5.

$$y = \sin x \sinh x$$

(a) Show that $\dfrac{d^4y}{dx^4} = -4y$

(4)

(b) Hence find the first three non-zero terms of the Maclaurin series for y, giving each coefficient in its simplest form.

(4)

(c) Find an expression for the nth non-zero term of the Maclaurin series for y.

(2)

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2. (a) Use the Maclaurin series expansion for $\cos x$ to determine the series expansion of $\cos^2\left(\frac{x}{3}\right)$ in ascending powers of x, up to and including the term in x^4

Give each term in simplest form.

(2)

(b) Use the answer to part (a) and calculus to find an approximation, to 5 decimal places, for

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{x}\cos^2\left(\frac{x}{3}\right)\right) \mathrm{d}x$$

(3)

(c) Use the integration function on your calculator to evaluate

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(\frac{1}{x}\cos^2\left(\frac{x}{3}\right)\right) \mathrm{d}x$$

Give your answer to 5 decimal places.

(1)

(d) Assuming that the calculator answer in part (c) is accurate to 5 decimal places, comment on the accuracy of the approximation found in part (b).

(1)

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3. $$f(x) = \arcsin x \qquad -1 \leqslant x \leqslant 1$$

(a) Determine the first two non-zero terms, in ascending powers of x, of the Maclaurin series for $f(x)$, giving each coefficient in its simplest form.

(4)

(b) Substitute $x = \frac{1}{2}$ into the answer to part (a) and hence find an approximate value for π

Give your answer in the form $\frac{p}{q}$ where p and q are integers to be determined.

(2)

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9. $$y = \cosh^n x \qquad n \geqslant 5$$

(a) (i) Show that

$$\frac{d^2y}{dx^2} = n^2\cosh^n x - n(n-1)\cosh^{n-2} x$$

(4)

(ii) Determine an expression for $\frac{d^4y}{dx^4}$

(2)

(b) Hence determine the first three non-zero terms of the Maclaurin series for y, giving each coefficient in simplest form.

(2)

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2. (a) Write down the Maclaurin series of e^x, in ascending power of x, up to and including the term in x^3

(1)

(b) Hence, without differentiating, determine the Maclaurin series of

$$e^{(e^x - 1)}$$

in ascending powers of x, up to and including the term in x^3, giving each coefficient in simplest form.

(5)

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