OCR
Oxford Cambridge and RSA

# Wednesday 07 October 2020 – Afternoon

## A Level Mathematics B (MEI)

**H640/01** Pure Mathematics and Mechanics

**Time allowed: 2 hours**

*8168829224*

**You must have:**
- the Printed Answer Booklet
- a scientific or graphical calculator

**INSTRUCTIONS**
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

**ADVICE**
- Read each question carefully before you start your answer.

## Formulae A Level Mathematics B (MEI) (H640)

### Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

### Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

### Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

### Differentiation

| $f(x)$ | $f'(x)$ |
|---|---|
| $\tan kx$ | $k\sec^2 kx$ |
| $\sec x$ | $\sec x \tan x$ |
| $\cot x$ | $-\operatorname{cosec}^2 x$ |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

### Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

### Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\left(f(x)\right)^n dx = \frac{1}{n+1}\left(f(x)\right)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

### Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where $\theta$ is measured in radians

**Trigonometric identities**

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$

**Numerical methods**

Trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

**Probability**

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

**Sample variance**

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

**The binomial distribution**

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1 - p$

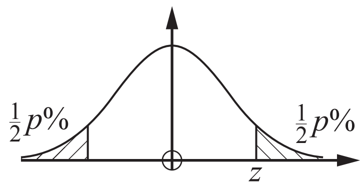
Mean of $X$ is $np$

**Hypothesis testing for the mean of a Normal distribution**

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

**Percentage points of the Normal distribution**

| $p$ | 10 | 5 | 2 | 1 |
|---|---|---|---|---|
| $z$ | 1.645 | 1.960 | 2.326 | 2.576 |




**Kinematics**

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

**Section A** (22 marks)

**1** Simplify $\left(\frac{27}{x^9}\right)^{\frac{2}{3}} \times \left(\frac{x^4}{9}\right)$. **[2]**

**2** Express $\frac{a+\sqrt{2}}{3-\sqrt{2}}$ in the form $p+q\sqrt{2}$, giving $p$ and $q$ in terms of $a$. **[3]**

**3** The points A and B have position vectors $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \\ 8 \end{pmatrix}$ respectively.

Show that the exact value of the distance AB is $\sqrt{101}$. **[3]**

**4** Find the second derivative of $(x^2+5)^4$, giving your answer in factorised form. **[5]**

**5** A child is running up and down a path. A simplified model of the child's motion is as follows:

- he first runs north for 5 s at $4\,\text{m}\,\text{s}^{-1}$;
- he then suddenly stops and waits for 8 s;
- finally he runs in the opposite direction for 7 s at $3.5\,\text{m}\,\text{s}^{-1}$.

**(a)** Taking north to be the positive direction, sketch a velocity-time graph for this model of the child's motion. **[2]**

Using this model,

**(b)** calculate the total distance travelled by the child, **[2]**

**(c)** find his final displacement from his original position. **[1]**

**6** A uniform ruler AB has mass 28 g and length 30 cm. As shown in Fig. 6, the ruler is placed on a horizontal table so that it overhangs a point C at the edge of the table by 25 cm.

A downward force of $F$ N is applied at A. This force just holds the ruler in equilibrium so that the contact force between the table and the ruler acts through C.


**Fig. 6**

**(a)** Complete the force diagram in the Printed Answer Booklet, labelling the forces and all relevant distances. **[2]**

**(b)** Calculate the value of $F$. **[2]**

Answer **all** the questions.

**Section B** (78 marks)

**7** **In this question you must show detailed reasoning.**

The function $f(x)$ is defined by $f(x) = x^3 + x^2 - 8x - 12$ for all values of $x$.

**(a)** Use the factor theorem to show that $(x+2)$ is a factor of $f(x)$. **[2]**

**(b)** Solve the equation $f(x) = 0$. **[4]**

**8** Fig. 8.1 shows the cross-section of a straight driveway 4 m wide made from tarmac.

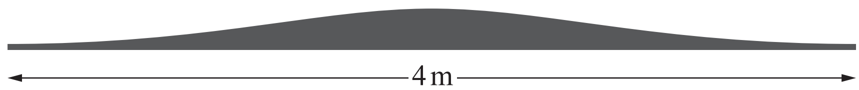



**Fig. 8.1**

The height $h$ m of the cross-section at a displacement $x$ m from the middle is modelled by $h = \dfrac{0.2}{1+x^2}$ for $-2 \leqslant x \leqslant 2$.

A lower bound of $0.3615\,\text{m}^2$ is found for the area of the cross-section using rectangles as shown in Fig. 8.2.

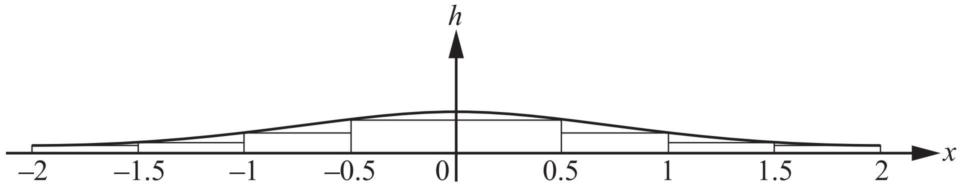



**Fig. 8.2**

**(a)** Use a similar method to find an upper bound for the area of the cross-section. **[3]**

**(b)** Use the trapezium rule with 4 strips to estimate $\displaystyle\int_0^2 \frac{0.2}{1+x^2}\,dx$. **[2]**

**(c)** The driveway is 10 m long. Use your answer in part **(b)** to find an estimate of the volume of tarmac needed to make the driveway. **[2]**

**9** A particle is moving in a straight line. The acceleration $a\,\mathrm{m\,s^{-2}}$ of the particle at time $t$ s is given by $a = 0.8t + 0.5$. The initial velocity of the particle is $3\,\mathrm{m\,s^{-1}}$ in the positive $x$-direction.

