Question	Scheme	Marks	AOs
2	When $n = 1$, $2^{3n+1} + 3\left(5^{2n+1}\right) = 16 + 375 = 391$ $391 = 17 \times 23$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $2^{3k+1} + 3\left(5^{2k+1}\right)$ is divisible by 17	M1	2.4
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 2^{3k+4} + 3\left(5^{2k+3}\right) - 2^{3k+1} - 3\left(5^{2k+1}\right)$	M1	2.1
	$= 7 \times 2^{3k+1} + 7 \times 3\left(5^{2k+1}\right) + 17 \times 3\left(5^{2k+1}\right)$		
	$= 7\mathrm{f}(k) + 17 \times 3\left(5^{2k+1}\right)$	A1	1.1b
	$\mathrm{f}(k+1) = 8\mathrm{f}(k) + 17 \times 3\left(5^{2k+1}\right)$	A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	A1	2.4
		(6)	
			(6 marks)

Notes:

B1: Shows the statement is true for $n = 1$
M1: Assumes the statement is true for $n = k$
M1: Attempts $\mathrm{f}(k+1) - \mathrm{f}(k)$
A1: Correct expression in terms of $\mathrm{f}(k)$
A1: Correct expression in terms of $\mathrm{f}(k)$
A1: Obtains a correct expression for $\mathrm{f}(k + 1)$
A1: Correct complete conclusion

A1*: Puts all the components together to form the given differential equation cso

(b)

M1: Uses the model to find the integrating factor and attempts the solution of the differential equation. Look for $I.F. = \mathrm{e}^{\int \frac{2}{100+t}\mathrm{d}t} \Rightarrow S \times \text{'their } I.F.\text{'} = \int 3 \times \text{'their } I.F.\text{'}\,\mathrm{d}t$

A1: Correct solution condone missing + c

For the next three mark there must be a constant of integration

M1: Interprets the initial conditions, $t = 0$ $S = 0$, and uses in their equation to find the constant of integration.

dM1: Dependent on having a constant of integration. Uses their solution to the problem to find the amount of salt after 10 minutes.

A1: Awrt 27 or $\frac{3310}{121}$. (If the units are stated they must be correct)

<u>Note:</u> If achieves $S(100+t)^2 = 30\,000t + 300t^2 + t^3 + c$ the constant of integration $c = 0$ and the correct amount of salt can be achieved. If there is no + c the maximum they can score is M1A1M0M0A0

Notes continued

(c)

Note: Look out for setting $S = 0.9$ in this part, which scores no marks.

M1: Uses their solution to the model and divides by 100 + t as an interpretation of the concentration and sets = 0.9.

Alternatively recognises that the amount of salt = 0.9(100 + t) and substitutes for S in their solution to the model.

dM1: Dependent on previous method mark. Solves their equation to obtain a value for t. May use a calculator.

A1: Awrt 115 (If the units are stated they must be correct) or 1hr 45 mins with units

(d)

B1: Evaluates the model by making a suitable comment – see scheme for examples.

Question	Scheme	Marks	AOs
6	**<u>Way 1</u>** f(k +1) – f(k)		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145 \times 5$) so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 3^{2k+6} - 2^{2k+2} - 3^{2k+4} + 2^{2k}$	M1	2.1
	either $8\mathrm{f}(k) + 5 \times 2^{2k}$ or $3\mathrm{f}(k) + 5 \times 3^{2k+4}$	A1	1.1b
	$\mathrm{f}(k+1) = 9\mathrm{f}(k) + 5 \times 2^{2k}$ or $\mathrm{f}(k+1) = 4\mathrm{f}(k) + 5 \times 3^{2k+4}$ o.e.	A1	1.1b
	<u>If true for $n = k$ then it is true for</u>	A1	2.4

A1*: Puts all the components together to form the given differential equation cso

(b)

M1: Uses the model to find the integrating factor and attempts the solution of the differential equation. Look for $I.F. = \mathrm{e}^{\int \frac{2}{100+t}\,\mathrm{d}t} \Rightarrow S \times \text{'their } I.F.\text{'} = \int 3 \times \text{'their } I.F.\text{'}\,\mathrm{d}t$

A1: Correct solution condone missing + c

For the next three mark there must be a constant of integration

M1: Interprets the initial conditions, $t = 0\ \ S = 0$, and uses in their equation to find the constant of integration.

dM1: Dependent on having a constant of integration. Uses their solution to the problem to find the amount of salt after 10 minutes.

