

4c	B1	B0 if incorrect extras
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Question	Scheme	Marks	AOs
	Allow column vectors throughout this question		
5(a)	Differentiate \mathbf{v} wrt t	M1	3.1a
	$\frac{3}{2}t^{-\frac{1}{2}}\mathbf{i} - 2\mathbf{j}$ isw	A1	1.1b
		(2)	
5(b)	$3t^{\frac{1}{2}} = 2t$	M1	2.1
	Solve for t	DM1	1.1b
	$t = \frac{9}{4}$	A1	1.1b
		(3)	
5(c)	Integrate \mathbf{v} wrt t	M1	3.1a
	$\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^2\mathbf{j} (+\mathbf{C})$	A1	1.1b
	$t = 1, \mathbf{r} = -\mathbf{j} \Rightarrow \mathbf{C} = -2\mathbf{i}$ so $\mathbf{r} = 2t^{\frac{3}{2}}\mathbf{i} - t^2\mathbf{j} - 2\mathbf{i}$	A1	2.2a
		(3)	
5(d)	$\sqrt{(3t^{\frac{1}{2}})^2 + (2t)^2} = 10$ or $(3t^{\frac{1}{2}})^2 + (2t)^2 = 10^2$	M1	2.1
	$9t + 4t^2 = 100$	M(A)1	1.1b
	$t = 4$	A1	1.1b
	$\mathbf{r} = 14\mathbf{i} - 16\mathbf{j}$	M1	1.1b
	$\sqrt{14^2 + (-16)^2}$	M1	3.1a
	$\sqrt{452} (2\sqrt{113})$ (m)	A1	1.1b
		(6)	

(14 marks)

Notes:

5a	M1	Both powers decreasing by 1 (M0 if vector(s) disappear but allow recovery)
	A1	cao
5b	M1	Complete method, using \mathbf{v} , to obtain an equation in t only, allow a sign error
	DM1	Dependent on M1, solve for t

Section B: MECHANICS

Question	Scheme	Marks	AOs
6.	Integrate \mathbf{v} w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j} (+ \mathbf{C})$	A1	1.1b
	Substitute $t = 4$ and $t = 1$ into their \mathbf{r}	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j} (+ \mathbf{C}); t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j} (+ \mathbf{C})$ or $(4, -32); (2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
(6 marks)			
Notes: Allow column vectors throughout			
<p>M1: At least one power increasing by 1.</p> <p>A1: Any correct (unsimplified) expression</p> <p>M1: Must have attempted to integrate \mathbf{v}. Substitute $t = 4$ and $t = 1$ into their \mathbf{r} to produce 2 vectors (or 2 points if just working with coordinates).</p> <p>A1: $4\mathbf{i} - 32\mathbf{j} (+ \mathbf{C})$ and $2\mathbf{i} - 2\mathbf{j} (+ \mathbf{C})$ or $(4, -32)$ and $(2, -2)$. These can be seen or implied.</p> <p>M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms.</p> <p>A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)</p>			

9MA0-32: Mechanics 1906

Mark scheme

Question	Scheme	Marks	AO
1(a)	Differentiate \mathbf{v}	M1	1.1a
	$(\mathbf{a} =)6\mathbf{i} - \frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$= 6\mathbf{i} - 15\mathbf{j} \text{ (m s}^{-2}\text{)}$	A1	1.1b
		(3)	
1(b)	Integrate \mathbf{v}	M1	1.1a
	$(\mathbf{r} =)(\mathbf{r}_0) + 3t^2\mathbf{i} - 2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$= (-20\mathbf{i} + 20\mathbf{j}) + (48\mathbf{i} - 64\mathbf{j}) = 28\mathbf{i} - 44\mathbf{j} \text{ (m)}$	A1	2.2a
		(3)	
		(6)	
Marks	Notes		
	N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers		
1a	M1	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ with attempt to differentiate (both powers decreasing by 1) M0 if \mathbf{i} 's and \mathbf{j} 's omitted and they don't recover	
	A1	Correct differentiation in any form	
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.	
1b	M1	Use of $\mathbf{r} = \int \mathbf{v} dt$ with attempt to integrate (both powers increasing by 1) M0 if \mathbf{i} 's and \mathbf{j} 's omitted and they don't recover	
	A1	Correct integration in any form. Condone \mathbf{r}_0 not present	
	A1	Correct and simplified.	

