Question	Scheme	Marks	AOs
8	$2+4\lambda-2(4-2\lambda)-6+\lambda=6 \Rightarrow \lambda=...$	M1	1.1b
	$\lambda=2 \Rightarrow$ Required point is $(2+2(4),\ 4+2(-2),\ -6+2(1))$ $(10,\ 0,\ -4)$	A1	1.1b
	$2+t-2(4-2t)-6+t=6 \Rightarrow t=...$	M1	3.1a
	$t=3$ so reflection of $(2,4,-6)$ is $(2+6(1),4+6(-2),-6+6(1))$	M1	3.1a
	$(8,\ -8,\ 0)$	A1	1.1b
	$\begin{pmatrix}10\\0\\-4\end{pmatrix}-\begin{pmatrix}8\\-8\\0\end{pmatrix}=\begin{pmatrix}2\\8\\-4\end{pmatrix}$	M1	3.1a
	$\mathbf{r}=\begin{pmatrix}10\\0\\-4\end{pmatrix}+k\begin{pmatrix}1\\4\\-2\end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r}-\begin{pmatrix}10\\0\\-4\end{pmatrix}\right)\times\begin{pmatrix}1\\4\\-2\end{pmatrix}=\mathbf{0}$	A1	2.5
		(7)	
		(7 marks)	

Notes:

M1: Substitutes the parametric equation of the line into the equation of the plane and solves for λ

A1: Obtains the correct coordinates of the intersection of the line and the plane

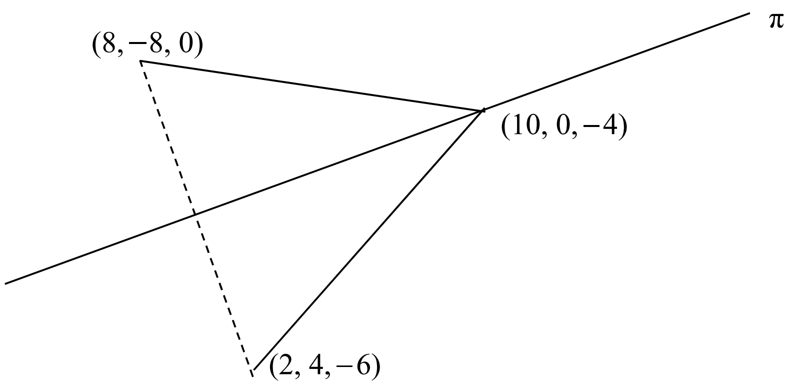
M1: Substitutes the parametric form of the line perpendicular to the plane passing through $(2,\ 4,\ -6)$ into the equation of the plane to find t

M1: Find the reflection of $(2,\ 4,\ -6)$ in the plane

A1: Correct coordinates

M1: Determines the direction of l by subtracting the appropriate vectors

A1: Correct vector equation using the correct notation



Question	Scheme	Marks	AOs
2(a)	$\begin{pmatrix}3\\-4\\2\end{pmatrix}\bullet\begin{pmatrix}6\\2\\12\end{pmatrix}=18-8+24$	M1	3.1a
	$d=\dfrac{18-8+24-5}{\sqrt{3^2+4^2+2^2}}$	M1	1.1b
	$=\sqrt{29}$	A1	1.1b
		(3)	
(b)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=\ldots$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\ldots$	M1	2.1
	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\5\end{pmatrix}=0$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\-2\end{pmatrix}=0$ $\therefore\ -\mathbf{i}-3\mathbf{j}+\mathbf{k}$ is perpendicular to Π_2	A1	2.2a
		(2)	
(c)	$\begin{pmatrix}-1\\-3\\1\end{pmatrix}\bullet\begin{pmatrix}3\\-4\\2\end{pmatrix}=-3+12+2$	M1	1.1b
	$\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}\cos\theta=11$ $\Rightarrow\cos\theta=\dfrac{11}{\sqrt{(-1)^2+(-3)^2+1^2}\sqrt{(3)^2+(-4)^2+2^2}}$	M1	2.1
	So angle between planes $\theta=52^\circ$ *	A1*	2.4
		(3)	
		(8 marks)	

Notes:

(a)

M1: Realises the need to and so attempts the scalar product between the normal and the position vector

M1: Correct method for the perpendicular distance

A1: Correct distance

(b)

M1: Recognises the need to calculate the scalar product between the given vector and both direction vectors

A1: Obtains zero both times and makes a conclusion

(c)

M1: Calculates the scalar product between the two normal vectors

M1: Applies the scalar product formula with their 11 to find a value for cos θ

A1*: Identifies the correct angle by linking the angle between the normal and the angle between the planes

Question	Scheme	Marks	AOs
7(a) **Way 1**	$1+2\lambda=1+t$ $-1-\lambda=-t$ $4+3\lambda=3+2t$ $\Rightarrow t=\ldots \text{or } \lambda=\ldots$	M1	3.1a
	Checks the third equation with $t=2$ and $\lambda=1$ Or shows that the coordinate (3, -2, 7) lies on both lines	A1	1.1b
	As the lines intersect at a point the lines lie in the same plane.	A1	2.4
		(3)	
(a) **Way 2**	$1=1+2\lambda+t$ $-1=\quad-\lambda-t$ $4=3+3\lambda+2t$ $\Rightarrow t=\ldots \text{or } \lambda=\ldots$ $1=1+2\lambda+t$ $0=-1-\lambda-t$ $3=4+3\lambda+2t$ $\Rightarrow t=\ldots \text{or } \lambda=\ldots$	M1	3.1a
	Checks the third equation with $t=2$ and $\lambda=-1$ Checks the third equation with $t=-2$ and $\lambda=1$	A1	1.1b
	Second coordinates lie on the plane; therefore, the lines lie on the same plane	A1	2.4
		(3)	
(a) **Way 3**	$x=1+t,\quad y=-t,\quad z=3+2t$ $\dfrac{1+t-1}{2}=\dfrac{-t+1}{-1}=\dfrac{3+2t-4}{3}$ Solves a pair of equations $t=\ldots$	M1	3.1a
	Solve two pairs of equations to find $t=2$	A1	1.1b
	As the lines intersect at a point the lines lie in the same plane.	A1	2.4
		(3)	
(a) **Way 4 (Using Further Pure 2 knowledge)**	$\begin{pmatrix}2\\-1\\3\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow 2x-y+3z=0$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow x-y+2z=0$ attempts to solve the equations to find a normal vector OR attempts the cross product $\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\ldots$ **AND** either finds the equation of one plane **OR** finds dot product between the normal and one coordinate	M1	3.1a