A1: Awrt 27 or $\frac{3310}{121}$. (If the units are stated they must be correct)

<u>**Note:**</u> If achieves $S(100+t)^2 = 30\,000t + 300t^2 + t^3 + c$ the constant of integration $c = 0$ and the correct amount of salt can be achieved. If there is no + c the maximum they can score is M1A1M0M0A0

Notes continued

(c)

Note: Look out for setting $S = 0.9$ in this part, which scores no marks.

M1: Uses their solution to the model and divides by 100 + t as an interpretation of the concentration and sets = 0.9.

Alternatively recognises that the amount of salt = 0.9(100 + t) and substitutes for S in their solution to the model.

dM1: Dependent on previous method mark. Solves their equation to obtain a value for t. May use a calculator.

A1: Awrt 115 (If the units are stated they must be correct) or 1hr 45 mins with units

(d)

B1: Evaluates the model by making a suitable comment – see scheme for examples.

Question	Scheme	Marks	AOs
6	<u>**Way 1**</u> f(k +1) – f(k)		
	When n = 1, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145 \times 5$) so the statement is true for n = 1	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 3^{2k+6} - 2^{2k+2} - 3^{2k+4} + 2^{2k}$	M1	2.1
	either $8\mathrm{f}(k) + 5 \times 2^{2k}$ or $3\mathrm{f}(k) + 5 \times 3^{2k+4}$	A1	1.1b
	$\mathrm{f}(k+1) = 9\mathrm{f}(k) + 5 \times 2^{2k}$ or $\mathrm{f}(k+1) = 4\mathrm{f}(k) + 5 \times 3^{2k+4}$ o.e.	A1	1.1b
	<u>If true for $n = k$ then it is true for</u>	A1	2.4

	$n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')		
		(6)	
	Way 2 $f(k+1)$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145 \times 5$) so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$f(k+1) = 3^{2(k+1)+4} - 2^{2(k+1)} \left(= 3^{2k+6} - 2^{2k+2}\right)$	M1	2.1
	$f(k+1) = 9f(k) + 5 \times 2^{2k}$ or $f(k+1) = 4f(k) + 5 \times 3^{2k+4}$ o.e.	A1 A1	1.1b 1.1b
	If true for $n = k$ then it is true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')	A1	2.4
		(6)	
	Way 3 $f(k) = 5M$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145 \times 5$) so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k} = 5M$	M1	2.4
	$f(k+1) = 3^{2(k+1)+4} - 2^{2(k+1)} \left(= 3^{2k+6} - 2^{2k+2}\right)$	M1	2.1
	$\left(f(k+1) = 3^2 \times 3^{2k+4} - 2^2 \times 2^{2k} = 3^2 \times \left(5M + 2^{2k+2}\right) - 2^2 \times 2^{2k}\right)$ $f(k+1) = 45M + 5 \times 2^{2k}$ o.e. OR $\left(f(k+1) = 3^2 \times 3^{2k+4} - 2^2 \times 2^{2k} = 3^2 \times 3^{2k+4} - 2^2 \times \left(3^{2k+4} - 5M\right)\right)$ $f(k+1) = 5 \times 3^{2k+4} + 20M$ o.e.	A1 A1	1.1b 1.1b
	If true for $n = k$ then it is true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all (positive integers) n. (Allow 'for all values')	A1	2.4
		(6)	
	Way 4 $f(k+1) + f(k)$		
	When $n = 1$, $3^{2n+4} - 2^{2n} = 729 - 4 = 725$ ($725 = 145 \times 5$) so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $3^{2k+4} - 2^{2k}$ is divisible by 5	M1	2.4
	$f(k+1) + f(k) = 3^{2k+6} - 2^{2k+2} + 3^{2k+4} - 2^{2k}$	M1	2.1
	$f(k+1) + f(k) = 3^2 \times 3^{2k+4} - 2^2 \times 2^{2k} + 3^{2k+4} - 2^{2k}$	A1	1.1b

