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} \Rightarrow \frac{123 - 2\frac{a}{2}(243-1)}{243} = -2 \Rightarrow a = \ldots$		
	$a = 1.5$	A1	1.1b
		(5)	
(11 marks)			

Notes:

(a)

B1: Shows that the result holds for $n = 1$. Must see substitution in the RHS minimum required is $\begin{pmatrix} 3 & \frac{a}{2}(3-1) \\ 0 & 1 \end{pmatrix}$ and reaches $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}$

M1: Assumes the result is true for some value of $n = k$. Assume (true for) $n = k$ is sufficient.

Alternatively states assume $\mathbf{M}^n$ or $\begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 3^k & \frac{a}{2}(3^k-1) \\ 0 & 1 \end{pmatrix}$

M1: Sets up a matrix multiplication of their assumed result multiplied by the original matrix, either way round. Allow a slip as long as the intention is clear.

A1: Achieves a correct un-simplified matrix

A1: Reaches a correct simplified matrix with **no errors, the correct un-simplified matrix seen previously and at least one intermediate line which must be correct.**

A1: Correct conclusion. This mark is dependent on all previous marks except B mark but n = 1 must have been attempted. It is gained by conveying the ideas of **all four bold** points either at the end of their solution or as a narrative in their solution. Condone $n \in \mathbb{Z}$

(b)(i)

B1: States correct determinant. This can be implied by a correct equation

M1: Correct method to find a value of n using $5 \times$ 'their $det(\mathbf{M}^n)$' $= 1215$ which involves solving an index equation of the form $p^n = q$ where $n > 1$

A1: $n = 5$

(ii)

M1: Sets up an equation by multiplying the matrix $\mathbf{M}^n$ by $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ setting equal to $\begin{pmatrix} 123 \\ -2 \end{pmatrix}$ and reaches a value for a. You may just see $2(3^n) - 2\frac{a}{2}(3^n - 1) = 123 \Rightarrow a = \ldots$

Follow through on their value for n.

A1: $a = 1.5$

Question	Scheme	Marks	AOs
3(a)	$\mathbf{M}^2+11\mathbf{M}=\begin{pmatrix}a&0\\0&a\end{pmatrix}\Rightarrow\begin{pmatrix}34&5k-10\\6k-12&k^2+30\end{pmatrix}+\begin{pmatrix}-22&55\\66&11k\end{pmatrix}=\begin{pmatrix}a&0\\0&a\end{pmatrix}$	M1	1.1b
	$\Rightarrow a=12$	A1	2.2a
	$5k-10+55=0\Rightarrow 5k=-45\Rightarrow k=-9^*$ or $6k-12+66=0\Rightarrow 6k=-54\Rightarrow k=-9^*$ or $k^2+11k+30=12\Rightarrow k^2+11k+18=0\Rightarrow k=-2,-9^*$, $k\neq-2$ as $5\times-2-10+55\neq0$ or $6\times-2-12+66\neq0$	A1*	2.1
		(3)	
	Alternative Using $k=-9$ $\mathbf{M}^2+11\mathbf{M}=\begin{pmatrix}a&0\\0&a\end{pmatrix}\Rightarrow\begin{pmatrix}34&-55\\-66&111\end{pmatrix}+\begin{pmatrix}-22&55\\66&-99\end{pmatrix}=\begin{pmatrix}12&0\\0&12\end{pmatrix}$	M1	1.1b
	$\Rightarrow a=12$	A1	2.2a
	Conclusion: therefore $k=-9$	A1*	2.1
		(3)	
(b)	$\begin{pmatrix}-2&5\\6&-9\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}X\\mX+c\end{pmatrix}\Rightarrow\begin{cases}-2x+5(mx+c)=X\\6x-9(mx+c)=mX+c\end{cases}$	M1	1.1b
	$\Rightarrow 6x-9mx-9c=-2mx+5m^2x+5mc+c$ $\left\{\Rightarrow\left(5m^2+7m-6\right)x+\left(5m+10\right)c=0\right\}$	M1 A1	3.1a 1.1b
	$\Rightarrow 5m^2+7m-6=0\{\Rightarrow(m+2)(5m-3)\}\Rightarrow m=-2,\frac{3}{5}$	M1	1.1b
	$m=\frac{3}{5}\Rightarrow 5m+10\neq0$ so need $c=0$ hence $y=\frac{3}{5}x$ is a fixed line	A1	2.2a
	$m=-2\Rightarrow 5m+10=0$ so c can be anything, so $y=-2x+c$ for any c is fixed.	A1	2.2a
		(6)	
(c)	$((0,c)\to(5c,-9c)$ so need c = 0,) $(1,m)\to(-2+5m,6-9m)$ so need or $5m=3$ hence $y=\frac{3}{5}x$ contains fixed points.	B1	3.2a