	$\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ **OR** $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$ or $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=...$		
	Achieves the correct planes containing **each** line $\mathbf{r}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ or $x-y-z=-2$ o.e. **OR** Shows that $\begin{pmatrix}1\\-1\\4\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ **and** $\begin{pmatrix}1\\0\\3\end{pmatrix}.\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2$ o.e.	A1	1.1b
	Both planes are the same, therefore the lines lie in the same plane.	A1	2.4
		(3)	
(b)	e.g. $\mathbf{r}=\begin{pmatrix}1\\0\\3\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}1\\-1\\4\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}2\\-1\\3\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}=\begin{pmatrix}3\\-2\\7\end{pmatrix}+p\begin{pmatrix}0\\-1\\1\end{pmatrix}+q\begin{pmatrix}1\\-1\\2\end{pmatrix}$ or $\mathbf{r}.k\begin{pmatrix}1\\-1\\-1\end{pmatrix}=-2k$	B1	2.5
		(1)	
(c) **Way 1**	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\2\end{pmatrix}=2+1+6$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\cos\theta=9$ $\Rightarrow\cos\theta=\dfrac{9}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	

Way 2 (Using Further Pure 2 knowledge)	$\begin{pmatrix}2\\-1\\3\end{pmatrix}\times\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}1\\-1\\-1\end{pmatrix}$	M1	1.1b
	$\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}\sin\theta=\sqrt{1^2+(-1)^2+(-1)^2}$ $\Rightarrow \sin\theta=\dfrac{\sqrt{1^2+(-1)^2+(-1)^2}}{\sqrt{2^2+(-1)^2+3^2}\sqrt{1^2+(-1)^2+2^2}}$	dM1	2.1
	$\theta=11$ cao	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

Allow using $\begin{pmatrix}1\\3\\0\end{pmatrix}$ instead of $\begin{pmatrix}1\\0\\3\end{pmatrix}$ for the method mark.

Way 1

M1: Starts by attempting to find where the two lines intersect. They must set up a parametric equation for line 1 (allow sign slips and as long as the intention is clear), forms simultaneous equations by equating coordinates and attempts to solve to find a value for $t=\ldots$ or $\lambda=\ldots$.

A1: Shows that there is a unique solution by checking the third equation or shows that the coordinate (3, -2, 7) lies on both lines.

A1: Achieves the correct values $t=2$ and $\lambda=1$, checks the third equation and concludes that either

- a common point,
- the lines intersect
- the equations are consistent

therefore, the lines lie in the same plane

Way 2

M1: Finds the vector equation of the plane with the both direction vectors and one coordinate (allow a sign slip), sets equal to the other coordinate, forms simultaneous equations and attempts to solve to find a value for $t=\ldots$ or $\lambda=\ldots$.

A1: Shows that the other coordinate lies on the plane by checking the third equation.

A1: Achieves the correct values $t=-2$ and $\lambda=1$ or $t=2$ and $\lambda=-1$ and concludes that the second coordinate lie on the plane; therefore, the lines lie on the same plane

Way 3

M1: Substitutes line 2 into line 1 and solves a pair of equations to find a value for *t*. Allow slip with the position of 0 and sign slips as long as the intention is clear.

Question	Scheme	Marks	AOs
4(a)	Attempts normal vector: E.g. let $\mathbf{n} = a\mathbf{i} + b\mathbf{j} + \mathbf{k}$ then $a + 2b - 3 = 0, -a + 2b + 1 = 0$ $\Rightarrow a = ..., b = ...$ or $\mathbf{n} = (\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \times (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	M1	3.1a
	$\mathbf{n} = k(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	1.1b
	$(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}).(2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) = ...$	M1	1.1b
	$4x + y + 2z = 10$	A1	2.5
		(4)	
	Alternative:		
	$x = 2 + \lambda - \mu$ $y = 4 + 2\lambda + 2\mu \Rightarrow$ $2x + y = 8 + 4\lambda$, $y - 2z = 6 + 8\lambda$ $z = -1 - 3\lambda + \mu$	M1 A1	3.1a 1.1b
	$2(2x + y - 8) = y - 2z - 6$ $(4x + y + 2z = 10)$	M1 A1	1.1b 2.5
		(4)	
(b)	$\frac{x-1}{5} = \frac{y-3}{-3} = \frac{z+2}{4} \Rightarrow \mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$ $4(1+5\lambda) + 3 - 3\lambda + 2(4\lambda - 2) = 10 \Rightarrow \lambda = ...$	M1	3.1a
	$\lambda = \frac{7}{25} \Rightarrow \mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \frac{7}{25}(5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$	dM1	1.1b
	$\left(\frac{12}{5}, \frac{54}{25}, -\frac{22}{25}\right)$	A1	1.1b
		(3)	
	Alternative: $4x + \left(-\frac{3}{5}(x-1) + 3\right) + 2\left(\frac{4}{5}(x-1) - 2\right) = 10 \Rightarrow x = ...$	M1	3.1a
	$\Rightarrow y = ..., z = ...$	M1	1.1b
	$\left(\frac{12}{5}, \frac{54}{25}, -\frac{22}{25}\right)$	A1	1.1b
		(3)	
(c)	$(4\mathbf{i} + \mathbf{j} + 2\mathbf{k}).(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 8 - 1 + 6 = 13$ $13 = \sqrt{14}\sqrt{21}\cos\theta \Rightarrow \theta = ...$	M1	1.1b
	$\theta = 41°$	A1	1.1b
		(2)	
			(9 marks)

Question	Scheme	Marks	AOs
7(a)	$\begin{pmatrix}-1\\2\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=-2+6-4=0$ and $\begin{pmatrix}2\\0\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=4+0-4=0$ Alt: $\begin{pmatrix}-1\\2\\1\end{pmatrix}\times\begin{pmatrix}2\\0\\1\end{pmatrix}=\begin{pmatrix}2\times1-1\times0\\-(-1\times1-1\times2)\\-1\times0-2\times2\end{pmatrix}=\ldots$	M1	1.1b
	As 2**i** + 3**j** – 4**k** is perpendicular to both direction vectors (two non-parallel vectors) of *Π* then it must be perpendicular to *Π*	A1	2.2a
		(2)	
(b)	$\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}=\begin{pmatrix}3\\3\\2\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix}\Rightarrow\ldots$	M1	1.1a
	$2x+3y-4z=7$	A1	2.2a
		(2)	
(c)	$\dfrac{\lvert 2(4+t)+3(-5+6t)-4(2-3t)-7\rvert}{\sqrt{2^2+3^2+(-4)^2}}=2\sqrt{29}\Rightarrow t=\ldots$	M1	3.1a
	$t=-\dfrac{9}{8}$ and $t=\dfrac{5}{2}$	A1	1.1b
	$\boldsymbol{r}=\begin{pmatrix}4\\-5\\2\end{pmatrix}-\dfrac{9}{8}\begin{pmatrix}1\\6\\-3\end{pmatrix}=\ldots$ or $\boldsymbol{r}=\begin{pmatrix}4\\-5\\2\end{pmatrix}+\dfrac{5}{2}\begin{pmatrix}1\\6\\-3\end{pmatrix}=\ldots$	M1	1.1b
	$\left(\dfrac{23}{8},-\dfrac{47}{4},\dfrac{43}{8}\right)$ and $\left(\dfrac{13}{2},10,-\dfrac{11}{2}\right)$	A1	2.2a
		(4)	
			(8 marks)

