Mark Scheme (Results)

June 2015

Pearson Edexcel International A Level in Statistics 2 (WST02/01)

Edexcel and BTEC Qualifications

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- d… or dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper or ag- answer given
- ⎿ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

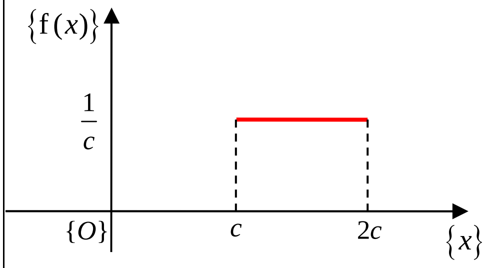
June 2015 WMST02/01
Statistics 2 Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\{\mathrm{P}(X>4)=\}\,1-\mathrm{F}(4)$ $1-\mathrm{F}(4)$ seen or used	M1
	$\left\{=1-\frac{3}{5}\right\}=\frac{2}{5}$ $\frac{2}{5}$ or 0.4	A1
		[2]
(b)	$\mathrm{P}(3<X<a)=0.642$	
	$\mathrm{F}(a)-\mathrm{F}(3)=0.642$ $\mathrm{F}(a)-\mathrm{F}(3)=0.642$	M1 o.e.
	$\mathrm{F}(a)-\frac{1}{20}(3^2-4)=0.642\ \{\Rightarrow \mathrm{F}(a)=0.892\}$ Correct equation	A1 o.e.
	$\frac{1}{5}(2a-5)-\text{"}\frac{1}{20}(3^2-4)\text{"}=0.642\Rightarrow a=...$ Solving this equation o.e., leading to $a=...$ (or $x=...$). Follow through their F(3)	dM1
	$\left\{\frac{1}{5}(2a-5)=0.892\ \Rightarrow\right\}\ a=4.73$ $a=4.73$ (or $x=4.73$)	A1 **cao**
		[4]
(b)	**Alternative Method for Part (b)**	
	$\int_{3}^{4}\left(\frac{1}{10}x\right)\{\mathrm{d}x\}$ Correct expression for finding the probability between $x=3$ and $x=4$	M1
	$\left\{=\left[\frac{x^2}{20}\right]_3^4\right\}=\frac{4^2}{20}-\frac{3^2}{20}\ \left\{=\frac{7}{20}\right\}$ Correct $\frac{4^2}{20}-\frac{3^2}{20}$, simplified or un-simplified.	A1
	$\int_{3}^{4}\left(\frac{1}{10}x\right)\{\mathrm{d}x\}+\int_{4}^{a}\left(\frac{2}{5}\right)\{\mathrm{d}x\}=0.642\Rightarrow a=...$ Writes a correct equation and attempts to solve leading to $a=...$ (or $x=...$)	dM1
	$\left\{\frac{7}{20}+\frac{2}{5}a-\frac{8}{5}=0.642\ \Rightarrow\right\}\ a=4.73$ $a=4.73$ (or $x=4.73$)	A1 **cao**
		[4]
(c)	$\mathrm{f}(x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{20}(x^2-4)\right)=\frac{1}{10}x$ $\mathrm{f}(x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{5}(2x-5)\right)=\frac{2}{5}$ Attempt at differentiation. **See notes.**	M1
	At least one of $\frac{1}{10}x$ or $\frac{2}{5}$	A1
	Both $\frac{1}{10}x$ and $\frac{2}{5}$	A1
	$\mathrm{f}(x)=\begin{cases}\frac{1}{10}x, & 2\leqslant x\leqslant 4\\ \frac{2}{5}, & 4<x\leqslant 5\\ 0, & \text{otherwise}\end{cases}$ **This mark is dependent on M1** All three lines with limits correctly followed through from their $\mathrm{F}'(x)$	dB1ft
		[4]
		10

	Question 1 Notes	
1. (a)	**M1**	$1-\text{F}(4)$ seen or used.
	Note	Can be implied by either $1-\frac{3}{5}$ or $1-\frac{1}{5}\left(2(4)-5\right)$ or $1-\frac{1}{20}\left(4^2-4\right)$ The probability statements $1-\text{P}(X\leqslant 4)$ or $1-\text{P}(X<4)$ are not sufficient for M1
	A1	$\frac{2}{5}$ or 0.4
	Note	Give M1A1 for the correct answer from no working.
(b)	**NOTE**	In part (b), candidates are allowed to write - $\text{F}(a)$ as either $\text{P}(X<a)$ or $\text{P}(X\leqslant a)$. Also condone $\text{F}(a)$ written as $\text{F}(x)$ - $\text{F}(3)$ as either $\text{P}(X<3)$ or $\text{P}(X\leqslant 3)$
	M1	For writing $\text{F}(a)-\text{F}(3)=0.642$ or equivalent (see NOTE above)
	A1	For an un-simplified $\text{F}(a)-\frac{1}{20}(3^2-4)=0.642$ or equivalent (see NOTE above)
	Note	Give 1st M1 1st A1 for $\text{F}(a)=0.892$ or $\text{P}(X\geqslant a)=0.108$
	SC	Allow SC 1st M1 1st A1 for $\frac{1}{20}(a^2-4)-\frac{1}{20}(3^2-4)=0.642$
	Note	Give 1st M0 for $\text{F}(a-1)-\text{F}(3)=0.642$ o.e. without a correct acceptable statement
	dM1	**dependent on the FIRST method mark being awarded.** Attempts to solve $\frac{1}{5}(2a-5)-\text{"their F(3)"}=0.642$ leading to $a=\ldots$(or $x=\ldots$)
	Note	dM1 can be given for either $\frac{1}{5}(2a-5)=0.892$ or $1-\frac{1}{5}(2a-5)=0.108$ leading to $a=\ldots$(or $x=\ldots$)
	A1	$a=4.73$ (or $x=4.73$) **cao**
	Note	Give M0A0M0A0 for $\text{F}(a)-(1-\text{F}(3))=0.642$ $\left\{\Rightarrow \text{F}(a)=1.392\right\}$
	Note	Give M0A0M0A0 for $\int_3^a\left(\frac{1}{10}x\right)\text{d}x=0.642$ (this solves to give awrt 4.67)
(c)	**M1**	At least one of either $\frac{1}{20}(x^2-4)\rightarrow \pm\alpha x\pm\beta,\ \alpha\neq 0,\beta$ can be 0 $\frac{1}{5}(2x-5)\rightarrow\pm\delta,\ \delta\neq 0$
	1st A1	At least one of $\frac{1}{10}x$ or $\frac{2}{5}$. Can be simplified or un-simplified.
	2nd A1	Both $\frac{1}{10}x$ and $\frac{2}{5}$. Can be simplified or un-simplified.
	dB1ft	**dependent on the FIRST method mark being awarded.** All three lines with limits correctly followed through from their $\text{F}'(x)$
	Note	Condone the use of $<$ rather than $\leqslant$ or vice versa.
	Note	0, otherwise is equivalent to 0, $x<2$ **and** 0, $x>5$
	Note	In part (c), accept f being expressed consistently in another variable eg. u

