

OCR

Oxford Cambridge and RSA

...day June 20XX – Morning/Afternoon

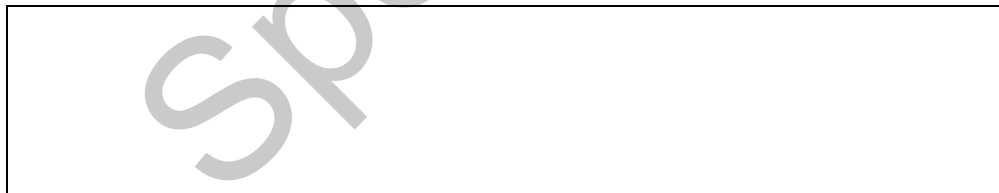
A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

SAMPLE MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100



This document consists of 20 pages

Text Instructions

1. Annotations and abbreviations

| Annotation in scoris | Meaning |
|------------------------------------|---|
| ✓ and ✕ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| Other abbreviations in mark scheme | Meaning |
| E1 | Mark for explaining a result or establishing a given result |
| dep* | Mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This indicates that the instruction In this question you must show detailed reasoning appears in the question. |

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

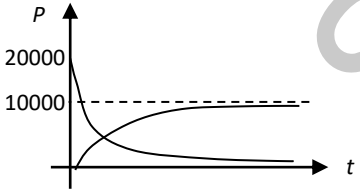
- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g . E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question | Answer | Marks | AOs | Guidance |
|----------|---|--|--|--|
| 1 | $\text{Area} = 5 = \frac{1}{2} \times 7^2 \times \theta$ $\theta = \frac{10}{49} [= 0.204]$ | M1 A1 [2] | 3.1a 1.1 | |
| 2 | $\frac{3}{1-r} = 8$ $\Rightarrow 3 = 8(1-r)$ $\Rightarrow r = \frac{5}{8}$ | M1 M1 A1 [3] | 1.1 1.1 1.1 | Use of correct formula Clearing fraction |
| 3 | <p>Either</p> $ 2x-1 \geq 4$ $\Rightarrow 2x-1 \geq 4$ <p>or $2x-1 \leq -4$</p> <hr/> <p>Or</p> $(2x-1)^2 \geq 16$ $4x^2 - 4x - 15 \geq 0$ $(2x-5)(2x+3) \geq 0$ $\Rightarrow x \geq 2\frac{1}{2}$ $\Rightarrow x \leq -1\frac{1}{2}$ $\{x: x \leq -1\frac{1}{2}\} \cup \{x: x \geq 2\frac{1}{2}\}$ | M1 M1 M1 M1 A1 A1 [4] | 1.1 1.1 1.1 1.1 1.1 2.5 | M1 for sketch graph of $y = (2x-1)^2$ and $y = 16$ M1 for $x = 2\frac{1}{2}, -1\frac{1}{2}$ OR $x \geq 2\frac{1}{2}$ or $x \leq -1\frac{1}{2}$ If final ans not in one of these forms then withhold final A1 |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|--|---|--|--|
| 4 | (a) | $\frac{dy}{dx} = \frac{1}{2} (1 - 3x^2)^{-\frac{1}{2}} \cdot (-6x)$ $= \frac{-3x}{\sqrt{(1 - 3x^2)}}$ | B1 M1 A1 [3] | 1.1 1.1 1.1 | $\frac{1}{2} u^{-\frac{1}{2}}$ soi Chain rule oe, but must simplify $\frac{1}{2} \times 6$ |
| 4 | (b) | $\frac{dy}{dx} = \frac{(3x+2) \cdot 2x - x^2 \cdot 3}{(3x+2)^2}$ $= \frac{3x^2 + 4x}{(3x+2)^2}$ | M1 A1 A1 [3] | 1.1 1.1 1.1 | Quotient rule or product rule oe, but must simplify numerator |
| 5 | (a) | Horizontal component = $100 \times \cos 32^\circ$ 84.8 N | M1 A1 [2] | 3.3 1.1 | |
| 5 | (b) | Frictional force is $100 \cos \theta$ [with $\theta < 32^\circ$] So frictional force increases. (oe) | M1 E1 [2] | 3.4 2.2a | |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|---|---|---|--|
| 6 | (a) | $A3 * \pi$ oe | B1 [1] | 2.2a | Or $0.125 \times \pi$ oe |
| 6 | (b) | $\frac{1}{2} D2 + D3 + D4 + D5 + \frac{1}{2} D6$ | B1 [1] | 2.2a | Or equivalent expressed in words. |
| 6 | (c) | $5.0766 \times 0.3927 = 1.9935...$ $1.99 \text{ (units}^2\text{) (to 3sf)}$ | M1 A1 [2] | 1.1 1.1 | Or $5.0766 \times \frac{\pi}{8}$ |
| 7 | (a) | $y = ut \sin \theta - \frac{1}{2} gt^2$ stated and used $y = 26 \times \frac{12}{13} t - 5t^2$ $= 24t - 5t^2$ $x = 26 \times \frac{5}{13} t$ $= 10t$ | M1 E1 M1 A1 [4] | 3.3 2.1 3.4 1.1 | AG Use of $\frac{5}{13}$ Accept any form |
| 7 | (b) | We require $16 = 24t - 5t^2$ Solving $5t^2 - 24t + 16 = 0$ $((5t - 4)(t - 4) = 0 \text{ or } ...)$ $t = 0.8 \text{ or } 4$ Distances are $10 \times 0.8 = 8 \text{ m}$ and $10 \times 4 = 40 \text{ m}$. | M1 M1 A1 B1FT [4] | 3.4 1.1 1.1 3.2a | Equating their y expression to 16 Method that could give 2 correct roots for their quadratic. Implied by 2 correct roots for their quadratic Cao FT only their t |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|--|---|--|---|
| 7 | (c) | E.g. Air resistance should be included E.g. The balloon should not be treated as a particle E.g. Horizontal force due to wind should be considered | B1 B1 [2] | 3.5c 3.5c | Any two appropriate factors that would have an impact on the model. |
| 8 | | let $u = x^2$, $u' = 2x$, $v' = e^{2x}$, $v = \frac{1}{2}e^{2x}$ $\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \int 2x \cdot \frac{1}{2}e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \int x e^{2x} dx$ let $u = x$, $u' = 1$, $v' = e^{2x}$, $v = \frac{1}{2}e^{2x}$ $\int x e^{2x} dx = \frac{1}{2}x e^{2x} - \int \frac{1}{2}e^{2x} dx$ $= \frac{1}{2}x e^{2x} - \frac{1}{4}e^{2x} (+c)$ so $\int x^2 e^{2x} dx = \frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x} + c$ | M1A1 A1 M1 A1 A1 A1 [7] | 1.1a 1.2 1.1 1.1a 1.1 1.1 2.5 | Do not award if no '+ c' |