Notes:

(a)
M1: Attempts the scalar product of each direction vector and the vector 2**i** + 3**j** – 4**k.** Some numerical calculation is required, just "= 0" is insufficient. Alternatively, attempts the cross product (allow sign slips) with the two direction vectors.
A1: Shows that both scalar products = 0 (minimum $-2+6-4=0$ and $4-4=0$) and makes a minimal conclusion with no erroneous statements. If using cross product, the calculation must be correct, and a minimal conclusion given with no erroneous statements.

Question	Scheme	Marks	AOs
8(a)	A complete method to use the scalar product of the direction vectors and the angle 120° to form an equation in a $$\frac{\begin{pmatrix}2\\a\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}}{\sqrt{2^2+a^2}\sqrt{1^2+(-1)^2}}=cos\,1\,20$$	M1	3.1b
	$$\frac{a}{\sqrt{4+a^2}\sqrt{2}}=-\frac{1}{2}$$	A1	1.1b
	$2a=-\sqrt{4+a^2}\sqrt{2}\Rightarrow 4a^2=8+2a^2\Rightarrow a^2=4\Rightarrow a=\ldots$	M1	1.1b
	$a=-2$	A1	2.2a
		(4)	
(b)	Any two of **i**: $-1+2\lambda=4$ (1) **j**: $5+\text{'their}-2\text{'}\lambda=-1+\mu$ (2) **k**: $2=3-\mu$ (3)	M1	3.4
	Solves the equations to find a value of $\lambda\left\{=\frac{5}{2}\right\}$ **and** $\mu\{=1\}$	M1	1.1b
	$r_1=\begin{pmatrix}-1\\5\\2\end{pmatrix}+\frac{5}{2}\begin{pmatrix}2\\\text{'their}-2\text{'}\\0\end{pmatrix}$ or $r_2=\begin{pmatrix}4\\-1\\3\end{pmatrix}+1\begin{pmatrix}0\\1\\-1\end{pmatrix}$	dM1	1.1b
	(4,0,2) or $\begin{pmatrix}4\\0\\2\end{pmatrix}$	A1	1.1b
	Checks the third equation e.g. $\lambda=\frac{5}{2}$: **L HS** $=5-2\lambda=5-5=0$ $\mu=1$: **R HS** $=-1+\mu=-1+1=0$ therefore **common point/intersect/consistent/tick** or substitutes the values of λ and μ into the relevant lines and achieves the same coordinate	B1	2.1
		(5)	
(c)	Full attempt to find the minimum distance from the point of intersection (nest) to the plane (ground) E.g. Minimum distance $=\frac{\lvert 2\times\text{'4'}+(-3)\times\text{'0'}+1\times\text{'2'}-2\rvert}{\sqrt{2^2+(-3)^2+1)^2}}=\ldots$ **Alternatively** $\mathbf{r}=\begin{pmatrix}\text{'4'}\\\text{'0'}\\\text{'2'}\end{pmatrix}+\lambda\begin{pmatrix}2\\-3\\1\end{pmatrix}$ $2(\text{'4'}+2\lambda)-3(\text{'0'}-3\lambda)+(\text{'2'}+\lambda)=2\Rightarrow$ $\lambda=\ldots\left\{-\frac{4}{7}\right\}$	M1 A1ft	3.1b 3.4

	$\mathbf{r} = \begin{pmatrix} '4' \\ '0' \\ '2' \end{pmatrix} + '-\frac{4}{7}'\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{20}{7} \\ \frac{12}{7} \\ \frac{10}{7} \end{pmatrix}$ Minimum distance = $\sqrt{\left(2 \times -\frac{4}{7}'\right)^2 + \left(-3 \times -\frac{4}{7}'\right)^2 + \left(1 \times -\frac{4}{7}\right)^2} =$ … $= \sqrt{\left('4' - '\frac{20}{7}'\right)^2 + \left('0' - '\frac{12}{7}'\right)^2 + \left('2' - '\frac{10}{7}'\right)^2} =$...		
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$or awrt 2.1	A1	2.2b
		(3)	
	Alternative Find perpendicular distance from plane to the origin $2x - 3y + z = 2$ $\|n\| = \sqrt{2^2 + (-3)^2 + 1^2} = \sqrt{14}$ shortest distance = $\frac{2}{\sqrt{14}}$ Find perpendicular distance from the plane containing the point of intersection to the origin $2x - 3y + z = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 10$ shortest distance = $\frac{10}{\sqrt{14}}$ Minimum distance = $\frac{10}{\sqrt{14}} - \frac{2}{\sqrt{14}}$	M1 A1ft	3.1b 3.4
	$\frac{8}{\sqrt{14}}$ or $\frac{4\sqrt{14}}{7}$or awrt 2.1	A1	2.2b
		(3)	
(d)	For example Not reliable as the birds will not fly in a straight line Not reliable as angle between flights paths will not always be 120° Not reliable/reliable as the ground will not be flat/smooth Not reliable as bird’s nest is not a point	B1	3.2b
		(1)	
			(13 marks)

Notes:

(a)

M1: See scheme, allow a sign slip and cos 60

A1: Correct simplified equation in a, cos 120 must be evaluated to – ½ and dot product calculated

Note: If the candidate states either $\left|\frac{a \square b}{|a||b|}\right| = cos\,\theta$ or $\left|\frac{a}{\sqrt{4+a^2}\sqrt{2}}\right| = cos\ 6\ 0$then has the equation $\frac{a}{\sqrt{4+a^2}\sqrt{2}} = \frac{1}{2}$ award this mark. If the module of the dot product is not seen then award A0 for this equation.