Question Number	Scheme		Marks
2. (a)	$X \sim \text{Po}(8)$		
	$\{\text{P}(X \neq 8)\} = 1 - \text{P}(X = 8)$	$1 - \text{P}(X = 8)$, can be implied	M1
	$= 0.860413...$ or 0.8605	0.86 or awrt 0.860 or awrt 0.861	A1
			[2]
(b)	$X \sim \text{Po}(8)$		
	$\{\text{P}(X \geqslant 8)\} = 1 - 0.453$	$1 - 0.453$ or awrt 0.547	B1
	$\left\{\left[\text{P}(X \geqslant 8)\right]^4\right\} = (1 - 0.453)^4 \ \left\{= (0.547)^4\right\}$	Applying $\left[\text{their P}(X \geqslant 8)\right]^4$	M1
	$= 0.089526...$	0.09 or awrt 0.090	A1
			[3]
(c)	Y = number of chocolate chips in the 9 biscuits		
	$\{Y \sim \text{Po}(72) \approx\} Y \sim \text{N}(72, 72)$	Normal or N $(72, 72)$	M1 A1
	$\{\text{P}(Y > 75)\} \approx \text{P}(Y > 75.5)$	For either 74.5 or 75.5	M1
	$= \text{P}\left(Z > \dfrac{75.5 - 72}{\sqrt{72}}\right)$	Standardising $(\pm)$ with their mean, their standard deviation and either 75.5 or 75 or 74.5	M1
	$= \text{P}(Z > 0.41...) = 1 - 0.6591$		
	$= 0.3409$ (from calculator 0.339994…)	awrt 0.341 or awrt 0.340	A1
			[5]
(d)	$\text{H}_0 : \lambda = 1.5,\ \ \text{H}_1 : \lambda > 1.5$ or $\text{H}_0 : \lambda = 6,\ \ \text{H}_1 : \lambda > 6$	Both hypotheses are stated correctly	B1
	{Under H_0, for 4 hours} $X \sim \text{Po}(6)$		
	Probability Method $\text{P}(X \geqslant 11) = 1 - \text{P}(X \leqslant 10)$ $= 1 - 0.9574$ **Note: Award 1st M1 for the use of $X \sim \text{Po}(6)$**	**Critical Region Method** $\text{P}(X \leqslant 9) = 0.9161$ or $\text{P}(X \geqslant 10) = 0.0839$ $\text{P}(X \leqslant 10) = 0.9574$ or $\text{P}(X \geqslant 11) = 0.0426$	M1
	$\text{P}(X \geqslant 11) = 0.0426$ $\text{CR} : X \geqslant 11$	Either $\text{P}(X \geqslant 11) = 0.0426$ or $\text{CR} : X \geqslant 11$ or $\text{CR} : X > 10$	A1
	Reject H_0 or significant or 11 lies in the CR	**dependent on previous M** See notes	dM1
	Conclude either • The **rate of sales** of packets of biscuits has **increased**. • The **mean** number of packets of biscuits **sold** has **increased**.	Correct conclusion in context.	A1 **cso**
			[5]
			15