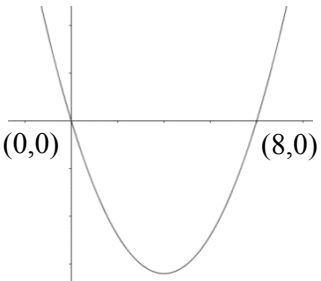
| Question | | | Answer | Marks | AOs | Guidance |
|----------|-----|--|---|---------------------------|---|---|
| 9 | (a) | (i) | Let acceleration be a in the direction of motion. N2L in direction of motion gives $-F = ma$ so $a = -\frac{F}{m}$, which is constant. | M1 A1 | 3.3 1.1 | Decide to use N2L to find acceleration No need to say 'constant' |
| | | (ii) | (As a constant) use <i>suvat</i> , giving $S = 0 \times T - \frac{1}{2} \times \left(-\frac{F}{m}\right) T^2$ so $S = \left(\frac{F}{2m}\right) T^2$ and $k = \frac{F}{2m}$ | B1 E1 [4] | 2.1 2.4 | Use appropriate (sequence of) <i>suvat</i> |
| 9 | (b) | As sliding, friction is limiting and $F = \mu R$ $R = mg$ $k = \frac{F}{2m}$ so $k = \frac{\mu mg}{2m}$ Hence $\mu = \frac{2k}{g} = \frac{2 \times 1.4}{9.8} = \frac{2}{7}$ | M1 A1 M1 A1 [4] | 3.3 3.4 1.1 2.2a | In $F = \mu R$, substitute for F & R in terms of m and g Or 0.286 (3s.f.) | |
| 10 | (a) | (i) |  | M1 M1 | 1.1 1.1 | P_G shape through O P_R shape through (0, 20000), [condone graphs for $-ve t$] |
| | | (ii) | asymptote for $P_G = 10\ 000$ Asymptote for $P_R = 0$ | A1 A1 [4] | 2.2a 2.2a | Or $p = 10\ 000$ Or $p = 0$ |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|---|-------|------|--|
| 10 | (d) | $\frac{dP_G}{dt} = 10\,000ke^{-kt}$ $\frac{dP_R}{dt} = -20\,000ke^{-kt}$ so $\frac{dP_R}{dt} = -2 \frac{dP_G}{dt}$ | M1 | 3.1b | Attempts to differentiate either or both |
| | | | A1 | 1.1 | |
| | | | A1 | 1.1 | Or in words |
| | | | E1 | 2.1 | |
| | | | [4] | | |
| 10 | (e) | $10\,000(1 - e^{-3k}) = 20\,000e^{-3k}$ $\Rightarrow 1 - e^{-3k} = 2e^{-3k}$ $\Rightarrow e^{-3k} = \frac{1}{3}$ $\Rightarrow -3k = \ln\left(\frac{1}{3}\right)$ $\Rightarrow k = -\frac{1}{3}\ln\left(\frac{1}{3}\right) = 0.366 \text{ or } \frac{1}{3}\ln 3$ | M1 | 1.1a | Taking natural logs of both sides |
| | | | A1 | 1.1 | |
| | | | M1 | 1.1 | cao |
| | | | A1 | 2.1 | |
| | | | [4] | | |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|---|---|--|---------------------|
| 11 | (a) | <p>P is $(\sqrt{2}, \frac{\sqrt{2}}{2})$</p> $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ $= \frac{\cos\theta}{-2\sin\theta}$ <p>When $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{1}{2}$</p> <p>Equation of tangent is $(y - \frac{\sqrt{2}}{2}) = -\frac{1}{2}(x - \sqrt{2})$</p> $\Rightarrow y = -\frac{1}{2}x + \frac{1}{2}\sqrt{2} + \frac{1}{2}\sqrt{2}$ $\Rightarrow x + 2y = 2\sqrt{2}$ | <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>E1</p> <p>[6]</p> | <p>1.1</p> <p>3.1a</p> <p>1.1</p> <p>1.1</p> <p>2.1</p> <p>1.1</p> | <p>oe</p> <p>AG</p> |
| 11 | (b) | <p>When $x = 0$, $y = \sqrt{2}$ so A is $(0, \sqrt{2})$</p> <p>When $y = 0$, $x = 2\sqrt{2}$ so B is $(2\sqrt{2}, 0)$</p> <p>Area of triangle = $\frac{1}{2}\sqrt{2} \times 2\sqrt{2} = 2$ units²</p> | <p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p> | <p>1.1</p> <p>1.1</p> <p>1.1</p> | |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|--|---------------------------------------|-------------------------------|---|
| 12 | (a) | Require both components zero at the same <i>time</i> i component zero only when $t = 1$ and j component only when $t = -1$ so there are no such times | M1 A1 [2] | 3.1b 2.4 | May be implied but must be clear Or say j component ≥ 2 since $t \geq 0$ |
| 12 | (b) | This requires use of the velocity vector Travelling due north means that the i component is zero and the j component +ve So we need $2t - 2 = 0$ for i component, giving $t = 1$. This gives j component $4 > 0$ so yes at $t = 1$. | M1 A1 [2] | 3.3 2.4 | Recognise velocity vector required Must test j component |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|---|------------|-------------|--|
| 12 | (c) | This requires use of the position vector either | M1 | 3.1b | Recognise position vector required |
| | | $\mathbf{r} = \int \mathbf{v} dt$ so $\mathbf{r} = \int ((2t-2)\mathbf{i} + (2t+2)\mathbf{j}) dt = (t^2 - 2t + C)\mathbf{i} + (t^2 + 2t + D)\mathbf{j}$ | M1 | 1.1 | May use + C instead |
| | | $\mathbf{r} = 3\mathbf{i} + 14\mathbf{j}$ when $t = 3$ so $C = 0$ and $D = -1$ so $\mathbf{r} = (t^2 - 2t)\mathbf{i} + (t^2 + 2t - 1)\mathbf{j}$ | A1 | 1.1 | |
| | | or | | | |
| | | $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j}$ when $t = 3$ $\mathbf{v} = 4\mathbf{i} + 8\mathbf{j}$, $\mathbf{r} = (4\mathbf{i} + 8\mathbf{j})(t-3) + \frac{1}{2}(2\mathbf{i} + 2\mathbf{j})(t-3)^2 + 3\mathbf{i} + 14\mathbf{j}$ and so $\mathbf{r} = (t^2 - 2t)\mathbf{i} + (t^2 + 2t - 1)\mathbf{j}$ | M1 | 1.1 | Must find a but may omit $3\mathbf{i} + 14\mathbf{j}$ |
| | | | A1 | 1.1 | |
| | | the boat is NE of O when the i and j components are equal and +ve we require $t^2 - 2t = t^2 + 2t - 1$ so $t = 0.25$ this gives components of -0.4375 so no. | M1 | 3.2b | Award even if +ve not mentioned |
| | | | A1 | 2.1 | Must be complete argument |
| | | | [5] | | |