Question	Scheme		Marks	AOs
3(i)(a)	Integrate \mathbf{a} wrt t to obtain velocity		M1	3.4
	$\mathbf{v} = (t - 2t^2)\mathbf{i} + \left(3t - \frac{1}{3}t^3\right)\mathbf{j} (+C)$		A1	1.1b
	$8\mathbf{i} - \frac{28}{3}\mathbf{j} \text{ (m s}^{-1}\text{)}$		A1	1.1b
			(3)	
3(i)(b)	Equate \mathbf{i} component of \mathbf{v} to zero		M1	3.1a
	$t - 2t^2 + 36 = 0$		A1ft	1.1b
	$t = 4.5$ (ignore an incorrect second solution)		A1	1.1b
			(3)	
3(ii)	Differentiate \mathbf{r} wrt to t to obtain velocity		M1	3.4
	$\mathbf{v} = (2t - 1)\mathbf{i} + 3\mathbf{j}$		A1	1.1b
	Use magnitude to give an equation in t only		M1	2.1
	$(2t - 1)^2 + 3^2 = 5^2$		A1	1.1b
	Solve problem by solving this equation for t		M1	3.1a
	$t = 2.5$		A1	1.1b
			(6)	
(12 marks)				
Notes: Accept column vectors throughout				
3(i)(a)	M1	At least 3 terms with powers increasing by 1 (but M0 if clearly just multiplying by t)		
	A1	Correct expression		
	A1	Accept $8\mathbf{i} - 9.3\mathbf{j}$ or better. Isw if speed found.		
3(i)(b)	M1	Must have an equation in t only (Must have integrated to find a velocity vector)		
	A1ft	Correct equation follow through on their \mathbf{v} but must be a 3 term quadratic		
	A1	cao		
3(ii)	M1	At least 2 terms with powers decreasing by 1 (but M0 if clearly just dividing by t)		
	A1	Correct expression		
	M1	Use magnitude to give an equation in t only, must have differentiated to find a velocity (M0 if they use $\sqrt{x^2 - y^2}$)		

Question	Scheme	Marks	AOs
6	Integrate \mathbf{a} w.r.t. time	M1	1.1a
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + \mathbf{C}$ (allow omission of \mathbf{C})	A1	1.1b
	$\mathbf{v} = \frac{5t^2}{2}\mathbf{i} - 10t^{\frac{3}{2}}\mathbf{j} + 20\mathbf{i}$	A1	1.1b
	When $t = 4$, $\mathbf{v} = 60\mathbf{i} - 80\mathbf{j}$	M1	1.1b
	Attempt to find magnitude: $\sqrt{(60^2 + 80^2)}$	M1	3.1a
	Speed = 100 m s^{-1}	A1ft	1.1b
			(6 marks)
Notes:			
<p>1st M1: for integrating \mathbf{a} w.r.t. time (powers of t increasing by 1)</p> <p>1st A1: for a correct \mathbf{v} expression without \mathbf{C}</p> <p>2nd A1: for a correct \mathbf{v} expression including \mathbf{C}</p> <p>2nd M1: for putting $t = 4$ into their \mathbf{v} expression</p> <p>3rd M1: for finding magnitude of their \mathbf{v}</p> <p>3rd A1: ft for 100 m s^{-1}, follow through on an incorrect \mathbf{v}</p>			

Question Number	Scheme	Marks	Notes
6(a)		M1	Integrate a to obtain v
	$v = t^2 - 3t (+c)$	A1	Condone missing C
	$t = 3, v = 2 \Rightarrow c = 2$	M1	Substitute to find C
	$v = t^2 - 3t + 2$	A1	
		(4)	
(b)	$0 = (t-2)(t-1)$	M1	Set their $v = 0$ and solve for t
	$t = 1, 2$	A1	
	$s = \int_1^2 (t^2 - 3t + 2) dt$	M1	Integrate v to obtain s
	$= \left[\frac{1}{3}t^3 - \frac{3}{2}t^2 + 2t \right]_1^2$	A1ft	Condone if limits not seen. Follow their v .
	$= -\frac{1}{6} \text{ m}$	dM1	Use their t values as limits. Dependent on the preceding M1.
	Dist = $\frac{1}{6}$ (m)	A1	0.17, 0.167 or better
		(6)	
		[10]	