	Question 2 Notes	
2. (a)	**M1**	$1-\text{P}(X=8)$ or $\text{P}(X<8)+\text{P}(X>8)$ or $\text{P}(X\le 7)+\text{P}(X\geqslant 9)$
	Note	Can be implied by either $1-\frac{\text{e}^{-8}8^8}{8!}$ or $1-\left(\text{P}(X\le 8)-\text{P}(X\le 7)\right)$ or $1-\left(0.5925-0.4530\right)$ or $1-0.1395$ or $\text{P}(X\le 7)+1-\text{P}(X\leqslant 8)$
	A1	0.86 or awrt 0.860 or awrt 0.861
(b)	**B1**	$1-0.453$ or awrt 0.547 (**Note:** calculator gives 0.5470391905…)
	M1	Applying $\left[\text{their P}(X\geqslant 8)\right]^4$
	A1	0.09 or awrt 0.090 (**Note:** calculator gives 0.08955168526…)
(c)	**1st M1**	For writing N or for using a normal approximation.
	1st A1	For a correct mean of 72 and a correct variance of 72
	Note	1st M1 and/or 1st A1 may be implied in applying the standardisation formula
	2nd M1	For either 74.5 or 75.5 (i.e. an attempt at a continuity correction)
	3rd M1	Standardising $(\pm)$ with their mean, their standard deviation and either 75.5 or 75 or 74.5
	Note	Award 2nd M1 3rd M0 for $\frac{75.5-72}{72}$ from a correct $Y\sim\text{N}(72, 72)$
	Note	You can recover the 1st A1 in part (c) for $\text{N}\left(72,\sqrt{72}\right)\Rightarrow z=\frac{75.5-72}{\sqrt{72}}$
	2nd A1	awrt 0.341 or awrt 0.340. (**Note:** calculator gives 0.339994…)
(d)	**B1**	$\text{H}_0:\lambda=1.5,\ \ \text{H}_1:\lambda>1.5$ correctly labelled or $\text{H}_0:\lambda=6,\ \ \text{H}_1:\lambda>6.$
	Note	Allow μ used instead of λ
	Note	B0 for either $\text{H}_0=6,\ \ \text{H}_1>6$ or $\text{H}_0:x=6,\ \ \text{H}_1:x>6$ or $\text{H}_0:p=6,\ \ \text{H}_1:p>6$
	1st M1	For use of $X\sim\text{Po}(6)$ (may be implied by 0.9161, 0.9574, 0.9799, 0.0839, 0.0426 or 0.0201). Condone by $\frac{\text{e}^{-6}(6)^{11}}{11!}$. Allow any value off the $\text{Po}(6)$ tables.
	1st A1	For either $\text{P}(X\geqslant 11)=0.0426$ or $\text{CR}:X\geqslant 11$ or $\text{CR}:X>10$ Condone $\text{CR}\geqslant 11$
	Note	Award 1st M1 1st A1 for writing down $\text{CR}:X\geqslant 11$ or $\text{CR}:X>10$ from no working.
	Note	Give A0 stating $\text{CR}:\text{P}(X\geqslant 11)$
	2nd dM1	**dependent on the FIRST method mark being awarded.** For a correct follow through comparison based on their probability or CR and their significance level compatible with their ***stated*** alternative hypothesis. Do not allow non-contextual conflicting statements. Eg. “significant” and “accept H_0”.
	Note	M1 can be implied by a correct contextual statement.
	Note	Give final M0A0 for $\text{P}(X=11)=0.9799-0.9574=0.0225\Rightarrow$ Reject H_0, etc.
	Note	Give final M0A0 for $\text{P}(X\leqslant 11)=0.9799\Rightarrow$ Accept H_0, etc
	2nd A1	Award for a correct solution only with all previous marks in part (d) being scored. Correct conclusion which is in context, using either the words rate of sales and increased **or** mean sold and increased

Question Number	Scheme	Marks
3. (a)	 A horizontal line drawn above the x-axis in the first quadrant	B1
	dependent on the first B mark Labels of c, $2c$ and $\frac{1}{c}$, marked on the graph. Ignore $\{O\}$, $\{x\}$ and $\{\mathrm{f}(x)\}$	dB1
		[2]
(b)	$\mathrm{E}(X)=\frac{3c}{2}$ $\mathrm{E}(X)=\frac{3c}{2}$, simplified or un-simplified.	B1
	$\left\{\mathrm{E}(X^2)=\right\}\int_c^{2c}\left(\frac{1}{2c-c}x^2\right)\{\mathrm{d}x\}$ $\int_c^{2c}x^2\mathrm{f}(x)\{\mathrm{d}x\}$ where $\mathrm{f}(x)$ is equivalent to $\frac{1}{c}$. (Limits are required)	M1
	$=\left[\frac{1}{c}\left(\frac{x^3}{3}\right)\right]_{\{c\}}^{\{2c\}}$ $\pm Ag(c)x^2\rightarrow\pm Bg(c)x^3,\ A\neq 0,\ B\neq 0$ (Ignore limits for this mark)	M1
	$=\left(\frac{(2c)^3}{3c}-\frac{c^3}{3c}\right)\left\{=\frac{7c^2}{3}\right\}$ **dependent on first M mark.** Applies limits of $2c$ and c to an ***integrated*** function in x and subtracts the correct way round.	dM1
	$\mathrm{Var}(X)=\mathrm{E}(X^2)-(\mathrm{E}(X))^2$	
	$=\frac{7c^2}{3}-\left(\frac{3c}{2}\right)^2$ **dependent on first M mark.** Applying the variance formula correctly with their $\mathrm{E}(X)$	dM1
	$=\frac{c^2}{12}$ * Correct proof	A1
		[6]
(c)	$X>2(2c-X)$ Correct un-simplified (or simplified) inequality statement. **Can be implied by** $X>\frac{4c}{3}$	M1
	$\Rightarrow\ X>4c-2X\ \ \Rightarrow 3X>4c$	
	$\Rightarrow\ X>\frac{4c}{3}$ **dependent on the first M mark.** Rearranges $X>2(2c-X)$ to give X >… or X <… **See notes**	dM1
	$\left\{\mathrm{P}(X>2(2c-X))=\mathrm{P}\left(X>\frac{4c}{3}\right)\right\}=\frac{2}{3}$ $\frac{2}{3}$	A1
		[3]
		11
	Note : In (c), give M2 for either $X>\frac{4c}{3}$ or $\mathrm{P}\left(X>\frac{4c}{3}\right)$ or $1-\mathrm{P}\left(X<\frac{4c}{3}\right)$	