Question	Scheme	Marks	AOs
5(a)	$2(\lambda-5)+3(-3\lambda-4)-2(5\lambda+3)=6 \Rightarrow \lambda=...(-2)$ $\lambda=\text{"}-2\text{"} \Rightarrow x=...$ or $y=...$ or $z=...$ or e.g. $2x+3(-3x-15-4)-2(5x+25+3)=6 \Rightarrow x=...$	**M1**	1.1b
	$(-7,\ 2,-7)$	**A1**	1.1b
		(2)	
(b)	E.g. $\mathbf{r}=\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2t\\3t\\-2t\end{pmatrix}$ meets the plane when $2(-5+2t)+3(-4+3t)-2(3-2t)=6 \Rightarrow t=...$	**M1**	3.1a
	$t=2 \Rightarrow$ mirror point is $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times4\\3\times4\\-2\times4\end{pmatrix}=...$	**M1**	1.1b
	$=\begin{pmatrix}3\\8\\-5\end{pmatrix}$	**A1**	1.1b
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+\mu\begin{pmatrix}3-(-7)\\8-2\\-5-(-7)\end{pmatrix}=...$	**ddM1**	1.1b
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+\mu\begin{pmatrix}10\\6\\2\end{pmatrix}$ *	**A1***	2.1
		(5)	
	(b) Alternative for first 2 marks:		
	Distance from (–5, −4, 3) to plane is $\dfrac{\lvert 2\times-5+3\times-4-2\times3-6\rvert}{\sqrt{2^2+3^2+2^2}}=2\sqrt{17}$	**M1**	3.1a
	$\begin{vmatrix}2k\\3k\\-2k\end{vmatrix}=4\sqrt{17} \Rightarrow 4k^2+9k^2+4k^2=16\times17 \Rightarrow k=4$ $k=4 \Rightarrow$ mirror point is $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times4\\3\times4\\-2\times4\end{pmatrix}=...$	**M1**	1.1b

(c)	Line joining mirror points intersects plane at $\begin{pmatrix}-5\\-4\\3\end{pmatrix}+\begin{pmatrix}2\times2\\3\times2\\-2\times2\end{pmatrix}$, so equation of line is $\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}-1-(-7)\\2-2\\-1-(-7)\end{pmatrix}=\ldots$	**M1**	3.1a
	$\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}6\\0\\6\end{pmatrix}$ oe e.g. $\mathbf{r}=\begin{pmatrix}-1\\2\\-1\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$	**A1**	2.5
		(2)	
	Alternative 1 to (c) (Not on spec)		
	Normal to Π_2: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-3&5\\10&6&2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ Direction of l_2: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-3&4&3\\2&3&-2\end{vmatrix}=\begin{pmatrix}-17\\0\\-17\end{pmatrix}$ equation of line is $\mathbf{r}=\begin{pmatrix}-7\\2\\-7\end{pmatrix}+s\begin{pmatrix}-17\\0\\-17\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	
	Alternative 2 to (c) (Not on spec)		
	Normal to Π_2: $\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&-3&5\\10&6&2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ $(-3\mathbf{i}+4\mathbf{j}+3\mathbf{k})\cdot(-7\mathbf{i}+2\mathbf{j}-7\mathbf{k})=8$ Π_2 is $3x-4y-3z=-8$ then e.g. solves simultaneously with Π_1 and $x=\lambda$ to give $y=2$, $z=\lambda$ So equation of line is $\mathbf{r}=\begin{pmatrix}0\\2\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	
	Alternative 3 to (c) (Not on spec)		
	As alternative 2 to find the equation of plane 2: $3x-4y-3z=-8$ Then solves simultaneously with plane 1 to give e.g. $y=2$, $x=z$ Hence $\mathbf{r}=\begin{pmatrix}s\\2\\s\end{pmatrix}$ oe	**M1** **A1**	3.1a 2.5
		(2)	

(d)	Line from (c) must lie in plane, so $\begin{pmatrix}1\\0\\1\end{pmatrix}$ is perpendicular to $\begin{pmatrix}1\\1\\a\end{pmatrix}$ $\Rightarrow \begin{pmatrix}1\\0\\1\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=0 \Rightarrow 1\times1+0\times1+1\times a=0 \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$b=\begin{pmatrix}-7\\2\\-7\end{pmatrix}.\begin{pmatrix}1\\1\\-1\end{pmatrix}=2$	**A1**	2.2a
		(3)	
	Alternative 1 to (d):		
	$\begin{pmatrix}-7\\2\\-7\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=b \Rightarrow -7+2-7a=b$ $\begin{pmatrix}-1\\2\\-1\end{pmatrix}.\begin{pmatrix}1\\1\\a\end{pmatrix}=b \Rightarrow -1+2-a=b$ $\Rightarrow a=...$ or $b=...$	**M1**	3.1a
	$a=-1$ or $b=2$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	
	Alternative 2 to (d) (Not on spec):		
	Normal to Π_2: $\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & -3 & 5\\10 & 6 & 2\end{vmatrix}=\begin{pmatrix}-36\\48\\36\end{pmatrix}$ $\begin{vmatrix}3 & -4 & -3\\2 & 3 & -2\\1 & 1 & a\end{vmatrix}=0 \Rightarrow 3(3a+2)+4(2a+2)-3(-1)=0 \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	
	Alternative 3 to (d):		
	$\mathbf{r}=\begin{pmatrix}0\\2\\0\end{pmatrix}+s\begin{pmatrix}1\\0\\1\end{pmatrix}$ lies in $\Pi_3 \Rightarrow s+2+as=b$ $(a+1)s+2=b \Rightarrow a=...$	**M1**	3.1a
	$a=-1$	**A1**	1.1b
	$a=-1$ and $b=2$	**A1**	2.2a
		(2)	