Q	Scheme	Marks	Notes
	Change in energy $= \pm \left(\frac{1}{2} \times 4 \times 6^2 - 4g \times 10 \sin \alpha \right)$	A2	-1 each error
	$= 72 - 40g \times \frac{1}{7} = 16 \text{ (J) *given answer*}$	A1	-16 is A0. Condone -16 becoming +16
		(4)	
3a alt	Complete strategy using <i>suvat</i> and N2L to find the work done	M1	
	$v^2 = u^2 + 2as \Rightarrow 36 = -20a \quad (a = -1.8)$	A1	
	$Fr + 4g \sin \theta = 4 \times (\text{their } 1.8)$ ($Fr = 1.6$)	A1	
	Work Done = $1.6 \times 10 = 16 \text{ (J)}$ *given answer*	A1	
		(4)	
	NB: For 3(b) must be using work-energy		
3b	Considering the whole journey: $\frac{1}{2} \times 4v^2 = \frac{1}{2} \times 4 \times 36 - 2 \times 16$	M1	Requires all 3 terms. Must be dimensionally correct. Condone sign errors
		A1	Correct unsimplified equation
	$v^2 = 20, \quad v = 4.47 \text{ (m s}^{-1}\text{)} \quad (4.5)$	A1	Accept $2\sqrt{5}$
		(3)	
3b alt	Working from B to A: $\frac{1}{2} \times 4 \times v^2 + 16 = 40g \sin \alpha$	M1	Requires all 3 terms. Must be dimensionally correct. Condone sign errors
		A1	Correct unsimplified equation
	$v^2 = 20, \quad v = 4.47 \text{ (m s}^{-1}\text{)} \quad (4.5)$	A1	Accept $2\sqrt{5}$
		(3)	
		[7]	
4a	Differentiate p to obtain v :	M1	
	$\mathbf{v} = (3t^2 - 9t - 24)\mathbf{i} + (-3t^2 + 6t + 12)\mathbf{j}$	A1	
	Equate coefficients and obtain quadratic in	DM1	Dependent on preceding M1

Q	Scheme	Marks	Notes
	$T:$ $3T^2 - 9T - 24 = -3T^2 + 6T + 12$ $6T^2 - 15T - 36 = 0$		
	Solve for T : $3(2T + 3)(T - 4) = 0,$	M1	Independent. Solve a 3 term quadratic in T
	$T = 4$	A1	
		(5)	
4b	Differentiate v to obtain a :	M1	
	$a = (6t - 9)\mathbf{i} + (-6t + 6)\mathbf{j}$	A1	
	Use their T : $a = (6T - 9)\mathbf{i} + (-6T + 6)\mathbf{j} = 15\mathbf{i} - 18\mathbf{j}$	DM1	Dependent on the preceding M1
	Use Pythagoras: $ a = \sqrt{15^2 + 18^2}$	M1	
	$= \sqrt{549} = 23.4 \text{ (m s}^{-2}\text{)}$	A1	23.4 or better
		(5)	
		[10]	

Question Number	Scheme	Marks
1	Impulse-momentum principle: $(7\mathbf{i} - 5\mathbf{j}) = 4\mathbf{v} - 4(2\mathbf{i} + 3\mathbf{j})$	M1A1
	$\left(\mathbf{v} = \frac{15}{4}\mathbf{i} + \frac{7}{4}\mathbf{j} \right)$	A1
	$ \mathbf{v} = \frac{1}{4}\sqrt{15^2 + 7^2}$	M1
	$= \frac{1}{4}\sqrt{274} = 4.1 \text{ (m s}^{-1}\text{) (or better)}$	A1 cso
		(5)
		[5]
	Notes	
	First M1 for use of Impulse-Momentum principle, dim correct, correct no. of terms and must be a <i>difference</i> of momenta. First A1 for a correct equation Second A1 for correct velocity vector Second M1 for attempt to find magnitude of their \mathbf{v} Third A1 cso for an exact answer or 4.1 or better	
2a	Use of $P = Fv$: $280 = F \times 2$ oe	M1
	Equation of motion: $F - 75g \sin \theta = R$	M1 A1
	$140 - 75 \times 9.8 \times \frac{1}{21} = R$	
	$R = 105$ (or 110)	A1
		(4)
	Notes	
	First M1 for $280 = F \times 2$ oe Second M1 for resolving parallel to the plane with $a = 0$ with usual rules	
	First A1 for a correct equation as shown	
	Second A1 for 105 or 110	
2b	Equation of motion: $75g \sin \theta + \frac{280}{3.5} - 60 = 75a$ or $-75a$	M1A2
	$a = 0.73$ (m s ⁻²) (0.733) or -0.73 (-0.733)	A1
		(4)
		[8]
	Notes	
	First M1 for resolving parallel to the plane with $a \neq 0$ with usual rules First A1 and Second A1 for a correct equation. Deduct 1 mark for each incorrect term. (A1A0 or A0A0) (Use of 280/2 is an A error) Third A1 for 0.73 or 0.733 (allow negative answers)	
3a	Integrate: $v = \int (4t - 8) dt = 2t^2 - 8t (+C)$	M1