	Question 3 Notes	
3. (a)	**1st B1**	A horizontal line drawn above the *x*-axis in the first quadrant
	2nd dB1	**dependent on the FIRST B mark being awarded.** Labels of *c*, 2*c* and $\frac{1}{c}$, marked on the graph.
	Note	Allow the label $\frac{1}{2c-c}$ as an alternative to $\frac{1}{c}$
	Note	Ignore {*O*}, {*x*} and {f(*x*)}
(b)	**B1**	$\mathrm{E}(X)=\frac{3c}{2}$, simplified or un-simplified. This mark can be implied.
	Note	B1 can be given for an un-simplified $\left(\frac{(2c)^2}{c}\right)-\left(\frac{c^2}{c}\right)$ or $\frac{3c^2}{2c}$ or $2c-\frac{c}{2}$ etc.
	Note	$\int_c^{2c}\frac{1}{c}x\,\mathrm{d}x$ or $\left[\frac{x^2}{2c}\right]_c^{2c}$ are not sufficient for B1.
	1st M1	Correct $\mathrm{E}(X^2)$ expression of $\int_c^{2c}x^2\mathrm{f}(x)\{\mathrm{d}x\}$ where $\mathrm{f}(x)$ is equivalent to $\frac{1}{c}$.
	Note	Must have limits of 2*c* and *c*. Note the d*x* is not required for this mark.
	2nd M1	$\pm Ag(c)x^2\to\pm Bg(c)x^3,\ A\neq 0,\ B\neq 0$, where $g(c)$ is a function of *c*
	Note	Limits are not required for the second 2nd M1 mark.
	3rd dM1	**dependent on the FIRST method mark being awarded.** Applies limits of $2c$ and c to an integrated function in *x* and subtracts the correct way round.
	4th M1	**dependent on the FIRST method mark being awarded.** Applying the variance formula correctly with their follow through $\mathrm{E}(X)$.
	Note	Allow 4th M1 for $\{\mathrm{Var}(X)=\}\int_c^{2c}\left(\frac{1}{2c-c}x^2\right)\{\mathrm{d}x\}-\left(\int_c^{2c}\left(\frac{1}{2c-c}x\right)\{\mathrm{d}x\}\right)^2$
	A1	Correctly proves that $\mathrm{Var}(X)=\frac{c^2}{12}$. **Note:** Answer is given
(c)	**1st M1**	For writing down a correctly un-simplified (or simplified) inequality statement. Eg: $X>2(2c-X)$ or $\mathrm{P}(X>2(2c-X))$ (**Note:** "P" is not required for this mark)
	2nd dM1	**dependent on the FIRST method mark being awarded.** Rearranges to give $\mathrm{P}(X>\pm\alpha c)$ or $\mathrm{P}(X<\pm\alpha c)$ or $X>\pm\alpha c$ or $X<\pm\alpha c,\ \alpha\neq 0$
	Note	"P" is not required for these cases above
	Note	Also allow, with P, the statements $1-\mathrm{P}(X<\pm\alpha c)$ or $1-\mathrm{P}(X>\pm\alpha c),\ \alpha\neq 0$
	NOTE	Give M2 for either $X>\frac{4c}{3}$ or $\mathrm{P}\left(X>\frac{4c}{3}\right)$ or $1-\mathrm{P}\left(X<\frac{4c}{3}\right)$
	A1	$\frac{2}{3}$ or $\frac{4}{6}$ or $0.\dot{6}$
	Note	Give M1M1A1 for a final answer of $\frac{2}{3}$ ***from any*** working.

Question Number	Scheme		Marks
3.	**Alternative Method 1 for Part (b)**		
(b)	$\{\mathrm{Var}(X) =\}$		
		Implied $\mathrm{E}(X) = \frac{3c}{2}$	**B1**
	$\int_c^{2c} \left(\frac{1}{2c-c} \left(x - \frac{3}{2}c \right)^2 \right) \{\mathrm{d}x\}$	$\int_c^{2c} x^2 \mathrm{f}(x) \{\mathrm{d}x\}$ where $\mathrm{f}(x)$ is equivalent to $\frac{1}{c}$. (Limits are required)	**1st M1**
		Applies $\int_c^{2c} \mathrm{f}(x) \left(x - \frac{3c}{2} \right)^2 \{\mathrm{d}x\}$ where $\mathrm{f}(x)$ is a is equivalent to $\frac{1}{c}$. (Limits are required)	**4th dM1**
	$= \frac{1}{c} \left[\frac{1}{3} \left(x - \frac{3c}{2} \right)^3 \right]_{\{c\}}^{\{2c\}}$	$\pm A\mathrm{g}(c)(x-\delta)^2 \to \pm B\mathrm{g}(c)(x-\delta)^3$, $A, B, \delta \neq 0$ (Ignore limits for this mark)	**2nd M1**
	$= \frac{1}{3c} \left(\left(\frac{c}{2} \right)^3 - \left(-\frac{c}{2} \right)^3 \right)$	**dependent on first M mark.** Applies limits of $2c$ and c to an integrated function in *x* and subtracts the correct way round.	**3rd dM1**
	$= \frac{1}{3c} \left(\frac{c^3}{4} \right) = \frac{c^2}{12}$ *	Correct proof	**A1**
			[6]
	Alternative Method 2 for Part (b)		
(b)	$\{\mathrm{Var}(X) =\}$		
	$\int_c^{2c} \left(\frac{1}{2c-c} \left(x - \frac{3}{2}c \right)^2 \right) \{\mathrm{d}x\}$	**Award as in Alt. Method 1**	**B1** **1st M1** **4th M1**
	$= \frac{1}{c} \int_c^{2c} \left(x^2 - 3cx + \frac{9}{4}c^2 \right) \{\mathrm{d}x\}$		
	$= \frac{1}{c} \left[\frac{1}{3}x^3 - \frac{3}{2}cx^2 + \frac{9}{4}c^2 x \right]_{\{c\}}^{\{2c\}}$	$\pm A\mathrm{g}(c)(x-\delta)^2 \to \pm B\mathrm{g}(c)(\pm\alpha x^3 \pm \beta x^2 \pm \delta x)^3$, $A, B, \alpha, \beta, \delta \neq 0$ (Ignore limits for this mark)	**2nd M1**
	$= \frac{1}{c} \left(\left(\frac{1}{3}(2c)^3 - \frac{3}{2}c(2c)^2 + \frac{9}{4}c^2(2c) \right) - \left(\frac{1}{3}(c)^3 - \frac{3}{2}c(c)^2 + \frac{9}{4}c^2(c) \right) \right)$	As earlier	**3rd dM1**
	$= \frac{1}{c} \left(\left(\frac{8}{3}c^3 - 6c^3 + \frac{9}{2}c^3 \right) - \left(\frac{1}{3}c^3 - \frac{3}{2}c^3 + \frac{9}{4}c^3 \right) \right)$		
	$= \frac{1}{c} \left(\left(\frac{7}{6}c^3 \right) - \left(\frac{13}{12}c^3 \right) \right) = \frac{1}{c} \left(\frac{c^3}{12} \right)$		
	$= \frac{c^2}{12}$ *	Correct proof	**A1**
			[6]