Question	Scheme	Marks	AOs
4(a)	Attempts the scalar product between the direction of W and the normal to the road and uses trigonometry to find an angle.	M1	3.1a
	$\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\left(\begin{pmatrix}-1\\-1\\-3\end{pmatrix}-\begin{pmatrix}1\\2\\-3\end{pmatrix}\right)\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$	M1 A1	1.1b 1.1b
	$\sqrt{(2)^2+(3)^3+(0)^2}\sqrt{(3)^2+(-5)^3+(-18)^2}\cos\alpha="-9"$ $\theta=90-\arccos\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ or $\theta=\arcsin\left(\frac{9}{\sqrt{13}\sqrt{358}}\right)$ Angle between pipe and road $=7.58^\circ$ (3sf) or 0.132 radians (3sf) (Allow -7.58° or -0.132 radians)	M1 A1	1.1b 3.2a
		(5)	
(b)	$W:\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}$ or $\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$C \text{ to } W:\left\{\begin{pmatrix}-1\\-1\\-3\end{pmatrix}+t\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$ or $\left\{\begin{pmatrix}1\\2\\-3\end{pmatrix}+\lambda\begin{pmatrix}2\\3\\0\end{pmatrix}-\begin{pmatrix}-1\\-2\\0\end{pmatrix}\right\}$	M1	3.4
	$\begin{pmatrix}2t\\3t+1\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow t=...$ or $\begin{pmatrix}2+2\lambda\\4+3\lambda\\-3\end{pmatrix}\bullet\begin{pmatrix}2\\3\\0\end{pmatrix}=0\Rightarrow\lambda=...$ or $(2t)^2+(3t+1)^2+(-3)^2=...$ or $(2+2t)^2+(4+3t)^2+(-3)^2=...$	M1	3.1b
	$t=-\frac{3}{13}$ or $\lambda=-\frac{16}{13}\Rightarrow(C \text{ to } W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ or $(2t)^2+(3t+1)^2+(-3)^2=13\left(t+\frac{3}{13}\right)^2+\frac{121}{13}$ or $(2+2t)^2+(4+3t)^2+(-3)^2=13\left(\lambda+\frac{16}{13}\right)^2+\frac{121}{13}$ or $\frac{d\left((2t)^2+(3t+1)^2+(-3)^2\right)}{dt}=0\Rightarrow t=-\frac{3}{13}\Rightarrow C \text{ to } W$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$ Or $\frac{d\left((2+2t)^2+(4+3t)^2+(-3)^2\right)}{dt}=0\Rightarrow t=-\frac{16}{13}\Rightarrow(C \text{ to } W)_{\min}$ is $-\frac{6}{13}\mathbf{i}+\frac{4}{13}\mathbf{j}-3\mathbf{k}$	A1	1.1b
	$d=\sqrt{\left(-\frac{6}{13}\right)^2+\left(\frac{4}{13}\right)^2+(-3)^2}$ or $d=\sqrt{\frac{121}{13}}$	**dd**M1	1.1b

	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	
			(11 marks)

Notes

(a)

M1: Realises the scalar product between the direction of *W* and the normal to the road is needed and so applies it and uses trigonometry to find an angle

M1: Calculates the scalar product between $\pm\left(\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}-1\\-1\\-3\end{pmatrix}\right)$ and $\pm\begin{pmatrix}3\\-5\\-18\end{pmatrix}$ (Allow sign slips as long as the intention is clear)

A1: $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=-9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-5\\-18\end{pmatrix}=9$ or $\begin{pmatrix}2\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=9$ or $\begin{pmatrix}-2\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}-3\\5\\18\end{pmatrix}=-9$

M1: A fully complete and correct method for obtaining the acute angle

A1: Awrt 7.58° or awrt 0.132 radians **(must see units)**. Do not isw and withhold this mark if extra answers are given.

(b)

B1ft: Forms the correct parametric form for the pipe *W*. Follow through their direction vector for *W* from part (a).

M1: Identifies the need to and forms a vector connecting *C* to *W* using a parametric form for *W*

M1: Uses the model to form the scalar product of *C* to *W* and the direction of *W* to find the value of their parameter **or** finds the distance *C* to *W* or $(C \text{ to } W)^2$ in terms of their parameter

A1: Correct vector or correct completion of the square

ddM1: Correct use of Pythagoras on their vector *CW* or appropriate method to find the shortest distance between the point and the pipe. **Dependent on both previous method marks.**

A1: Correct length for the required section of pipe is 305 or 305 cm or 3.05 m

<u>Alternatives for part (b):</u>

4(b) Way 2	$\mathbf{AC}=\begin{pmatrix}0\\1\\-3\end{pmatrix},\quad \mathbf{AB}=\begin{pmatrix}2\\3\\0\end{pmatrix}$	B1ft	1.1b
	$\mathbf{AC.AB}=\begin{pmatrix}0\\1\\-3\end{pmatrix}.\begin{pmatrix}2\\3\\0\end{pmatrix}=3$	M1	3.4
	$\Rightarrow \cos CAB=\frac{3}{\sqrt{10}\sqrt{13}}\Rightarrow CAB=...$	M1	3.1b
	$CAB=74.74....°$	A1	1.1b
	$d=\sqrt{10}\sin 74.74....°$	**dd**M1	1.1b
	Shortest length of pipe needed is 305 or 305 cm or 3.05 m	A1	3.2a
		(6)	

Question	Scheme	Marks	AOs
4.	$(\mathbf{r} =)\begin{pmatrix}-2+\lambda\\5-\lambda\\4-3\lambda\end{pmatrix}$ **or** $\begin{pmatrix}-2\\5\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\-1\\-3\end{pmatrix}$ (oe)	M1	1.1b
	So meet if $\begin{pmatrix}-2+\lambda\\5-\lambda\\4-3\lambda\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\1\end{pmatrix}=-7 \Rightarrow (-2+\lambda)\times1+(5-\lambda)\times-2+(4-3\lambda)\times1=-7$	M1 A1	3.1a 1.1b
	$\Rightarrow 0\lambda-8=-7 \Rightarrow -8=-7$ a contradiction so no intersection	A1ft	2.3
	Hence *l* is parallel to Π but not in it.	A1cso	3.2a
		(5)	
	(5 marks)		
Notes			
	M1 Forms a parametric form for the line. Allow one slip. **M1** Substitutes into the equation of the plane to an equation in λ. May use Cartesian form of plane to substitute into. **A1** Correct equation in λ **A1ft** Simplifies and derives a contradiction and deduces line and plane do not meet. Follow through in their initial equation in λ so - contradiction so no intersection if λ disappears and constants unequal - line lies in plane if a tautology is arrived at - meet in a point if a solution for λ is found. **But do not allow for incorrect simplification from a correct initial equation in λ** Note that a miscopy/misread of 7 instead of −7 can therefore score a maximum of M1M1A0A1A0. **A1cso** Correct deduction from correct working. This may be seen two separate statements in their working. You may see attempts at showing the line is parallel before/after deducing there is no intersection.		
Alt 1	Note that some may a attempt a mix of the main scheme and Alt 1. Mark under main scheme unless Alt 1 would score higher.		
	$\begin{pmatrix}1\\-1\\-3\end{pmatrix}\bullet\begin{pmatrix}1\\-2\\1\end{pmatrix}=1\times1+(-1)\times(-2)+(-3)\times1=0$	M1	3.1a
	Hence *l* is parallel to Π	A1	1.1b
	$(-2,5,4)$ on *l*, but $(1)(-2)+(-2)(5)+1(4)=-8$	M1	1.1b
	$-8\neq-7$ so $(-2,5,4)$ is not on the plane.	A1ft	2.3
	Hence *l* is (parallel to Π but) not in the plane.	A1cso	3.2a
		(5)	
	(5 marks)		
Alt 1 Notes			
	M1 Attempts the dot product between the two direction vectors. **A1** Shows dot product is zero and makes the correct deduction that line is parallel to plane. **M1** Finds a point on *l* and substitutes into the equation of Π (vector or Cartesian) **A1ft** Simplifies and derives a contradiction – follow through their equation, so if arrive at a tautology, they should deduce the line is in the plane. **A1cso** Correct deduction from correct working but may be split across working.		