| Question | Answer | Marks | AOs | Guidance |
|----------|--|---|---|---|
| 13 | <p>DR discriminant = $k^2 - 8k$ $\Rightarrow k^2 - 8k > 0$</p>  <p>$\Rightarrow k < 0$ or $k > 8$</p> | <p>B1 M1 E1</p> <p>A1</p> <p>[4]</p> | <p>1.2 1.1 2.4</p> <p>2.5</p> | <p>Or give table of values, oe</p> <p>or $(-\infty, 0) \cup (8, \infty)$ or $\{k : k < 0\} \cup \{k : k > 8\}$</p> |
| 14 (a) | <p>Using N2L in direction of motion, acceleration $a \text{ m s}^{-1}$ $(40 - 10 - 5) = 10a$ so $a = 2.5$ Mark force in bar as tension T [N] Either For A $T - 10 = 6a$</p> <hr/> <p>or For B $40 - 5 - T = 4a$ so tension is 25 [N]</p> | <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[4]</p> | <p>3.3</p> <p>1.1</p> <p>3.4</p> <p>3.4</p> <p>1.1</p> | <p>$F = ma$ and all forces present</p> <p>Not required, may be implied</p> <p>Allow their a</p> <p>Allow their a</p> <p>Or $T = 25$</p> |