Question Number	Scheme	Marks
	Use $t=0, v=6 : v=2t^2-8t+6$	M1A1
	Use factor theorem or factorise: $v=2(t-1)(t-3)$ \Rightarrow at rest for $t=1$	M1
	Second value $t=3$	A1
	Alternative: verify that $v=0$ when $t=1$ then find second solution.	(5)
	Notes	
	First M1 for attempt to integrate, at least one power increasing Second M1 for using initial conditions to find an expression for v First A1 for a correct expression for v Third M1 for showing that $v=0$ when $t=1$ Second A1 for $t=3$ (N.B. this is actually B1 mark) <i>but must come from a correct v.</i>	
3b	Integrate to find distance: $s = \int v dt = \frac{2}{3}t^3 - 4t^2 + Ct$ Follow their $C \neq 0$	M1A1 ft
	Correct strategy: $\left[\frac{2}{3}t^3 - 4t^2 + Ct \right]_1^3 + \left[\frac{2}{3}t^3 - 4t^2 + Ct \right]_3^4$	M1
	$-\left(0 - \frac{8}{3}\right) + \left(\frac{8}{3} - 0\right) = \frac{16}{3}$ (m) (5.33)	A1 (4)
	Notes	
	First M1 for attempt to integrate their v (M0 if they integrate a multiple of their v), at least one power increasing First A1 ft on their v (but must include a non-zero C) Second M1 (independent) for a complete method to find the total distance Second A1 for $16/3$ or 5.3 or better N.B. If they consider $0 < t < 4$ instead of $1 < t < 4$, then treat as a MR and they can score the second M1, if they use a correct strategy for $(0,4)$.	

June 2016
IAL WME02
Mark Scheme

Question Number	Scheme	Marks
1a	Impulse-momentum equation: $(4\mathbf{i} + 3\mathbf{j}) = 3(\mathbf{v} - (3\mathbf{i} + 5\mathbf{j}))$	M1A1
	$\mathbf{v} = \frac{5}{3}\mathbf{i} + 6\mathbf{j}$	A1
	Find the magnitude: speed = $\sqrt{\left(\frac{5}{3}\right)^2 + 6^2} = 6.23 \text{ (m s}^{-1}\text{)} \text{ (6.2 or better)}$	M1A1 (5)
1b	Gain in KE = $\frac{m}{2}(\mathbf{v} ^2 - \mathbf{u} ^2) = \frac{3}{2}((\text{their } 6.23)^2 - (3^2 + 5^2))$	M1A1 ft
	= 7.17 (J) (7.2 or better) (must be +ve)	A1 (3)
		[8]
	Notes	
1a	First M1 for $\pm(4\mathbf{i} + 3\mathbf{j}) = 3(\mathbf{v} - (3\mathbf{i} + 5\mathbf{j}))$ (M0 if 3 omitted or wrong mass used or term omitted) First A1 for a correct equation Second A1 for a correct \mathbf{v} Second M1 for finding the magnitude of their \mathbf{v} Third A1 for $\frac{\sqrt{349}}{3}$, 6.2 or better.	
1b	M1 for $\pm\frac{3}{2}((\text{their } 6.23)^2 - (3^2 + 5^2))$ (M0 if 3 omitted or wrong mass used or term omitted) Also M0 for $\pm\frac{3}{2}\left\{\left(\frac{5}{3}\mathbf{i} + 6\mathbf{j}\right)^2 - (3\mathbf{i} + 5\mathbf{j})^2\right\}$ unless it becomes $\pm\frac{3}{2}\left\{\left(\left(\frac{5}{3}\right)^2 + 6^2\right) - (3^2 + 5^2)\right\}$ First A1ft on their \mathbf{v} for a correct expression Second A1 for 43/6 oe, 7.2 or better.	