	or $$\begin{pmatrix}-2 & 5\\ 6 & -9\end{pmatrix}\begin{pmatrix}x\\ \frac{3}{5}x\end{pmatrix}=\begin{pmatrix}-2x+3x\\ 6x-\frac{27}{5}x\end{pmatrix}=\begin{pmatrix}x\\ \frac{3}{5}x\end{pmatrix}\Rightarrow y=\frac{3}{5}x$$ or $$\begin{pmatrix}-2 & 5\\ 6 & -9\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow\begin{cases}-2x+5y=x\\ 6x-9y=y\end{cases}\Rightarrow y=\frac{3}{5}x$$		
		(1)	
			(10 marks)

Notes:

(a)

M1: Evaluates $\mathbf{M}^2$ and uses in the equation given.

A1: Correct value of *a* deduced

A1*: Correct work to show $k=-9$. If off diagonals are used no further justification is needed (they are "given" the result is true). If the bottom right entry is used there must be a valid reason for rejecting -2 as a solution (ie checking the off diagonal).

Alternative: Using $k=-9$

M1: Evaluates $\mathbf{M}^2$ and uses in the equation given.

A1: Correct value of *a* deduced

A1*: Draws the conclusion that $k=-9$

(b)

M1: Sets up the matrix equation for invariant lines and extracts the simultaneous equations from the matrix equation.

M1: Eliminates "*X*" to get a linear equation in "*x*".

A1: Correct equation need not be simplified isw

M1: Solves their quadratic equation in *m* by any valid means including calculator

A1: Deduces $y=\frac{3}{5}x$ is a fixed line (where *c* = 0). If the value for *m* here is wrong, allow this A for $y=-2x$ if the general case for the final A is not scored.

A1: Deduces $y=-2x+c$ is a fixed line where *c* can be any value. Must include all the lines.

Question	Scheme	Marks	AOs
5(a)	Rotation	B1	1.1b
	120 degrees (anticlockwise) or $\frac{2\pi}{3}$ radians (anticlockwise) Or 240 degrees clockwise or $\frac{4\pi}{3}$ radians clockwise	B1	2.5
	About (from) the origin. Allow (0, 0) or *O* for origin.	B1	1.2
		(3)	
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	1.1b
		(1)	
(c)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	A1ft	1.1b
		(2)	
(d)	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix} = \ldots$ or $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \ldots$ Note: $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2}+\frac{1}{2}k \\ \frac{1}{2}+\frac{\sqrt{3}}{2}k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$ can score M1 (for the matrix equation) but needs an equation to be "extracted" to score the next A1	M1	3.1a
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y$ (Note that candidates may then substitute x = 1 which is acceptable)	A1ft	1.1b
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or}\frac{1}{2-\sqrt{3}}\right)$	A1	1.1b
	$\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or}\frac{1}{2-\sqrt{3}}\right)$	B1	1.1b
		(4)	
			(10 marks)

Question	Scheme	Marks	AOs
1. (a)	$(\det(\mathbf{M}) =)\ (4)(-7)-(2)(-5)$	M1	1.1a
	M is non-singular because $\det(\mathbf{M}) = -18$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	Area $R = \dfrac{\text{Area } S}{(\pm)\lvert\det \mathbf{M}\rvert} = \ldots$	M1	1.2
	$\text{Area}(R) = \dfrac{63}{\lvert -18 \rvert} = \dfrac{7}{2}$ oe	A1ft	1.1b
		(2)	
(c)	$\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \begin{pmatrix} 4x-10x \\ 2x-14x \end{pmatrix}$	M1	1.1b
	$= \begin{pmatrix} -6x \\ -12x \end{pmatrix}$ and so all points on $y = 2x$ map to points on $y = 2x$, hence the line is invariant. OR $= -6\begin{pmatrix} x \\ 2x \end{pmatrix}$ hence $y = 2x$ is invariant.	A1	2.1
		(2)	
	(6 marks)		