Question	Scheme	Marks	AOs
6(i)	When $n = 1$, $\sum_{r=1}^{1}(3r+1)(r+2) = 4\times 3 = 12$ $1(1+2)(1+3) = 12$ (so the statement is true for $n = 1$)	B1	2.2a
	Assume true for $n = k$ so $\sum_{r=1}^{k}(3r+1)(r+2) = k(k+2)(k+3)$	M1	2.4
	$\sum_{r=1}^{k+1}(3r+1)(r+2) = k(k+2)(k+3)+(3k+4)(k+3)$	M1	2.1
	$=(k+3)(k^2+5k+4)$	A1	1.1b
	$\sum_{r=1}^{k+1}(3r+1)(r+2) = (k+1)(k+3)(k+4)$	A1	1.1b
	$\sum_{r=1}^{k+1}(3r+1)(r+2) = (\underline{k+1})(\underline{k+1}+2)(\underline{k+1}+3)$ If the statement is <u>true for *n* = *k* then</u> it has been shown <u>true for *n* = *k* + 1</u> and as it is <u>true for *n* = 1</u>, the statement <u>is true for all (positive integers) *n*.</u>	A1	2.4
		(6)	
(ii) **Way 1**	When $n = 1$, $4^1+5^1+6^1 = 15$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^k+5^k+6^k$ is divisible by 15	M1	2.4
	$f(k+2) = 4^{k+2}+5^{k+2}+6^{k+2}$	M1	2.1
	$=16\times 4^k+16\times 5^k+16\times 6^k+9\times 5^k+20\times 6^k$ $=16f(k)+45\times 5^{k-1}+120\times 6^{k-1}$	A1 A1	1.1b 1.1b
	E.g As 15 divides f(*k*), 45 and 120, so 15 divides f(*k*+1). <u>If true for *n* = *k*</u> then <u>true for *n* = *k* + 2</u>, <u>true for *n* = 1</u> so <u>true for all positive odd integers *n*</u>	A1	2.4
		(6)	
(ii) **Way 2**	When $n = 1$, $4^1+5^1+6^1 = 15$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^k+5^k+6^k$ is divisible by 15	M1	2.4
	$f(k+2)-f(k) = 4^{k+2}+5^{k+2}+6^{k+2}-4^k-5^k-6^k$	M1	2.1
	$=15\times 4^k+24\times 5^k+35\times 6^k$ $=15f(k)+45\times 5^{k-1}+120\times 6^{k-1}$	A1	1.1b
	$f(k+2)=16f(k)+45\times 5^{k-1}+120\times 6^{k-1}$	A1	1.1b
	E.g $f(k+2)=16f(k)+15(3\times 5^{k-1}+8\times 6^{k-1})$ so <u>if true for *n* = *k*</u> then <u>true for *n* = *k* + 2</u>, <u>true for *n* = 1</u> so <u>true for all positive odd integers *n*</u>	A1	2.4
		(6)	