| Question | | Answer | Marks | AOs | Guidance |
|----------|-----|---|-------|------|--|
| 14 | (b) | Without the force from the string, N2L in direction of motion $-15 = 10a^*$ so $a^* = -1.5$ Mark force in bar as tension T^* [N] either For A $T^* - 10 = 6a^* = -9$ | M1 | 3.3 | |
| | | | A1 | 1.1 | Not required, may be implied |
| | | or For B $-5 - T^* = 4a^* = -6$ so $T^* = 1$ giving a force of 1 [N] which is a tension | M1 | 3.4 | Allow their a |
| | | | A1 | 1.1 | |
| | | | A1 | 2.2a | Must be made clear in some way |
| | | | [5] | | |
| 15 | | Let the reactions of the supports on the shelf be U N at C and V N at D Neither U nor V can be negative if the shelf does not tip Any position between C and D must give $U > 0$ and $V > 0$ Consider putting the load between A and C, x cm from C cw moments about C If $V \geq 0$ then $W \times 60 \geq 3W \times x$ so $x \leq 20$ Consider putting the load between D and B, y cm from D anti-cw moments about D If $U \geq 0$ then $W \times 30 \geq 3W \times y$ so $y \leq 10$ The load must be placed not closer than 10 cm to A and 50 cm to B oe | M1 | 2.2a | May be implied |
| | | | B1 | 2.2a | Need not show but must be stated |
| | | | M1 | 3.1b | moments about C |
| | | | A1 | 1.1 | allow < |
| | | | B1 | 3.1b | allow < |
| | | | A1 | 3.2a | Must be clear statement and include CD |
| | | [6] | | | |

| Question | AO1 | AO2 | AO3(PS) | AO3(M) | Total |
|---------------|-----------|-----------|-----------|-----------|------------|
| 1 | 1 | | 1 | | 2 |
| 2 | 3 | | | | 3 |
| 3 | 3 | 1 | | | 4 |
| 4 a | 3 | | | | 3 |
| 4 b | 3 | | | | 3 |
| 5 a | 1 | | | 1 | 2 |
| 5 b | | 1 | | 1 | 2 |
| 6 a | | 1 | | | 1 |
| 6 b | | 1 | | | 1 |
| 6 c | 2 | | | | 2 |
| 7 a | 1 | 1 | | 2 | 4 |
| 7 b | 2 | | 1 | 1 | 4 |
| 7 c | | | | 2 | 2 |
| 8 | 6 | 1 | | | 7 |
| 9 a | 1 | 2 | | 1 | 4 |
| 9 b | 1 | 1 | | 2 | 4 |
| 10 a | 2 | 2 | | | 4 |
| 10 b | | | | 2 | 2 |
| 10 c | | | | 1 | 1 |
| 10 d | 2 | 1 | 1 | | 4 |
| 10 e | 3 | 1 | | | 4 |
| 11 a | 4 | 1 | 1 | | 6 |
| 11 b | 3 | | | | 3 |
| 12 a | | 1 | 1 | | 2 |
| 12 b | | 1 | | 1 | 2 |
| 12 c | 2 | 1 | 2 | | 5 |
| 13 | 2 | 1 | 1 | | 4 |
| 14 a | 2 | | | 2 | 4 |
| 14 b | 2 | 1 | | 2 | 5 |
| 15 | 1 | 2 | 3 | | 6 |
| Totals | 50 | 21 | 11 | 18 | 100 |