Notes

(a)	**M1**	An attempt to find $\det(\mathbf{M})$. Just the calculation is sufficient. Site of −18 implies this mark, which may be embedded in an attempt at the inverse..
	A1	$\det(\mathbf{M}) = -18$ **and** reference to zero, e.g. $-18 \neq 0$ **and** conclusion. The conclusion may precede finding the determinant (e.g. "Non-singular if det(**M**) ≠ 0, det(**M**) = −18 ≠ 0" is sufficient or accept "Non-singular if det(**M**)≠ 0, det(**M**) = −18, therefore non-singular" or some other indication of conclusion.) Need not mention "det(**M**)" to gain both marks here, a correct calculation, statement −18≠0, and conclusion hence **M** is non-singular can gain M1A1.
(b)	**M1**	Recalls determinant is needed for area scale factor by dividing 63 by ±their determinant.
	A1ft	$\dfrac{7}{2}$ or follow through $\dfrac{63}{\lvert \text{their det} \rvert}$. Must be positive and should be simplified to single fraction or exact decimal. (Allow if made positive following division by a negative determinant.)
(c)	**M1**	Attempts the matrix multiplication shown or with equivalent, e.g $\begin{pmatrix} \frac{1}{2}y \\ y \end{pmatrix}$. May use $\begin{pmatrix} x \\ y \end{pmatrix}$ **and** substitute $y = 2x$ later and this is fine for the method.
	A1	Correct multiplication and working leading to conclusion that the line is invariant. If the −6 is not extracted, they must make reference to image points being on line $y = 2x$. If the −6 is extracted to show it is a multiple of $\begin{pmatrix} x \\ 2x \end{pmatrix}$ followed by a conclusion "invariant" as minimum.

Question	Scheme	Marks	AOs
6(i) (a)	Multiplies the matrix **A** by itself and sets equal to **I** to form one equation in a only and another equation involving both a and b. $\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}\begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow 4 + a(a-4) = 1$ and either $2a + ab = 0$ or $2(a-4) + b(a-4) = 0$ or $a(a-4) + b^2 = 1$	M1	3.1a
	Solves a 3TQ involving only the constant a. This could come after a value of b is found and this value substituted into an equation involving both a and b $a^2 - 4a + 3 = 0 \Rightarrow (a-3)(a-1) = 0 \Rightarrow a = ...$	dM1	1.1b
	$a = 1, a = 3$	A1	11b
	Substitutes a value for a into an equation involving both a and b and solves for b. e.g. $2(1) + (1)b \Rightarrow b = ...$ $2(1-4)b + (1-4) = 0 \Rightarrow b = ...$ $(1)(1-4) + b^2 = 1 \Rightarrow b = ...$ Alternatively uses $2a + ab = 0$ $a(2+b) = 0$ As $a \neq 0$ $2 + b = 0 \Rightarrow b = ...$	dM1	1.1b
	$b = -2$	A1	1.1b
		(5)	
	Alternative (i) (a) Finds $\mathbf{A}^{-1}$ in terms of a and b, sets equal to **A** and attempts to find at least two different equations. Allow a single sign slip $\frac{1}{2b - a(a-4)}\begin{pmatrix} b & -a \\ -(a-4) & 2 \end{pmatrix} = \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$ One equation from $\frac{b}{2b - a(a-4)} = 2$, $\frac{2}{2b - a(a-4)} = b$ One equation from $\frac{-a}{2b - a(a-4)} = a$, $\frac{-(a-4)}{2b - a(a-4)} = a - 4$	M1	3.1a
	Uses their value of b and their value of the determinant to form and solve a 3TQ involving only the constant a $a^2 - 4a + 3 = 0$ $\Rightarrow (a-3)(a-1) = 0$ $\Rightarrow a = ...$ Eliminates b from their equations and solve a 3TQ involving only the constant a $a^2 - 4a + 3 = 0$ $\Rightarrow (a-3)(a-1) = 0$ $\Rightarrow a = ...$	dM1	1.1b
	$a = 1, a = 3$	A1	1.1b