Question	Scheme	Marks	AOs
6	$\frac{dy}{dx} = 2e^{2x}\sinh x + e^{2x}\cosh x = ae^{2x}\sinh x + be^{2x}\cosh x$ or $e^{2x}(a\sinh x + b\cosh x)$	**M1**	2.2a
	$\frac{dy}{dx} = e^{2x}(2\sinh x + \cosh x)$ n = 1 then $\frac{dy}{dx} = e^{2x}\left(\frac{3+1}{2}\sinh x + \frac{3-1}{2}\cosh x\right)$ {so the result is true for n = 1}	**A1**	2.4
	(Assume the result is true for n = k, then) Must be an attempt at the product rule, with k's in all terms $\frac{d^{k+1}y}{dx^{k+1}} = Ae^{2x}(f(k)\sinh x + g(k)\cosh x) + e^{2x}(f(k)\cosh x + g(k)\sinh x)$ $\frac{d^{k+1}y}{dx^{k+1}} = 2e^{2x}\left(\frac{3^k+1}{2}\sinh x + \frac{3^k-1}{2}\cosh x\right) + e^{2x}\left(\frac{3^k+1}{2}\cosh x + \frac{3^k-1}{2}\sinh x\right)$	**M1**	2.1
	$=e^{2x}\left(\left(3^k+1+\frac{3^k-1}{2}\right)\sinh x + \left(3^k-1+\frac{3^k+1}{2}\right)\cosh x\right)$ or $= e^{2x}\left(\frac{3\times 3^k+1}{2}\sinh x + \frac{3\times 3^k-1}{2}\cosh x\right)$	**dM1**	1.1b
	$= e^{2x}\left(\frac{3^{k+1}+1}{2}\sinh x + \frac{3^{k+1}-1}{2}\cosh x\right)$	**A1**	2.1
	<u>If true for n = k then true for n = k + 1</u>, and as also <u>true for n = 1</u>, so the result is <u>true for all positive integers</u> or <u>true $n \in \mathbb{N}$</u>	**A1**	2.4
		(6)	
	Alternative using exponential definitions $y = e^{2x}\sinh x \Rightarrow y = e^{2x}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{1}{2}(e^{3x} - e^x)$ $\frac{dy}{dx} = \frac{1}{2}(3e^{3x} - e^x)$	**M1**	2.2a
	n = 1 then	**A1**	2.4

Question	Scheme	Marks	AOs
8(i)	$n=1, \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^1 = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}, \quad \begin{pmatrix}4\times1+1 & -8(1)\\ 2\times1 & 1-4(1)\end{pmatrix} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ So the result is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^k = \begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}$	M1	2.4
	$\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}$ or $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}^{k+1} = \begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}\begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}$	M1	1.1b
	$\begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix}\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix} = \begin{pmatrix}5(4k+1)-16k & -8(4k+1)+24k\\ 10k+2(1-4k) & -16k-3(1-4k)\end{pmatrix}$ or $\begin{pmatrix}5 & -8\\ 2 & -3\end{pmatrix}\begin{pmatrix}4k+1 & -8k\\ 2k & 1-4k\end{pmatrix} = \begin{pmatrix}5(4k+1)-16k & -40k-8(1-4k)\\ 2(1+4k)-6k & -16k-3(1-4k)\end{pmatrix}$	A1	1.1b
	$= \begin{pmatrix}4(k+1)+1 & -8(k+1)\\ 2(k+1) & 1-4(k+1)\end{pmatrix}$	A1	2.1
	If true for $n = k$ then true for $n = k + 1$, true for $n = 1$ so true for all (positive integers) n (Allow "for all values")	A1	2.4
		(6)	
(ii) Way 1	$f(k + 1) - f(k)$		
	When $n = 1$, $4^{n+1} + 5^{2n-1} = 16 + 5 = 21$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $4^{k+1} + 5^{2k-1}$ is divisible by 21	M1	2.4
	$f(k+1) - f(k) = 4^{k+2} + 5^{2k+1} - 4^{k+1} - 5^{2k-1}$	M1	2.1
	$= 4\times4^{k+1} + 25\times5^{2k-1} - 4^{k+1} - 5^{2k-1}$		
	$= 3f(k) + 21\times5^{2k-1}$ or e.g. $= 24f(k) - 21\times4^{k+1}$	A1	1.1b
	$f(k+1) = 4f(k) + 21\times5^{2k-1}$ or e.g. $f(k+1) = 25f(k) - 21\times4^{k+1}$	A1	1.1b
	If true for $n = k$ then true for $n = k + 1$, true for $n = 1$ so true for all (positive integers) n (Allow "for all values")	A1	2.4
		(6)	