Question Number	Scheme		Marks
4. (a)	$\mathrm{P}(X=0 \mid k=3)=0.0498$ $\mathrm{P}(X=0 \mid k=4)=0.0183$ $\mathrm{P}(X=0 \mid k=5)=0.0067$ $\left\{\mathrm{e}^{-k}<0.025 \Rightarrow k>\right\}$ 3.688...	At least one of these 9 probabilites **or** awrt 3.7 seen in their working.	B1
	$\mathrm{P}(X \leqslant 8 \mid k=3)=0.9962,\ \mathrm{P}(X \geqslant 9 \mid k=3)=0.0038$ $\mathrm{P}(X \leqslant 8 \mid k=4)=0.9786,\ \mathrm{P}(X \geqslant 9 \mid k=4)=0.0214$ $\mathrm{P}(X \leqslant 8 \mid k=5)=0.9319,\ \mathrm{P}(X \geqslant 9 \mid k=5)=0.0681$	**Both** $\mathrm{P}(X=0)=0.0183$ **or** awrt 3.7 **and either** $\mathrm{P}(X \geqslant 9)=0.0214$ **or** $\mathrm{P}(X \leqslant 8)=0.9786$	B1
	Both tails less than 2.5% when $\underline{k=4}$	Final answer given as $\underline{k=4}$	B1
			[3]
(b)	Actual sig. level = $0.0214+0.0183$	See notes	M1
	$=0.0397$	0.0397	A1 **cao**
			[2]
			5

		Question 4 Notes
4. (a)		
	1st B1	For any of 0.0498, 0.0183, 0.0067, 0.9962, 0.9786, 0.9319, 0.0038, 0.0214, 0.0681 or awrt 3.7 seen in their working.
	2nd B1	For both $\mathrm{P}(X=0)=0.0183$ **or** awrt 3.7 **and either** $\mathrm{P}(X \geqslant 9)=0.0214$ **or** $\mathrm{P}(X \leqslant 8)=0.9786$
	Note	These must be written as probability statements.
	3rd B1	Final answer given as $\underline{k=4}$. Also allow $\lambda=4$
	Note	**Do not recover working for part (a) in part (b)**
(b)	**M1**	For the addition of two probabilities for two tails, where each tail < 0.05
	A1	0.0397 **cao**

Question Number	Scheme	Marks
5.	$Y = \frac{2X_1 + X_2}{3}$ where $\begin{array}{|c|c|c|}\hline x & 6 & 9 \\ \hline \text{P}(X = x) & 0.35 & 0.65 \\ \hline\end{array}$	
	Note: You can mark parts (a) and (b) together for this question.	
(a)	$\frac{2(6)+6}{3} = 6$ $\frac{2(9)+9}{3} = 9$ At least three correct values for *y* of either 6, 7, 8 or 9	B1
	$\frac{2(6)+9}{3} = 7$ $\frac{2(9)+6}{3} = 8$ Correct values for *y* of 6, 7 8 and 9 only	B1
		[2]
(b)	$\{(6, 6) \Rightarrow \text{P}(Y=6)\} = (0.35)^2$ $\{(6, 9) \Rightarrow \text{P}(Y=7)\} = (0.65)(0.35)$ At least one of either $(0.35)^2$, $(0.65)(0.35), (0.35)(0.65)$ or $(0.65)^2$	M1
	$\{(9, 6) \Rightarrow \text{P}(Y=8)\} = (0.35)(0.65)$ $\{(9, 9) \Rightarrow \text{P}(Y=9)\} = (0.65)^2$ At least two of either $(0.35)^2$, $(0.65)(0.35), (0.35)(0.65)$ or $(0.65)^2$	M1
	$\begin{array}{|c|c|c|c|c|}\hline \text{sample} & (6, 6) & (6, 9) & (9, 6) & (9, 9) \\ \hline y & 6 & 7 & 8 & 9 \\ \hline \text{P}(Y = y) & 0.1225 & 0.2275 & 0.2275 & 0.4225 \\ \hline \text{or P}(Y = y) & \frac{49}{400} & \frac{91}{400} & \frac{91}{400} & \frac{169}{400} \\ \hline\end{array}$ **See notes**	A1
	At least 3 correct	A1
	See notes	B1ft
		[5]
(c)	$\{\text{E}(Y)\} = 6(0.1225) + 7(0.2275) + 8(0.2275) + 9(0.4225) = 7.95$ or $\frac{159}{20}$	M1;A1 **cao**
		[2]
		9
(c)	**<u>Alternative Method for Part (c)</u>**	
	$\left\{\text{E}(Y) = \frac{2}{3}\text{E}(X_1) + \frac{1}{3}\text{E}(X_2) = \frac{2}{3}\text{E}(X) + \frac{1}{3}\text{E}(X) = \text{E}(X)\right\}$	
	$= 6(0.35) + 9(0.65); = 7.95$ or $\frac{159}{20}$	M1; A1 **cao**
		[2]