Determine whether the particle is ever stationary. **[6]**

**10** **In this question you must show detailed reasoning.**

Fig. 10 shows the curve given parametrically by the equations $x = \frac{1}{t^2}$, $y = \frac{1}{t^3} - \frac{1}{t}$, for $t > 0$.

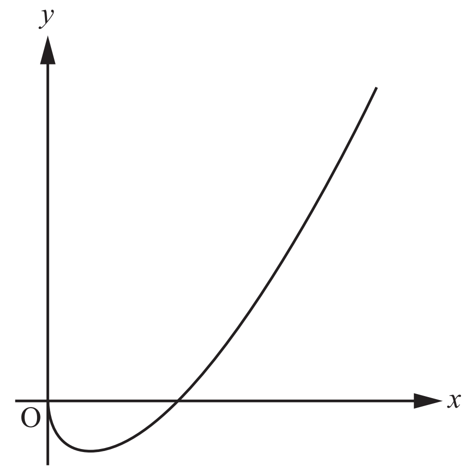



**Fig. 10**

**(a)** Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3 - t^2}{2t}$. **[3]**

**(b)** Find the coordinates of the point on the curve at which the tangent to the curve is parallel to the line $4y + x = 1$. **[3]**

**(c)** Find the cartesian equation of the curve. Give your answer in factorised form. **[3]**

**11** A block of mass 2 kg is placed on a rough horizontal table. A light inextensible string attached to the block passes over a smooth pulley attached to the edge of the table. The other end of the string is attached to a sphere of mass 0.8 kg which hangs freely.

The part of the string between the block and the pulley is horizontal. The coefficient of friction between the table and the block is 0.35. The system is released from rest.

**(a)** Draw a force diagram showing all the forces on the block and the sphere. **[3]**

**(b)** Write down the equations of motion for the block and the sphere. **[2]**

**(c)** Show that the acceleration of the system is $0.35\,\mathrm{m\,s^{-2}}$. **[4]**

**(d)** Calculate the time for the block to slide the first 0.5 m. Assume the block does not reach the pulley. **[2]**

**12** A function is defined by $\mathrm{f}(x) = x^3 - x$.

**(a)** By considering $\dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$, show from first principles that $\mathrm{f}'(x) = 3x^2 - 1$. **[4]**

**(b)** Sketch the gradient function $\mathrm{f}'(x)$. **[2]**

**(c)** Show that the curve $y = \mathrm{f}(x)$ has a single point of inflection which is not a stationary point. **[3]**

**13** A projectile is fired from ground level at $35\,\mathrm{m\,s^{-1}}$ at an angle of $\theta°$ above the horizontal.

**(a)** State a modelling assumption that is used in the standard projectile model. **[1]**

**(b)** Find the cartesian equation of the trajectory of the projectile. **[4]**

The projectile travels above horizontal ground towards a wall that is 110 m away from the point of projection and 5 m high. The projectile reaches a maximum height of 22.5 m.

**(c)** Determine whether the projectile hits the wall. **[6]**

**14** Douglas wants to construct a model for the height of the tide in Liverpool during the day, using a cosine graph to represent the way the height changes.

He knows that the first high tide of the day measures 8.55 m and the first low tide of the day measures 1.75 m.

Douglas uses $t$ for time and $h$ for the height of the tide in metres. With his graph-drawing software set to degrees, he begins by drawing the graph of $h = 5.15 + 3.4\cos t$.

**(a)** Verify that this equation gives the correct values of $h$ for the high and low tide. **[1]**

Douglas also knows that the first high tide of the day occurs at 1 am and the first low tide occurs at 7.20 am. He wants $t$ to represent the time in hours after midnight, so he modifies his equation to $h = 5.15 + 3.4\cos(at + b)$.

**(b) (i)** Show that Douglas's modified equation gives the first high tide of the day occurring at the correct time if $a + b = 0$. **[1]**

**(ii)** Use the time of the first low tide of the day to form a second equation relating $a$ and $b$. **[1]**

**(iii)** Hence show that $a = 28.42$ correct to 2 decimal places. **[2]**

**(c)** Douglas can only sail his boat when the height of the tide is at least 3 m.

Use the model to predict the range of times that morning when he cannot sail. **[3]**

**(d)** The next high tide occurs at 12.59 pm when the height of the tide is 8.91 m.

Comment on the suitability of Douglas's model. **[2]**

**15** Fig. 15 shows a particle of mass $m$ kg on a smooth plane inclined at 30° to the horizontal. Unit vectors **i** and **j** are parallel and perpendicular to the plane, in the directions shown.

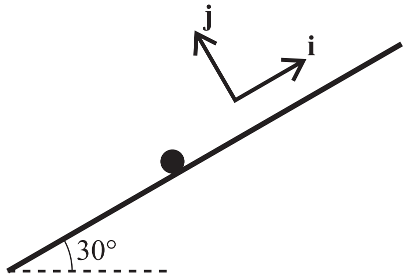



**Fig. 15**

**(a)** Express the weight **W** of the particle in terms of $m$, $g$, **i** and **j**. **[2]**

The particle is held in equilibrium by a force **F**, and the normal reaction of the plane on the particle is denoted by **R**. The units for both **F** and **R** are newtons.

**(b)** Write down an equation relating **W**, **R** and **F**. **[1]**

**(c)** Given that $\mathbf{F} = 6\mathbf{i} + 8\mathbf{j}$,

- show that $m = 1.22$ correct to 3 significant figures,
- find the magnitude of **R**. **[6]**

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