Question	Scheme	Marks	AOs
3	$n=1,\ \sum_{r=1}^{1}\frac{1}{(2r-1)(2r+1)}=\frac{1}{1\times3}=\frac{1}{3}$ and $\frac{n}{2n+1}=\frac{1}{2\times1+1}=\frac{1}{3}$ (true for n=1)	B1	2.2a
	Assume general statement is true for $n=k$. So assume $\sum_{r=1}^{k}\frac{1}{(2r-1)(2r+1)}=\frac{k}{2k+1}$ is true.	M1	2.4
	$\left(\sum_{r=1}^{k+1}\frac{1}{(2r-1)(2r+1)}=\right)\text{"}\frac{k}{2k+1}\text{"}+\frac{1}{(2k+1)(2k+3)}$	M1	2.1
	$=\frac{k(2k+3)+1}{(2k+1)(2k+3)}$	dM1	1.1b
	$=\frac{2k^2+3k+1}{(2k+1)(2k+3)}=\frac{(2k+1)(k+1)}{(2k+1)(2k+3)}=\frac{(k+1)}{2(k+1)+1}$ or $\frac{k+1}{2k+3}$	A1	1.1b
	As $\sum_{r=1}^{k+1}\frac{1}{(2r-1)(2r+1)}=\frac{(k+1)}{2(k+1)+1}$ then the general result is true for $n=k+1$ As the general result has been shown to be <u>true for $n=1$</u>, and <u>true for $n=k$ implies true for $n=k+1$</u>, so the result <u>is true for all n</u> $\in\mathbb{N}$	A1cso	2.4
		(6)	
	(6 marks)		

Notes

	B1	Substitutes $n=1$ into both sides of the statement to show they are eqaul. As a minimum expect to see $\frac{1}{1\times3}$ and $\frac{1}{2+1}$ for the substitutions. (No need to state true for n = 1 for this mark.)
	M1	Assumes (general result) true for $n=k$. (Assume (true for) n = k is sufficient – note that this may be recovered in their conclusion if they say e.g. if true for n = k then ... etc.)
	M1	Attempts to add $(k+1)$th term to their sum of k terms. Must be adding the (k+1)th term but allow slips with the sum.
	dM1	Depends on previous M. Combines their two fractions over a correct common denominator for their fractions, which may be $(2k+1)^2(2k+3)$ (allow a slip in the numerator).
	A1	Correct algebraic work leading to $\frac{(k+1)}{2(k+1)+1}$ or $\frac{k+1}{2k+3}$
	A1	**cso** Depends on all except the **B** mark being scored (but must have an attempt to show the n = 1 case). Demonstrates the expression is the correct for n = k + 1 (both sides must have been seen somewhere) and gives a correct induction statement with **all** three underlined statements (or equivalents) seen at some stage during their solution (so true for n = 1 may be seen at the start). For demonstrating the correct expression, accept giving in the form $\frac{(k+1)}{2(k+1)+1}$, or reaching $\frac{k+1}{2k+3}$ and stating "which is the correct form with n = k + 1" or similar – but some indication is needed. Note: if mixed variables are used in working (r's and k's mixed up) then withhold the final A. Note: If n is used throughout instead of k allow all marks if earned.