	$\frac{-a}{2b-a(a-4)}=a$ $\Rightarrow 2b-a(a-4)=-1\Rightarrow \frac{b}{-1}=2$ Or $\frac{-(a-4)}{2b-a(a-4)}=a-4$ $\Rightarrow 2b-a(a-4)=-1$ $\Rightarrow \frac{2}{-1}=b$	Substitutes a value for a into an equation to find a value for b	dM1	1.1b
	$b=-2$		A1	1.1b
(b)	Uses their smallest value of **a** and their value for **b** to form two equations $\begin{pmatrix}2 & 'a'\\ 'a-4' & 'b'\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow 2x+ay=x$ and $(a-4)x+by=y$ $\begin{pmatrix}2 & 1\\ -3 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}\Rightarrow 2x+y=x$ and $-3x-2y=y$		M1	3.1a
	$2x+y=x\Rightarrow x+y=0$ o.e. and $-3x-2y=y\Rightarrow x+y=0$ o.e.		M1	1.1b
	$x+y=0$ o.e.		A1	2.1
			(3)	
(ii)(a)	Area of the triangle $T=3$		B1	1.1b
	Complete method to find a value for p. Need to see an attempt at the determinant and setting equal to 15 divided by their area of T. The resulting 3TQ needs to be solved to find a value of p. Determinant $3p\times p-(-1)\times 2p=\frac{15}{\text{'their area'}}\Rightarrow p=...$		M1	3.1a
	$3p^2+2p-5(=0)$		A1	1.1b
	$p=1$ must reject $p=-\frac{5}{3}$		A1	1.1b
			(4)	
(b)	$\begin{pmatrix}3 & 0\\ 0 & -2\end{pmatrix}$		B1 B1	1.1b 1.1b
			(2)	
(c)	(their matrix found in part (b))$\begin{pmatrix}'p' & 2'p'\\ -1 & 3'p'\end{pmatrix}=\begin{pmatrix}... & ...\\ ... & ...\end{pmatrix}$ $\begin{pmatrix}3 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}1 & 2\\ -1 & 3\end{pmatrix}=\begin{pmatrix}... & ...\\ ... & ...\end{pmatrix}$		M1	1.1b

	$\begin{pmatrix} 3 & 6 \\ 2 & -6 \end{pmatrix}$	A1ft	1.1b
		(2)	
			(16 marks)

Notes:

(i)(a)

M1: Forming two equations, one involving *a* only and one involving *a* and *b*

dM1: Dependent on previous mark, solves a 3TQ involving *a*

A1: Correct values for *a*

dM1: Dependent on first method mark Substitutes one of their values of *a* into an equation involving *a* and *b* and solve to find a value for *b*. Alternatively factorises either $2a + ab = 0$ and uses $a \neq 0$ to find a value for *b*.

A1: Correct value for *b*

Alternative(i)(a)

M1: Finds $\mathbf{A}^{-1}$ and sets equal to **A** and forms two different equations

dM1: Dependent on previous mark. Eliminates *b* from their equations and solves a 3TQ involving only the constant *a.* Alternatively if the value of b is found first substitutes their value for *b* into their determinant = – 1 to form and solve a 3TQ for *a*

A1: Correct value for *a*

dM1: Dependent on first method mark. Substitutes a value for *a* into an equation to find a value for *b*. Alternatively uses one equation to find the determinant = – 1 and uses this to find a value of *b*.

A1: Correct values for *b*

(b)

M1: Extracts simultaneous equations using their matrix **A** with their smaller value of *a*.

M1: Gathers terms from their two equations.

A1: Achieves the correct equations and deduces the correct line. Accept equivalent equations as long as both have been shown to be the same.

(ii)(a)

B1: Area of the triangle $T = 3$

M1: Full method. Finds the determinant, sets equal to 15/their area and solves the resulting 3TQ

A1: Correct quadratic

A1: $p = 1$ only

(b)

B1 One correct row or column

B1: All correct

(c)

M1: Multiplies the matrices **QP** in the correct order (if answer only then evidence can be taken from 3 correct or 3 correct ft elements)

A1ft: Correct matrix following through on their answer to part (b) and their value of *p* as long as it is a positive constant

Question	Scheme	Marks	AOs
1(a)(i)	Rotation	B1	1.1b
	90 degrees anticlockwise about the origin	B1	1.1b
(ii)	Stretch	B1	1.1b
	Scale factor 3 parallel to the y-axis	B1	1.1b
		(4)	
(b)	$\mathbf{QP} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$	B1	1.1b
		(1)	
(c)(i)	$\lvert\mathbf{R}\rvert = 3$	B1ft	1.1b
(ii)	The **area** scale factor of the transformation	B1	2.4
		(2)	
			(7 marks)

Notes

(a)(i)
B1: Identifies the transformation as a rotation
B1: Correct angle (allow equivalents in degrees or radians), direction and centre the origin
(ii)
B1: Identifies the transformation as a stretch
B1: Correct scale factor and parallel to/in/along the y-axis/y direction
(b)
B1: Correct matrix
(c)(i)
B1ft: Correct value for the determinant (follow through their **R**)
(ii)
B1: Correct explanation, must include **area**
Note: scale factor of the transformation is B0