	Question 5 Notes	
5. (a)	**Note**	**You can mark parts (a) and (b) together for this question.**
	1st B1 **2nd B1**	At least three correct values for *y* of either 6, 7, 8 or 9 Correct values for *y* of 6, 7 8 and 9 only. **Note:** Any extra value(s) given is 2nd B0.
(b)	**1st M1**	At least one of either $(0.35)^2$, $(0.65)(0.35)$, $(0.35)(0.65)$ or $(0.65)^2$. **Can be implied.**
	2nd M1	At least two of either $(0.35)^2$, $(0.65)(0.35)$, $(0.35)(0.65)$ or $(0.65)^2$. **Can be implied.**
	1st A1	At least two correct probabilities given which either must be linked to a correct sample (x_1, x_2) or their followed through *y*-value.
	2nd A1 **B1ft**	At least 3 correct probabilities corresponding to the correct value of *y*. Either • all 4 correct probabilities corresponding to the correct value of *y* • 6, 7, 8 and 9 with two correct probabilities, two other probabilities and $\sum \mathrm{p}(y) = 1$
	Note **Note** **Note**	B1ft is dependent on 1st M1 2nd M1 1st A1. A table is not required but *y*-values must be linked with their probabilities for 2nd A1 B1 Eg: (6, 6) by itself does not count as an acceptable value of *y*
(c)	**M1** **Note**	A correct follow through expression for $\mathrm{E}(Y)$ using their distribution Also allow M1 for a correct expression for $\mathrm{E}(X)$
	A1	7.95 **cao** Allow $\frac{159}{20}$

Question Number	Scheme		Marks
6. (a)	$X \sim \mathrm{B}(30, 0.4)$	$X \sim \mathrm{B}(30, 0.4)$	B1
			[1]
(b)	Eg: Any one of either • Constant probabilty of buying insurance • Customers buy insurance independently of each other	Any one of these two assumptions in context which refers to insurance.	B1
			[1]
(c)	$\mathrm{P}(X < r) < 0.05$		
	$\{\mathrm{P}(X \leqslant 8) = \mathrm{P}(X < 9)\} = 0.0940$ $\{\mathrm{P}(X \leqslant 7) = \mathrm{P}(X < 8)\} = 0.0435$	For at least one of either 0.094(0) or 0.0435 seen in part (c)	M1
	So $r = 8$	$r = 8$	A1
			[2]
(d)	$\{Y \sim \mathrm{B}(100, 0.4) \approx\} Y \sim \mathrm{N}(40, 24)$	Normal or N	M1
		(40, 24)	A1
	$\{\mathrm{P}(Y \geqslant t)\} \approx \mathrm{P}(Y > t - 0.5)$	For either $t - 0.5$ or $t + 0.5$	M1
	$\left\{= \mathrm{P}\left(Z > \frac{(t-0.5)-40}{\sqrt{24}}\right) = 0.938\right\}$		
	$\frac{(t-0.5)-40}{\sqrt{24}} = -1.54$	Standardising $(\pm)$ with their mean and their standard deviation and either $t - 0.5$ or t or $t + 0.5$ or $t - 1.5$	M1
		-1.54 or 1.54 or awrt -1.54 or awrt 1.54	B1
	So, $\{\text{So, } t = 32.955571...\} \Rightarrow \underline{t = 33}$	$\underline{t = 33}$	A1 **cao**
			[6]
(e)	$\mathrm{H}_0 : p = 0.4$, $\mathrm{H}_1 : p < 0.4$	Both hypotheses are stated correctly	B1
	$\{\text{Under } \mathrm{H}_0,\ X \sim \mathrm{B}(25, 0.4)\}$		
	Probability Method $\mathrm{P}(X \leqslant 6); = 0.0736$ **Critical Region Method** $\mathrm{P}(X \leqslant 6); = 0.0736$ $\{\mathrm{P}(X \leqslant 7) = 0.1536\}$ $\mathrm{CR}: X \leqslant 6$	$\mathrm{P}(X \leqslant 6)$	M1
		Either 0.0736 or $\mathrm{CR}: X \leqslant 6$ or $\mathrm{CR}: X < 7$	A1
	$\{0.0736 < 0.10\}$		
	Reject H_0 or significant or 6 lies in the CR	**Dependent on 1st M1** See notes	dM1
	So **percentage** (or **proportion**) who buy **insurance** has **decreased**.		A1 **cso**
			[5]
			15

Question Number	Scheme	Marks
6. (e)	**<u>Alternative Method: Normal approximation to the Binomial Distribution</u>** • Normal Approximation gives 0.0764 (or 0.07652…) and loses all A marks	
	$H_0 : p = 0.4,\ \ H_1 : p < 0.4$ Both hypotheses are stated correctly	B1
	$\left\{Y \sim B(25, 0.4) \approx Y \sim N(10, 6)\right\}$	
	$P(X \leqslant 6) \approx P(X < 6.5)$ $P(X \leqslant 6)$ or $P(X < 6.5)$	M1
	$= P\left(Z < \frac{6.5 - 10}{\sqrt{6}}\right)$ $= P(Z < -1.4288...)$	
	$\left\{= 1 - 0.9236\right\} = 0.0764$ ***Award A0 here***	*A0*
	$\left\{0.0764 < 0.10\right\}$	
	Reject H_0 or significant As in the main scheme	M1
	So **<u>percentage</u>** (or **<u>proportion</u>**) who buy **<u>insurance</u>** has **<u>decreased</u>**. ***Award A0 here***	***A0***