Question	Scheme	Marks	AOs
8	**Way 1: $\mathbf{f}(\boldsymbol{k}+1)-\mathbf{f}(\boldsymbol{k})$** When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ Shows the statement is true for $n=1$, allow 5(7)	B1	2.2a
	Assume true for $\boldsymbol{n}=\boldsymbol{k}$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\text{f}(k+1)-\text{f}(k)=2^{k+3}+3^{2k+3}-\left(2^{k+2}+3^{2k+1}\right)$	M1	2.1
	$=2\times2^{k+2}+9\times3^{2k+1}-2^{k+2}-3^{2k+1}$ $=2^{k+2}+8\times3^{2k+1}$ $=\text{f}(k)+7\times3^{2k+1}$ or $8\text{f}(k)-7\times2^{k+2}$	A1	1.1b
	$\text{f}(k+1)=2\text{f}(k)+7\times3^{2k+1}$ or $9\text{f}(k)-7\times2^{k+2}$	A1	1.1b
	If **true for $\boldsymbol{n}=\boldsymbol{k}$** then **true for $\boldsymbol{n}=\boldsymbol{k}+1$** and as it is **true for $\boldsymbol{n}=1$** the statement is **true for all** (positive integers) ***n***	A1	2.4
		(6)	
	Way 2: $\mathbf{f}(\boldsymbol{k}+1)$ When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\text{f}(k+1)=2^{(k+1)+2}+3^{2(k+1)+1}$	M1	2.1
	$\text{f}(k+1)=2^{k+3}+3^{2k+3}=2\times2^{k+2}+9\times3^{2k+1}$ $=2\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}$ $=2\text{f}(k)+7\times3^{2k+1}$or $9\text{f}(k)-7\times2^{k+2}$	A1 A1	1.1b 1.1b
	If true for $\boldsymbol{n}=\boldsymbol{k}$ then **true for $\boldsymbol{n}=\boldsymbol{k}+1$** and as it is **true for $\boldsymbol{n}=1$** the statement is **true for all** (positive integers) ***n***	A1	2.4
		(6)	
	Way 3: $\mathbf{f}(\boldsymbol{k}+1)-\boldsymbol{m}\,\mathbf{f}(\boldsymbol{k})$ When $n=1$, $2^{n+2}+3^{2n+1}=2^3+3^3=35$ So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$, so $2^{k+2}+3^{2k+1}$ is divisible by 7	M1	2.4
	$\text{f}(k+1)-m\text{f}(k)=2^{k+3}+3^{2k+3}-m\left(2^{k+2}+3^{2k+1}\right)$	M1	2.1
	$=2\times2^{k+2}+9\times3^{2k+1}-m\times2^{k+2}-m\times3^{2k+1}$ $=(2-m)2^{k+2}+9\times3^{2k+1}-m\times3^{2k+1}$ $=(2-m)\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}$	A1	1.1b
	$\text{f}(k+1)=(2-m)\left(2^{k+2}+3^{2k+1}\right)+7\times3^{2k+1}+m\text{f}(k)$	A1	1.1b

M1: Uses (z – 3) and (z – their δ) and their conjugate pair correctly as factors and makes an attempt to expand Alternatively attempts to find the pair sum, triple sum and product A1: Establishes at least 2 of the required coefficients correctly A1: Correct quartic or correct constants (d) B1ft: For $-\frac{3}{2}$ and $-\frac{\delta}{2}$ as the real roots B1ft: For $-1-\frac{\mathrm{i}}{2}$ and $-\frac{\gamma}{2}$ as the complex roots