Question	Scheme	Marks	AOs
8. (a)	**Note:** Allow alternative vector forms throughout, e.g row vectors, **i**, **j**, **k** notation $\mathbf{b} = \pm\left[\begin{pmatrix}300\\300\\-50\end{pmatrix} - \begin{pmatrix}-300\\400\\-150\end{pmatrix}\right] = \pm\begin{pmatrix}600\\-100\\100\end{pmatrix}$	M1	1.1b
	So $\mathbf{r} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix}$ oe $\left(\text{e.g. } \mathbf{r} = \begin{pmatrix}300\\300\\-50\end{pmatrix} + \lambda\begin{pmatrix}6\\-1\\1\end{pmatrix}\right)$	A1	2.5
		(2)	
(b)(i)	$k = 200$	B1	2.2a
	If M is the point on mountain, and X a general point on the line then eg. $\overrightarrow{MX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \lambda\begin{pmatrix}600\\-100\\100\end{pmatrix} - \begin{pmatrix}100\\k\\100\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\400-k-100\lambda\\-250+100\lambda\end{pmatrix} = \begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix}$ May be in terms of k or with $k = 200$ used.	M1	3.1b
	e.g. $\begin{pmatrix}-400+600\lambda\\200-100\lambda\\-250+100\lambda\end{pmatrix} \bullet \begin{pmatrix}600\\-100\\100\end{pmatrix} = 0 \Rightarrow \lambda = \ldots$	dM1	1.1b
	So e.g. $\overrightarrow{OX} = \begin{pmatrix}-300\\400\\-150\end{pmatrix} + \frac{3}{4}\begin{pmatrix}600\\-100\\100\end{pmatrix} = \ldots$	M1	3.4
	So coordinates of X are $(150, 325, -75)$ Accept as $\begin{pmatrix}150\\325\\-75\end{pmatrix}$	A1	1.1b
		(5)	
(ii)	Length of tunnel is $\sqrt{(150-100)^2 + (325-200)^2 + (-75-100)^2} = \ldots$	M1	1.1b
	Awrt 221m from correct working, so λ must have been correct. (Must include units)	A1	1.1b
		(2)	
(c)	$\lvert\overrightarrow{OP}\rvert = \sqrt{(-300)^2 + 400^2 + (-150)^2} \approx 522$ $\lvert\overrightarrow{OQ}\rvert = \sqrt{300^2 + 300^2 + 50^2} \approx 427$	M1	1.1b
	New tunnel length is signficantly shorter than these values so it is likely that the company will decide to build the accessway. Reason and conclusion needed.	A1ft	2.2b
		(2)	
(d)	E.g. The mountainside is not likely to be flat so a plane may not be a good model. The tunnel and/or pipeline will not have negligible thickness so modelling as lines may not be appropriate. A shortest length tunnel may not be possible, or most practical, as the strata of the rock in the mountain have not been considered by the model.	B1	3.5b
		(1)	
	(12 marks)		

Question	Scheme	Marks	AOs
4(a)	Finds any two vectors $\pm\overrightarrow{LM}, \pm\overrightarrow{LN}$ or $\pm\overrightarrow{MN}$ $\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$ two out of three values correct is sufficient to imply the correct method	M1	3.3
	Applies the vector equation of the plane formula $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ Where **a** is any coordinate from L, M & N and vectors **b** and **c** come from an attempt at finding any two vectors that lie on the plane.	M1	1.1b
	A correct equation for the plane $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ $\mathbf{a} = \begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-2\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ **b** and **c** are any two vectors from $\pm\begin{pmatrix}8\\1\\1\end{pmatrix}$ or $\pm\begin{pmatrix}4\\3\\1\end{pmatrix}$ or $\pm\begin{pmatrix}-4\\2\\0\end{pmatrix}$	A1	1.1b
		(3)	
(b)(i) **(ii)**	Applies 'their' $\mathbf{b}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **AND** 'their' $\mathbf{c}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **Alternative 1** Finds 'their **b**' – 'their **c**' or vice versa and applies the dot product with $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ **AND** one of their **b** or **c** **Alternative 2** Applies 'their' $\mathbf{b}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ **AND** 'their' $\mathbf{c}.\begin{pmatrix}x\\y\\z\end{pmatrix}$ and solves to find values of *x*, *y* and *z* **Alternative 3** Applies the dot product between their answer to part (a) and the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$	M1	1.1b
	Show that both dot product(s) **= 0** therefore the lawn is **perpendicular** **Alternative 1** Shows results is **parallel** to $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ therefore the lawn is **perpendicular** **Alternative 2** Achieves the value 2 and concludes as a **constant** therefore the lawn is **perpendicular**	A1	2.4
	Outside Specification for this paper – using the cross product Finds the cross product between 'their **b**' and 'their **c**' and either	M1	1.1b