Jan 2018
Mechanics WME02
Mark Scheme

Q	Scheme	Marks	Notes
1.	Impulse- momentum equation	M1	Must be subtracting velocities (or equivalent). Dimensionally correct.
	$4\mathbf{i} + 5\mathbf{j} = \frac{1}{2}(\mathbf{v} - (2\mathbf{i} - 3\mathbf{j}))$	A1	Correct unsimplified equation.
	$\mathbf{v} = 10\mathbf{i} + 7\mathbf{j}$	A1	Seen or implied
	KE Gain	M1	Dimensionally correct. Condone \pm Must be difference of two KE terms.
	$= \frac{1}{2} 0.5(10^2 + 7^2 - (2^2 + (-3)^2))$	A1ft	Correct unsimplified expression Follow their v . Condone \pm
	$= 34 \text{ J}$	A1	CSO
		(6)	
2(a)	Use of $a = \frac{dv}{dt}$	M1	Usual rules for differentiation. Condone slip in multiplying brackets
	$v = 3t - 2t^2 - 1, a = \frac{dv}{dt} = 3 - 4t$	A1	
	$t = \frac{1}{2}, a = 1 \text{ (m s}^{-2}\text{)}$	A1	CSO
		(3)	
2(b)	$v = 0 \Rightarrow t = 0.5$	B1	Seen or implied
	$s = \int 3t - 2t^2 - 1 dt$	M1	Usual rules for integration
	$= \frac{3t^2}{2} - \frac{2t^3}{3} - t(+C)(= F(t))$	A1ft	Follow their v
	Correct strategy for distance	M1	For their "0.5" in (0,1) Must take account of change in direction
	$-[F(t)]_0^{0.5} + [F(t)]_{0.5}^1 = F(1) - 2F(0.5) + F(0)$	A1	Or equivalent, accept \pm . For their $F(t)$
	$\left(= \frac{5}{24} + \frac{1}{24} \right) = 0.25 \text{ m}$	A1	CSO
			NB Candidates who show no working and use their calculator to integrate must be starting with the correct function and show no errors in order to be able to score any marks. Full marks are available for a correct answer with no error seen.
		(6)	
		[9]	

Question Number	Scheme	Marks	Notes
3.	$v = (2t - 3)(t - 2) = 0$	M1	Solve for $v = 0$
	$t = \frac{3}{2}$ or 2	A1	Both values
			The first two marks could be implied by the use of 2 and $\frac{3}{2}$ as limits in the integration
	$\int 2t^2 - 7t + 6 dt$	M1	Use of $s = \int v dt$
	$= \frac{2}{3}t^3 - \frac{7}{2}t^2 + 6t (+C)$	A1	Correct integration
	$s = \int_0^{\frac{3}{2}} v dt - \int_{\frac{3}{2}}^2 v dt + \int_2^3 v dt$	M1	Correct strategy for distance. Accept equivalent e.g. $s = \int_0^3 v dt + 2 \left \int_{\frac{3}{2}}^2 v dt \right $
	$= \left[\frac{2}{3}t^3 - \frac{7}{2}t^2 + 6t \right]_0^{\frac{3}{2}}$ $- \left[\frac{2}{3}t^3 - \frac{7}{2}t^2 + 6t \right]_{\frac{3}{2}}^2$ $+ \left[\frac{2}{3}t^3 - \frac{7}{2}t^2 + 6t \right]_2^3$		$= \frac{27}{8} + \frac{1}{24} + \frac{7}{6}$
	$= \frac{55}{12}$	A1	4.6 or better from correct working
		6	

NB Marks changed - 3rd M1 is shown as A1 on open.

Question Number	Scheme	Marks	
3a	Use $\mathbf{v} = \lambda(\mathbf{i} + \mathbf{j})$: $6T^2 + 6T = 3T^2 + 24$	M1	Form an equation in t, T or λ $\lambda^2 - 108\lambda + 2592 = 0$
	Solve for T $3T^2 + 6T - 24 = 0$,	M1	Simplify to quadratic in t, T or λ and solve.
	$(T + 4)(T - 2) = 0, T = 2$	A1	$T = 2$ only
		(3)	
	If they score M1 and then state $T = 2$ allow 3/3		
	If they guess $T = 2$ and show that it works then allow 3/3.		
	If all we see is $T = 2$ with no equation then 0/3 for (a) but full marks are available for (b) and (c).		
3b	Differentiate: $\mathbf{a} = (12t + 6)\mathbf{i} + 6t\mathbf{j}$	M1	Majority of powers going down Need to be considering both components
		A1	Correct in t or T
	$= 30\mathbf{i} + 12\mathbf{j}$ (m s ⁻²)	A1	CAO
		(3)	
3c	Integrate : $\mathbf{r} = (2t^3 + 3t^2(+A))\mathbf{i} + (t^3 + 24t(+B))\mathbf{j}$	M1	Clear evidence of integration. Need to be considering both components Do not need to see the constant(s)
		A2	-1 each error
	If the integration is seen in part (a) it scores no marks at that stage, but if the result is used in part (c) then the M1A2 is available in part (c)		
	$\mathbf{OA} = 28\mathbf{i} + 56\mathbf{j}$ Use their T		
	Distance = $28\sqrt{5} = 62.6$ (m)	DM1	Dependent on previous M1 Use of Pythagoras on their \mathbf{OA}
		A1	63 or better , $\sqrt{3920}$
		(5)	
	NB: Incorrect T can score 2/3 in (b) and 4/5 in (c)		
		[11]	