	Question 6 Notes	
6. (a)	**B1** **Note**	$X \sim B(30, 0.4)$ or $X \sim Bin(30, 0.4)$. Condone $X \sim b(30, 0.4)$ $X \sim B(30, 0.4)$ o.e. must be seen in part (a) only.
(b)	**B1** **Note**	For any one of the two acceptable assumptions listed anywhere in part (b). A contextual statement, which refers to insurance, is required for this mark.
(c)	**Note**	Award M1 A1 for $r = 8$ seen from no incorrect working.
(d)	**1st M1** **1st A1** **Note**	For writing N or for using a normal approximation. For a correct mean of 40 and a correct variance of 24 1st M1 and/or 1st A1 may be implied in applying the standardisation formula
	2nd M1	For either $t - 0.5$ or $t + 0.5$ (i.e. an attempt at a continuity correction)
	3rd M1	As described on the mark scheme.
	B1	-1.54 or 1.54 or awrt -1.54 or awrt 1.54. Note: Calculator gives -1.5382
	2nd A1	$\underline{t = 33}$ **cao** (The integer value is required).
(e)	**B1**	$H_0 : p = 0.4,\ \ H_1 : p < 0.4$ corectly labelled. Also allow $H_0 : \pi = 0.4,\ \ H_1 : \pi < 0.4$ Also allow $H_0 : \pi = 0.4,\ \ H_1 : \pi < 0.4$ or $H_0 : p(x) = 0.4,\ \ H_1 : p(x) < 0.4$
	Note	B0 for $H_0 = 0.4,\ \ H_1 < 0.4$
	1st M1	**Probability Method & CR Method:** Stating $P(X \leqslant 6)$
	1st A1 **Note** **Note**	Either 0.0736 or $CR : X \leqslant 6$ or $CR : X < 7$ **Note:** Condone $CR \leqslant 6$ Award 1st M1 1st A1 for writing down $CR : X \leqslant 6$ or $CR : X < 7$ from no working. Give A0 for stating $CR : P(X \leqslant 6)$
	2nd dM1 **Note**	**dependent on the FIRST method mark being awarded.** For a correct follow through comparison based on their probability or CR and their significance level compatible with their ***stated*** alternative hypothesis. Do not allow non-contextual conflicting statements. Eg. "significant" and "accept H_0". M1 can be implied by a correct contextual statement.
	2nd A1	Award for a correct solution only with all previous marks in part (e) being scored. Correct conclusion which is in context, using the words <u>percentage</u> (or <u>proportion</u>), <u>insurance</u> and <u>decreased</u> (or equivalent words for decreased).

Question Number	Scheme	Marks
7. (a)	$\int_0^k \left(\frac{2x}{15}\right)\left\{\mathrm{d}x\right\} + \int_5^k \frac{1}{5}(5-x)\left\{\mathrm{d}x\right\} = 1$ Complete method of writing a correct equation for the area ***with correct limits*** and setting the result equal to 1	M1
	$\left[\frac{x^2}{15}\right]_{\{0\}}^{\{k\}} + \left[x - \frac{x^2}{10}\right]_{\{k\}}^{\{5\}} = 1$ Evidence of $x^n \to x^{n+1}$	M1
	Both $\frac{2x}{15} \to \frac{x^2}{15}$ **and** $\frac{1}{5}(5-x) \to x - \frac{x^2}{10}$	A1 o.e.
	$\left(\frac{k^2}{15}\right) + \left(5 - \frac{5^2}{10} - \left(k - \frac{k^2}{10}\right)\right) = 1$	
	$2k^2 + 150 - 75 - 30k + 3k^2 = 30$	
	$k^2 - 6k + 9 = 0$ or $\frac{k^2}{6} - k + \frac{3}{2} = 0$	
	$(k-3)(k-3) = 0 \Rightarrow k = ...$ **Dependent on the 1st M mark** Attempt to solve a 3 term quadratic equation leading to $k = ...$	dM1
	$k = 3$ $k = 3$	A1
		[5]
(b)	{mode =} 3 3 **or** states their *k* value from part (a)	B1 ft
		[1]
(c)	$\left\{ \mathrm{P}\left(X \leqslant \frac{k}{2} \middle\vert X \leqslant k\right) = \frac{\mathrm{P}\left(X \leqslant \frac{k}{2} \cap X \leqslant k\right)}{\mathrm{P}\left(X \leqslant k\right)} \right\}$	
	$= \frac{\mathrm{P}\left(X \leqslant \frac{k}{2}\right)}{\mathrm{P}\left(X \leqslant k\right)}$ Either $\frac{\mathrm{P}\left(X \leqslant \frac{k}{2}\right)}{\mathrm{P}\left(X \leqslant k\right)}$ or $\frac{\mathrm{F}\left(\frac{k}{2}\right)}{\mathrm{F}(k)}$ seen or implied.	M1
	$= \frac{\int_0^{\frac{k}{2}} \left(\frac{2x}{15}\right)\left\{\mathrm{d}x\right\}}{\int_0^k \left(\frac{2x}{15}\right)\left\{\mathrm{d}x\right\}}$ **see notes**	dM1
	$= \frac{\frac{1}{15}\left(\frac{k}{2}\right)^2}{\frac{k^2}{15}}$ Correct substitution of their limits or their *k* into conditional probability formula.	A1ft
	$\left\{ = \frac{\left(\frac{9}{60}\right)}{\left(\frac{9}{15}\right)} = \frac{0.15}{0.6} \right\} = \frac{1}{4}$ $\frac{1}{4}$ or 0.25	A1 **cao**
		[4]
		10