	compares with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel or applies the dot product formula with the vector $\begin{pmatrix}1\\2\\-10\end{pmatrix}$ to show parallel		
	Concludes **parallel** therefore the lawn is **perpendicular**	A1	2.4
	Attempts $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ where $\mathbf{a}=\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ or $\begin{pmatrix}6\\-3\\0\end{pmatrix}$ or $\begin{pmatrix}2\\0\\0\end{pmatrix}$ Allow $\mathbf{r}.\begin{pmatrix}1\\2\\-10\end{pmatrix}=\mathbf{a}.\begin{pmatrix}1\\2\\-10\end{pmatrix}$ for this mark	M1	1.1b
	$x+2y-10z=2$ or $x+2y-10z-2=0$	A1	1.1b
		(4)	
(c)	Finds the vector $\overrightarrow{PQ}$ or $\overrightarrow{QP}$ and uses it as the direction vector in the formula $\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ Two out three values correct is sufficient to imply the correct method	M1	3.3
	$\mathbf{r}=\mathbf{a}+\lambda\mathbf{d}$ where $\mathbf{a}=\begin{pmatrix}-10\\8\\2\end{pmatrix}$ or $\begin{pmatrix}6\\4\\3\end{pmatrix}$ and $\mathbf{d}=\pm\begin{pmatrix}16\\-4\\1\end{pmatrix}$	A1	1.1b
		(2)	
(d)	For example: The lawn will not be flat The washing line will not be straight	B1	3.5b
		(1)	
(e)	Applies the distance formula $\dfrac{\lvert(2\times1)+5\times2+(2.75\times-10)-2\rvert}{\sqrt{1^2+2^2+(-10)^2}}$	M1	3.4
	= 1.71 m or 171 cm	A1	2.2b
		(2)	
(f)	Must have an answer to part (e). Compares their answer to part (e) with 1.5 m and makes an appropriate comment about the model that is consistent with their answer to part (e). If their answer to part (e) is close to 1.5 (e.g. 1.4 to 1.6) they must compare and conclude that the model therefore is good If their answer to part (e) is significantly different to 1.5 they must compare and conclude that the model therefore it is not a good model.	B1ft	3.5a

A1: Correct expression M1: Fully correct strategy for the required area. Must be subtracting the area of the minor segment from the annulus area. A1: Correct exact answer Note: 6.968

Question	Scheme	Marks	AOs
6(a)	Any two of: $\begin{cases} \pm k\overrightarrow{AB} = \pm k(5\mathbf{i}+25\mathbf{j}+5\mathbf{k}), \\ \pm k\overrightarrow{AC} = \pm k(-15\mathbf{i}+15\mathbf{j}-10\mathbf{k}), \\ \pm k\overrightarrow{BC} = \pm k(-20\mathbf{i}-10\mathbf{j}-15\mathbf{k}) \end{cases}$	M1	3.3
	Let normal vector be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ $(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(\mathbf{i}+5\mathbf{j}+\mathbf{k})=0,\ (a\mathbf{i}+b\mathbf{j}+c\mathbf{k})\bullet(-3\mathbf{i}+3\mathbf{j}-2\mathbf{k})=0$ $\Rightarrow a+5b+c=0,\ -3a+3b-2c=0 \Rightarrow a=...,\ b=...,\ c=...$ Alternative: cross product $\begin{vmatrix} 1 & 5 & 1 \\ -3 & 3 & -2 \end{vmatrix} = (-10-3)\mathbf{i}-(-2+3)\mathbf{j}+(3+15)\mathbf{k}$	M1	1.1b
	$\mathbf{n}=k(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})$	A1	1.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet(10\mathbf{i}+5\mathbf{j}-50\mathbf{k})=...$	M1	1.1b
	$\mathbf{r}\bullet(13\mathbf{i}+\mathbf{j}-18\mathbf{k})=1035$ o.e. $\mathbf{r}\bullet(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})=-1035$ $\mathbf{r}\bullet(325\mathbf{i}+25\mathbf{j}-450\mathbf{k})=25875$	A1	2.5
		(5)	
(b)	Attempts the scalar product between their normal vector and the vector **k** and uses trigonometry to find an angle	M1	3.1b
	$(-13\mathbf{i}-\mathbf{j}+18\mathbf{k})\bullet\mathbf{k}=-18=\sqrt{13^2+1^2+18^2}\cos\alpha$	M1	1.1b
	$\cos\alpha=\frac{-18}{\sqrt{494}} \Rightarrow \alpha=144.08... \Rightarrow \theta=36°$	A1	3.2a
		(3)	
(c)	Distance required is $\lvert\lambda\rvert$ where $\begin{pmatrix} 13 \\ 1 \\ -18 \end{pmatrix}\bullet\begin{pmatrix} 5 \\ 12 \\ \lambda \end{pmatrix}=1035$	M1	3.4
	$\lvert\lambda\rvert=53.2\text{m}$	A1	1.1b
		(2)	
(d)	E.g. • The mineral layer will **not** be perfectly **flat/smooth** and will not form a plane • The mineral layer will have a **depth** and this should be taken into account	B1	3.5b

Question	Scheme	Marks	AOs
6(a)	Need **k** component to be zero at ground, so $0.84+0.8\lambda-\lambda^2=0 \Rightarrow \lambda=...$	M1	1.1b
	$\lambda=-\frac{3}{5},\frac{7}{5}$, but $\lambda\geqslant 0$ so $\lambda=\frac{7}{5}$	A1	1.1b
		(2)	
(b)	Direction is $(9-4.6\times1.4)\mathbf{i}+15\mathbf{j}+(0.8-2\times1.4)$ $=2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}$ or $\frac{64}{25}\mathbf{i}+15\mathbf{j}-2\mathbf{k}$	B1ft	2.2a
		(1)	
(c)	Direction perpendicular to ground is $a\mathbf{k}$, so angle to perpendicular is given by $(\cos\theta)=\frac{a\mathbf{k}.(2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k})}{a\times\lvert 2.56\mathbf{i}+15\mathbf{j}-2\mathbf{k}\rvert}$ or $\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}0\\0\\a\end{pmatrix}}{\begin{vmatrix}2.56\\15\\-2\end{vmatrix}\begin{vmatrix}0\\0\\a\end{vmatrix}}$ or angle between $\begin{pmatrix}2.56\\15\\-2\end{pmatrix}$ and $\begin{pmatrix}2.56\\15\\0\end{pmatrix}$ is given by $(\cos\theta)=\frac{\begin{pmatrix}2.56\\15\\-2\end{pmatrix}\bullet\begin{pmatrix}2.56\\15\\0\end{pmatrix}}{\begin{vmatrix}2.56\\15\\-2\end{vmatrix}\begin{vmatrix}2.56\\15\\0\end{vmatrix}}$	M1	1.1b
	$=\frac{-2}{\sqrt{2.56^2+15^2+(-2)^2}}\ (=-0.130\ldots)$ Or $=\frac{231.5536}{\sqrt{2.56^2+15^2+(-2)^2}\sqrt{2.56^2+15^2+(0)^2}}=0.991...$	M1	1.1b
	$90°-\arccos('-0.130...')=-7.48...$ or $\arccos(0.991...)$	ddM1	3.1b
	So the tennis ball hits ground at angle of $7.5°$ (1d.p.) cao	A1	3.2a
	Alternative Finds the length of the vector in the **ij** plane $=\sqrt{2.56^2+15^2}$	M1	1.1b
	$\tan\theta=\frac{2}{\sqrt{2.56^2+15^2}}$	M1	1.1b
	$\theta=\arctan\left(\frac{2}{\sqrt{2.56^2+15^2}}\right)$ or $\theta=90-\arctan\left(\frac{\sqrt{2.56^2+15^2}}{2}\right)$	ddM1	3.1b

