3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b - 1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

Answer ALL questions. Write your answers in the spaces provided.

1. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

3. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

Leave blank

6. Given that a and b are positive constants, solve the simultaneous equations

$$ab = 25$$

$$\log_4 a - \log_4 b = 3$$

Show each step of your working, giving exact values for a and b.

(6)

Leave blank

3. Solve, giving each answer to 3 significant figures, the equations

(a) $4^a = 20$

(2)

(b) $3 + 2\log_2 b = \log_2(30b)$

(5)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Leave blank

6. Given that

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1), \quad x > \frac{1}{2}$$

(a) show that

$$4x^2 - 16x - 9 = 0$$

(5)

(b) Hence solve the equation

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1), \quad x > \frac{1}{2}$$

(2)

Leave blank

14. (i) Given that

$$\log_a x + \log_a 3 = \log_a 27 - 1, \text{ where } a \text{ is a positive constant}$$

find, in its simplest form, an expression for x in terms of a.

(4)

(ii) Solve the equation

$$(\log_5 y)^2 - 7(\log_5 y) + 12 = 0$$

showing each step of your working.

(4)

Leave blank

3. Answer this question without a calculator, showing all your working and giving your answers in their simplest form.

(i) Solve the equation

$$4^{2x+1} = 8^{4x}$$

(3)

(ii) (a) Express

$$3\sqrt{18} - \sqrt{32}$$

in the form $k\sqrt{2}$, where k is an integer.

(2)

(b) Hence, or otherwise, solve

$$3\sqrt{18} - \sqrt{32} = \sqrt{n}$$

(2)

Leave blank

13. (a) Show that the equation

$$2\log_2 y = 5 - \log_2 x \qquad x > 0, y > 0$$

may be written in the form $y^2 = \frac{k}{x}$ where k is a constant to be found.

(3)

(b) Hence, or otherwise, solve the simultaneous equations

$$2\log_2 y = 5 - \log_2 x$$

$$\log_x y = -3$$

for $x > 0, y > 0$

(5)

Leave blank

2. Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

Leave blank

2. Find the exact solutions, in their simplest form, to the equations

(a) $2 \ln (2x + 1) - 10 = 0$

(2)

(b) $3^x e^{4x} = e^7$

(4)

Leave blank

6. Find algebraically the exact solutions to the equations

(a) $\ln(4 - 2x) + \ln(9 - 3x) = 2\ln(x + 1), \qquad -1 < x < 2$

(5)

(b) $2^x e^{3x+1} = 10$

Give your answer to (b) in the form $\dfrac{a + \ln b}{c + \ln d}$ where a, b, c and d are integers.

(5)

Leave blank

7. (i) $2\log(x + a) = \log(16a^6)$, where a is a positive constant

Find x in terms of a, giving your answer in its simplest form.

(3)

(ii) $\log_3(9y + b) - \log_3(2y - b) = 2$, where b is a positive constant

Find y in terms of b, giving your answer in its simplest form.

(4)

Leave blank

8. (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0$$

giving your answer to 2 decimal places.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)