Question	Scheme	Marks	AOs
8(a)	$n=1,\quad \text{lhs}=1(2)(3)=6,\quad \text{rhs}=\frac{1}{2}(1)(2)^2(3)=6$ (true for n = 1)	B1	2.2a
	Assume true for $n = k$ so $\sum_{r=1}^{k} r(r+1)(2r+1)=\frac{1}{2}k(k+1)^2(k+2)$	M1	2.4
	$\sum_{r=1}^{k+1} r(r+1)(2r+1)=\frac{1}{2}k(k+1)^2(k+2)+(k+1)(k+2)(2k+3)$	M1	2.1
	$=\frac{1}{2}(k+1)(k+2)\left[k(k+1)+2(2k+3)\right]$	dM1	1.1b
	$=\frac{1}{2}(k+1)(k+2)\left[k^2+5k+6\right]=\frac{1}{2}(k+1)(k+2)(k+2)(k+3)$ Shows that $=\frac{1}{2}(\underline{k+1})(\underline{k+1}+1)^2(\underline{k+1}+2)$ Alternatively shows that $\sum_{r=1}^{k+1} r(r+1)(2r+1)=\frac{1}{2}(k+1)(k+1+1)^2(k+1+2)$ $=\frac{1}{2}(k+1)(k+2)^2(k+3)$ Compares with their summation and concludes true for $n = k$ +1, may be seen in the conclusion.	A1	1.1b
	If the statement is **true for *n* = *k* then** it has been shown **true for *n* = *k* + 1** and as it is **true for *n* = 1**, the statement is **true** for all **positive integers *n*.**	A1	2.4
		(6)	
(b)	$\sum_{r=n}^{2n} r(r+1)(2r+1)=\frac{1}{2}(2n)(2n+1)^2(2n+2)-\frac{1}{2}(n-1)n^2(n+1)$	M1	3.1a
	$=\frac{1}{2}n(n+1)\left[4(2n+1)^2-n(n-1)\right]$	M1	1.1b
	$=\frac{1}{2}n(n+1)(15n^2+17n+4)$ $=\frac{1}{2}n(n+1)(3n+1)(5n+4)$	A1	1.1b
		(3)	
			(9 marks)

Question	Scheme	Marks	AOs
7	For $n=1$: $\begin{pmatrix}1-6\times1 & 9\times1\\ -4\times1 & 1+6\times1\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^1$ So the statement is true for $n=1$	B1	2.2a
	Assume true for $\boldsymbol{n=k}$, or **Assume** $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k=\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}$	M1	2.5
	$\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^{k+1}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$ OR $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k$	M1	2.1
	$=\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}=\begin{pmatrix}-5+30k-36k & 9-54k+63k\\ 20k-4-24k & -36k+7+42k\end{pmatrix}$ OR $=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}=\begin{pmatrix}-5+30k-36k & -45k+9+54k\\ -4+24k-28k & -36k+7+42k\end{pmatrix}$	M1	1.1b
	Achieves from fully correct working $=\begin{pmatrix}-5-6k & 9+9k\\ -4-4k & 7+6k\end{pmatrix}$	A1	1.1b
	$=\begin{pmatrix}1-6(k+1) & 9(k+1)\\ -4(k+1) & 1+6(k+1)\end{pmatrix}$ Hence the result is true for $n=k+1$. Since it is true for $n=1$, and if true for $n=k$ then true for $n=k+1$, thus by mathematical induction the result holds for all $n \in \mathbb{N}$	A1cso	2.4
		(6)	
		(6 marks)	

Notes:

(a)

B1: Shows the statement is true for $n=1$. Accept as minimum $\begin{pmatrix}1-6 & 9\\ -4 & 1+6\end{pmatrix}=\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$

M1: Makes the inductive assumption, **assume** true $\boldsymbol{n=k}$**.** This may appear in the conclusion.

M1: A correct statement for $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^{k+1}$ in terms of $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}^k$, can be either way round.

Can be implied by $\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}\times\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}$ or $\begin{pmatrix}-5 & 9\\ -4 & 7\end{pmatrix}\times\begin{pmatrix}1-6k & 9k\\ -4k & 1+6k\end{pmatrix}$

M1: Carries out the multiplication correctly, condone sign slips

A1: Correct simplified matrix **from fully correct working**

A1: Completes the inductive argument by showing clearly the matrix has the correct form (must have $(k+1)$ factors in terms) or uses the result with $n=k+1$ and shows that their result is the same.

Conclusion conveying **all** three underlined points or equivalent at some point in their argument. Depends on all three M's and A marks but can be scored without the B mark as long as it is stated true for $n = 1$