Question	Scheme	Marks	AOs
3(a)	Coordinates of Q are (8,−3,2)	B1	2.2a
		(1)	
(b)	Coordinates of R are $\begin{pmatrix}\cos 120° & 0 & \sin 120° \\ 0 & 1 & 0 \\ -\sin 120° & 0 & \cos 120°\end{pmatrix}\begin{pmatrix}8\\3\\2\end{pmatrix}=\ldots$ or $\begin{pmatrix}-0.5 & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & -0.5\end{pmatrix}\begin{pmatrix}8\\3\\2\end{pmatrix}=\ldots$	M1	1.1a
	So R is $\left(-4+\sqrt{3},\ 3, -4\sqrt{3}-1\right)$	A1	1.1b
		(2)	
(c)	Finds the distance $PR=\sqrt{\left(8-'\left(-4+\sqrt{3}\right)'\right)^2+(3-'3')^2+\left(2-'\left(-4\sqrt{3}-1\right)'\right)^2}$ Alternatively finds their $\overrightarrow{PR}$ or their $\overrightarrow{RP}$ then applies length of a vector formula. $\sqrt{\left(12-\sqrt{3}\right)^2+\left(3+4\sqrt{3}\right)^2}$ or $\sqrt{\left(-12+\sqrt{3}\right)^2+\left(-3-4\sqrt{3}\right)^2}$	M1	2.1
	$=\sqrt{204}\quad\left(=2\sqrt{51}\right)$ cso	A1	1.1b
		(2)	
(d)	$\overrightarrow{PR}.\overrightarrow{PQ}=\left(-12+\sqrt{3},0,-3-4\sqrt{3}\right).\left(0,-6,0\right)=0$ hence perpendicular	B1ft	1.1b
		(1)	
(e)	PQ is perpendicular to PR so Area = $\frac{1}{2}\times PQ\times PR$	M1	1.1b
	$=\frac{1}{2}\times 6\times\sqrt{204}=6\sqrt{51}$ cso	A1	1.1b
		(2)	
		(8 marks)	

Notes:

(a)

B1: Coordinates of Q correctly stated, accept as a column vector.

(b)

M1: Correct attempt to find coordinates of R using the given matrix with $\theta=120$. Must be multiplying in the correct way round. With no working two correct values or (-2.27, 3, -7.93) implies this mark.

A1: Correct exact coordinates as shown in scheme. Accept as a column vector. Cos 120 and sin 120 must have been evaluated.

(c)

M1: Applies the distance formula with the coordinates of P and their R. Alternatively finds the vector $\overrightarrow{PR}$ or $\overrightarrow{RP}$ then applies length of a vector formula.

A1: Correct answer following correct coordinates of R, must be a surd but need not be fully simplified.

Question	Scheme	Marks	AOs
3 (a)	Rotation	B1	1.1b
	30 degrees or $\frac{\pi}{6}$ about the x – axis Ignore any reference to direction	B1	1.1b
		(2)	
(b)	They have found **AB** when they should find **BA** Multiplication is the wrong way round It should be **BA** Matrix B should be on the left instead of the right Student has done transformation B followed by transformation A It should be $\begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	B1	2.3
		(1)	
(c)	$\left\{\begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\right\} = \left\{\begin{pmatrix} 1 & \frac{3\sqrt{3}}{2} & -\frac{3}{2} \\ \sqrt{3} & \frac{5\sqrt{3}}{2} & \frac{15}{2} \\ 1 & \sqrt{3} & -1 \end{pmatrix}\right\}$ $\left\{\begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\right\} = \left\{\begin{pmatrix} 1 & \frac{3\sqrt{3}}{2} & -1.5 \\ \sqrt{3} & \frac{5\sqrt{3}}{2} & 7.5 \\ 1 & \sqrt{3} & -1 \end{pmatrix}\right\}$	B1	1.1b
		(1)	
			(4 marks)

Notes:

(a)
B1: Identifies the single transformation as a rotation only
B1: Correct angle and axis. Ignore any reference to direction.
Note x-plane, zy-plane and $x = 0$ are 2nd B0
Any additional incorrect statements is 2nd B0