	Question 7 Notes	
7. (a)	**1st M1**	$\int_0^k \left(\frac{2x}{15}\right)\left\{\mathrm{d}x\right\} + \int_5^k \frac{1}{5}(5-x)\left\{\mathrm{d}x\right\} = 1$. (***with correct limits*** and =1) $\left\{\mathrm{d}x\right\}$not needed.
	2nd M1	Evidence of $x^n \to x^{n+1}$
	1st A1	**Both** $\frac{2x}{15} \to \frac{x^2}{15}$ **and** $\frac{1}{5}(5-x) \to x - \frac{x^2}{10}$
	3rd dM1	**dependent on the FIRST method mark being awarded.** Attempt to solve a **three term** quadratic equation. Please see table on page 20
	2nd A1	$k = 3$ from correct working.
	Note **Note**	**WARNING:** $\frac{2x}{15} = \frac{1}{5}(5-x)$ to get $k = 3$ is M0M0A0M0A0. It is possible to give M0M1A1M0A0 in part (a).
(b)	**B1 ft**	Mode = 3 or candidate states their *k* value from part (a), where $0 <$ their $k < 5$
(c)	**1st M1** **Note**	Either $\frac{\mathrm{P}\left(X \leqslant \frac{k}{2}\right)}{\mathrm{P}\left(X \leqslant k\right)}$ or $\frac{\mathrm{F}\left(\frac{k}{2}\right)}{\mathrm{F}\left(k\right)}$, seen or implied by their later working. Without reference to a correct conditional probability statement give 1st M0 for either $\frac{\mathrm{f}\left(\frac{k}{2}\right)}{\mathrm{f}(k)}$ or $\frac{\mathrm{F}\left(k\right) - \mathrm{F}\left(\frac{k}{2}\right)}{\mathrm{F}\left(k\right)}$ or $\frac{\mathrm{P}\left(X \leqslant \frac{k}{2}\right) \times \mathrm{P}\left(X \leqslant k\right)}{\mathrm{P}\left(X \leqslant k\right)}$
	2nd dM1 **Note**	**dependent on the FIRST method mark being awarded.** **Applies** the conditional probability statement by writing down • $\frac{\int_0^{\frac{k}{2}} \left(\frac{2x}{15}\right)\left\{\mathrm{d}x\right\}}{\int_0^k \left(\frac{2x}{15}\right)\left\{\mathrm{d}x\right\}}$ with limits. • $\frac{\mathrm{F}\left(\frac{k}{2}\right)}{\mathrm{F}\left(k\right)}$ where $\mathrm{F}(x)$ is defined as $\mathrm{F}(x) = \frac{x^2}{15}$ These statements can be implied by later working. Finding $\mathrm{P}(X \leqslant 1.5) = 0.15$ and $\mathrm{P}(X \leqslant 3) = 0.6$ without applying $\frac{0.15}{0.6}$ is 2nd M0
	1st A1ft **Note**	Correct substitution of their limits or their *k* into conditional probability formula. Candidates can work in terms of *k* for this 1st A1 mark.
	2nd A1 **Note**	$\frac{1}{4}$ or 0.25 **cao** Condone giving 2nd A1 for achieving a correct answer of 0.25 where at least one of their stated $\mathrm{P}\left(X \leqslant \frac{k}{2}\right)$ or $\mathrm{P}\left(X \leqslant k\right)$ is greater than 1
	Note	Alternative method using similar triangles. Area up to $\frac{k}{2}$ is $\frac{1}{4}$ of the area up to *k*. This can score 4 marks.

7. (a)	**Alternative Method 1 for Part (a) Using the CDF**		
	$0 \leqslant x \leqslant k,\ \mathrm{F}(x)=\int_0^k \frac{2t}{15}\{\mathrm{d}t\}=\left[\frac{2t^2}{30}\right]_0^x=\frac{x^2}{15}$	Evidence of $x^n \to x^{n+1}$	**2nd M1**
	$k<x\leqslant 5,\ \mathrm{F}(x)=\mathrm{F}(k)+\int_k^x \frac{1}{5}(5-t)\{\mathrm{d}t\}$ $=\frac{k^2}{15}+\left[\frac{1}{5}\left(5t-\frac{t^2}{2}\right)\right]_k^x$	**Both** $\frac{2x}{15}\to\frac{x^2}{15}$ **and** $\frac{1}{5}(5-x)\to x-\frac{x^2}{10}$	**1st A1** o.e.
	$=\frac{k^2}{15}+\frac{1}{5}\left(5x-\frac{x^2}{2}\right)-\frac{1}{5}\left(5k-\frac{k^2}{2}\right)$ $=x-\frac{x^2}{10}-k+\frac{k^2}{6}$		
	$\{\mathrm{F}(5)=1\Rightarrow\}\ 5-\frac{5^2}{10}-k+\frac{k^2}{6}=1$	Complete method of writing a correct equation for the area ***with correct limits*** and setting $\mathrm{F}(5)=1$	**1st M1**
	then apply the main scheme		
7. (a)	**Alternative Method 2 for Part (a) Use of Area**		
	$\frac{1}{2}k\left(\frac{2k}{15}\right)+\frac{1}{2}\left(\frac{5-k}{5}\right)\left(5-k\right)=1$	Complete area expression put = 1	M1
		At least one term correct on LHS	M1
		Correct LHS	A1 o.e.
	then apply the main scheme		

General	**Note**	The c.d.f is defined as $\mathrm{F}(x)=\begin{cases} 0, & x<0 \\ \frac{x^2}{15}, & 0\leqslant x\leqslant 3 \\ x-\frac{x^2}{10}-\frac{3}{2}, & 3<x\leqslant 5 \\ 1, & x>5 \end{cases}$

7. (a)	**Method mark for solving a 3 term quadratic of the form** $x^2+bx+c=0$ **Factorising/Solving a quadratic equation is tested in Question 7(a).** **1. Factorisation** $(x^2+bx+c)=(x+p)(x+q)$, where $\lvert pq\rvert=\lvert c\rvert$, leading to $x=\ldots$ $(ax^2+bx+c)=(mx\pm p)(nx\pm q)$, where $\lvert pq\rvert=\lvert c\rvert$ and $\lvert mn\rvert=\lvert a\rvert$, leading to $x=\ldots$ **2. Formula** Attempt to use correct formula (with values for *a*, *b* and *c*) **3. Completing the square** Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0$, $q\neq 0$, leading to $x=\ldots$

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