	So the tennis ball hits ground at angle of $7.5°$ (1d.p.)	A1	3.2a
		(4)	
(d)	In same plane as net when $\mathbf{r}.\mathbf{j} = 0$, $\begin{pmatrix} -4.1+9\lambda-2.3\lambda^2 \\ -10.25+15\lambda \\ 0.84+0.8\lambda-\lambda^2 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ leading to $-10.25+15\lambda = 0 \Rightarrow \lambda = ...$ $\left(= \frac{41}{60} = 0.683333...\right)$	M1	3.1b
	So is at position $\left(-4.1+9\times\frac{41}{60}-2.3\left(\frac{41}{60}\right)^2\right)\mathbf{i}+0\mathbf{j}+\left(0.84+0.8\times\frac{41}{60}-\left(\frac{41}{60}\right)^2\right)\mathbf{k}$	M1	1.1b
	$=$ awrt $0.976\mathbf{i}+$ awrt $0.920\mathbf{k}$ or $=$ awrt $0.976\mathbf{i}+0.92\mathbf{k}$ (to 3 s.f.) or $=$ awrt $0.976\mathbf{i}+\frac{3311}{3600}\mathbf{k}$	A1	1.1b
		(3)	
(e)	Modelling as a line, height of net is 0.9m along its length so as **0.92 > 0.9** the ball **will** pass over the net according to the model.	B1ft	3.2a
		(1)	
(f)	Identifies a suitable feature of the model that affects the outcome And uses it to draw a compatible conclusion. For example • The ball is **not a particle** and will have **diameter/radius**, therefore it will hit the net and not pass over. • As above, but so the **ball will clip the net** but it's momentum will take it over as it is mostly above the net. • The **model** says that the ball will clear the net by 2cm which may be smaller than the balls **diameter** • The **net will not be a straight line/taut** so will not be 0.9m high, so the ball **will** have enough clearance to **pass** over the net.	M1 A1	3.2b 2.2b
		(2)	
			(13 marks)

Notes:

Accept any alternative vector notations throughout.

(a)

M1: Attempts to solve the quadratic from equating the **k** component to zero.

A1: Correct value, must select positive root, so accept 1.4 oe.

Correct answer only M1 A1

(b)

B1ft: For $(2.56, 15, -2)$ o.e or follow through $(9-4.6\times'\lambda', 15, 0.8-2\times'\lambda')$ for their λ.

(c)

M1: Recognises the angle between the perpendicular and direction vector is needed, and identifies the perpendicular as $a\mathbf{k}$ for any non-zero a (including 1), and attempts dot product

Question	Scheme	Marks	AOs
6(a)	$\begin{pmatrix}3\\0\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=3\{+0\}-3$	M1	1.1b
	= **0** therefore the lines are **perpendicular.**	A1	2.4
		(2)	
(b)	$\mathbf{r}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\2\\0\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\ldots\{2\}$	M1	1.1b
	$x+2y-3z=2$ o.e.	A1	2.5
		(2)	
(c)	$3+2(1)-3(1)=2$ (therefore lies on the plane)	B1	1.1b
		(1)	
(d)	$\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}2\\3\\2\end{pmatrix}+\mu\begin{pmatrix}1\\2\\-3\end{pmatrix}$ **or** $\begin{pmatrix}p\\q\\r\end{pmatrix}+\mu\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}2\\3\\2\end{pmatrix}$ leading to $p=2-\mu$, $q=3-2\mu$, $r=2+3\mu$	M1	3.1a
	$(p-3)^2+(q-1)^2+(r-1)^2=(2\sqrt{5})^2$ $((2+\mu)-3)^2+((3+2\mu)-1)^2+((2-3\mu)-1)^2=(2\sqrt{5})^2$ $(-1+\mu)^2+(2+2\mu)^2+(1-3\mu)^2=20$ **or** $(p-3)^2+(q-1)^2+(r-1)^2=(2\sqrt{5})^2$ $((2-\mu)-3)^2+((3-2\mu)-1)^2+((2+3\mu)-1)^2=(2\sqrt{5})^2$ $(-1-\mu)^2+(2-2\mu)^2+(1+3\mu)^2=20$	M1	3.1a
	$14\mu^2-14=0$ o.e	A1	1.1b
	Solves their quadratic $\{\mu=-1 \text{ or } \mu=1\}$	M1	1.1b
	Uses $\mu=-1$ Using $\mu=1$	ddM1	1.1b

	$p=2+(-1)=...$ or $p=2-(1)=...$ $q=3+2(-1)=...$ or $q=3-2(1)=...$ $r=2-3(-1)=...$ or $r=2+3(1)=...$		
	$(1,1,5)$ only	A1	3.2a
		(6)	
	Alternative $\lvert AX\rvert=\sqrt{(3-2)^2+(1-3)^2+(1-2)^2}=\sqrt{6}$	M1	3.1a
	Correctly uses Pythagoras to find the length of XB $\lvert XB\rvert=\sqrt{(2\sqrt{5})^2-6}=\sqrt{14}$	M1	3.1a
	Find the magnitude of the direction vector and compares to the length of XB to find a value for μ	M1	1.1b
	$\mu=-1$ or $\mu=1$	A1	1.1b
	Uses $\mu=-1$ or Using $\mu=1$ $p=2+(-1)=...$ or $p=2-(1)=...$ $q=3+2(-1)=...$ or $q=3-2(1)=...$ $r=2-3(-1)=...$ or $r=2+3(1)=...$	ddM1	1.1b
	$(1,1,5)$ only	A1	3.2a
		(6)	
			(11 marks)

Notes:

(a)

M1: Applies the dot product to the direction vectors. Minimum requirement is 3 – 3

A1: Shows that the dot product **= 0** and concludes that the lines are **perpendicular**.

(b)

M1: Applies $\mathbf{r}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\2\\0\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=...$ or $\begin{pmatrix}x\\y\\z\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\2\\0\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=...$

A1: Correct Cartesian equation $x+2y-3z=2$ o.e.

Note: $\mathbf{i}+2\mathbf{j}-3\mathbf{k}=2$ is M1A0

(c)

B1: See scheme, no conclusion required

(d)

M1: Uses the point of intersection to find the coordinates of B as functions of a parameter

M1: Uses the distance between the point A and the point B to form an equation for their parameter only.

A1: Correct simplified quadratic equation