Question	Scheme	Marks	AOs
9(i)	$\begin{vmatrix} k & -2 & 7 \\ -3 & -5 & 2 \\ k & k & 4 \end{vmatrix} = k(-20-2k)+2(-12-2k)+7(-3k+5k)$ or $\begin{vmatrix} k & -2 & 7 & k & -2 \\ -3 & -5 & 2 & -3 & -5 \\ k & k & 4 & k & k \end{vmatrix} = k(-5)(4)-2(2)(k)+7(-3)(k)$ $-7(-5)(k)-k(2)(k)-(-2)(-3)(4)$	M1	1.1b
	$-2k^2-10k-24(=0)$ isw	A1	1.2
	$b^2-4ac=(10)^2-4(-2)(-24)=...$ $b^2-4ac=(5)^2-4(-1)(-12)=...$ Or $k^2+5k+12=0 \Rightarrow (k+2.5)^2+5.75=0 \Rightarrow (k+2.5)^2=-5.75$ $-2k^2-10k-25=0 \Rightarrow -2(k+2.5)^2-12.5=0 \Rightarrow (k+2.5)^2=-5.75$ Or $k^2+5k+12 \Rightarrow (k+2.5)^2+5.75 \Rightarrow (k+2.5)^2 \geqslant 0$ or $-2k^2-10k-25=0 \Rightarrow -2(k+2.5)^2-12.5=0 \Rightarrow -2(k+2.5)^2 \leqslant 0$ Or $\frac{\mathrm{d}(-2k^2-10k-24)}{\mathrm{d}k}=-4k-10=0 \Rightarrow k=-2.5 \Rightarrow \text{determinant}=-5.75$ Or $k=\frac{10\pm\sqrt{(-10)^2-4(-2)(-25)}}{2(-2)}=\frac{-5\pm\sqrt{23}\mathrm{i}}{2}$	M1	1.1b
	$b^2-4ac=-92<0$ therefore **no real roots** so **non-singular** $b^2-4ac=-23<0$ therefore **no real roots** so **non-singular** Or Square of negative is not real therefore **non-singular** Or $(k+2.5)^2+5.75>0$ therefore **no real roots** so **non-singular** $-2(k+2.5)^2-12.5<0$ therefore **no real roots** so **non-singular** Or As negative quadratic maximum value of determinant = – 5.25 therefore **no real roots** so **non-singular** Or Imaginary roots therefore **no real roots** so **non-singular**	A1	2.4
		(4)	

(ii)	$\begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} a & 4 \\ 2 & -a \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$ can be done separately for each point	M1	3.1a
	$\begin{pmatrix} 2a-2 & 8+a \\ -3a & -12 \end{pmatrix}$ or $(2a-2,\ -3a)$ and $(8+a, -12)$	A1	1.1b
	$\sqrt{[(2a-2)-(8+a)]^2+[-3a-(-12)]^2} = \sqrt{58}$ or $\overrightarrow{AB} = \begin{pmatrix} 8+a \\ -12 \end{pmatrix} - \begin{pmatrix} 2a-2 \\ -3a \end{pmatrix} = \begin{pmatrix} 10-a \\ -12+3a \end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix} 2a-2 \\ -3a \end{pmatrix} - \begin{pmatrix} 8+a \\ -12 \end{pmatrix} = \begin{pmatrix} a-10 \\ 12-3a \end{pmatrix}$ $(a-10)^2+(12-3a)^2 = 58$ or $(10-a)^2+(3a-12)^2 = 58$ leading to a 3TQ	M1	3.1a
	$10a^2 - 92a + 186 = 0$	A1	1.1b
	$a = 3, \dfrac{31}{5}$ o.e. cso	A1	1.1b
		(5)	
			(9 marks)

Notes:

(i)

M1: Correct method to find the determinant**,** condone a single sign slip but not on second term must be +2 (…)

Note: May expand along any row or column.

A1: Correct simplified determinant

M1: Either

- Finds the value of the discriminant or sufficient working seen to identify the sign e.g. 100 – 192
- Completes the square an rearranges so that $(k \pm a)^2 = -b$
- Completes the square and states that $(k \pm a)^2 \geqslant 0$
- Completes the square and states that $-\alpha(k \pm a)^2 \leqslant 0$
- Differentiates the determinant to find the coordinates of the vertex
- Use the quadratic formula to find the imaginary roots

A1: Correct solution only

Either

- Correct value for the discriminant (may be implied), concludes less than 0, therefore no real roots and non singular.
- Correct completing the square and conclude no real roots as square root of negative therefore non singular
- Correct completing the square and shows > 0 therefore no real roots and non singular.
- Correct completing the square and shows < 0 therefore no real roots and non singular.