12. (a) Use the substitution $u = 1 + \sqrt{x}$ to show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = \int_p^q \frac{2(u-1)^3}{u}\,\mathrm{d}u$$

where p and q are constants to be found.

(3)

(b) Hence show that

$$\int_0^{16} \frac{x}{1+\sqrt{x}}\,\mathrm{d}x = A - B\ln 5$$

where A and B are constants to be found.

(4)

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7. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

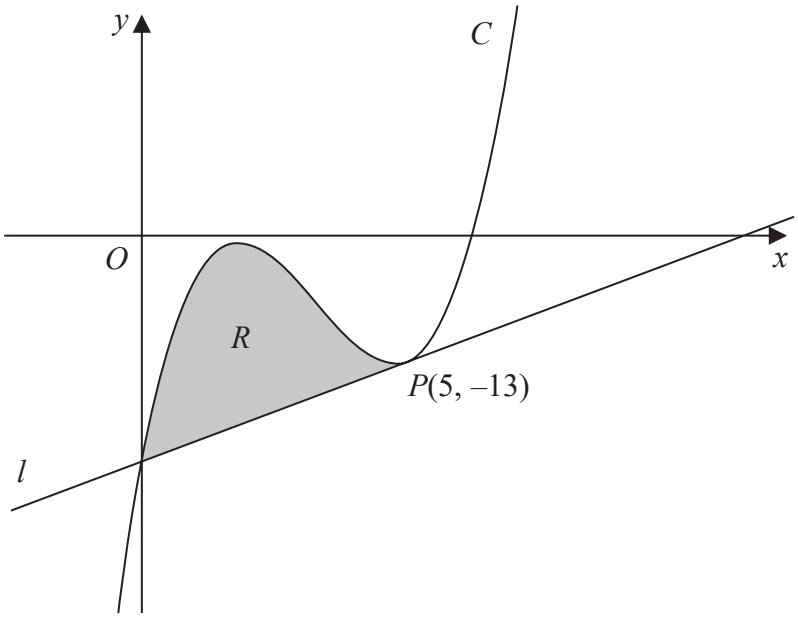

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + 27x - 23$$

The point $P(5, -13)$ lies on C

The line l is the tangent to C at P

(a) Use differentiation to find the equation of l, giving your answer in the form $y = mx + c$ where m and c are integers to be found.

(4)

(b) Hence verify that l meets C again on the y-axis.

(1)

The finite region R, shown shaded in Figure 2, is bounded by the curve C and the line l.

(c) Use algebraic integration to find the exact area of R.

(4)

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7. Given that $k \in \mathbb{Z}^+$

(a) show that $\displaystyle\int_k^{3k} \frac{2}{(3x-k)}\,\mathrm{d}x$ is independent of k,

(4)

(b) show that $\displaystyle\int_k^{2k} \frac{2}{(2x-k)^2}\,\mathrm{d}x$ is inversely proportional to k.

(3)

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13. Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

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13.

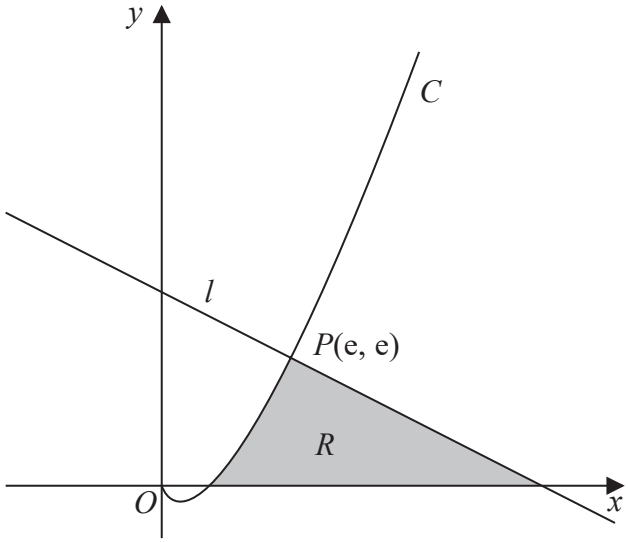

Figure 2

Figure 2 shows a sketch of part of the curve C with equation $y = x\ln x, \quad x > 0$

The line l is the normal to C at the point $P(\mathrm{e}, \mathrm{e})$

The region R, shown shaded in Figure 2, is bounded by the curve C, the line l and the x-axis.

Show that the exact area of R is $A\mathrm{e}^2 + B$ where A and B are rational numbers to be found.

(10)

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8.

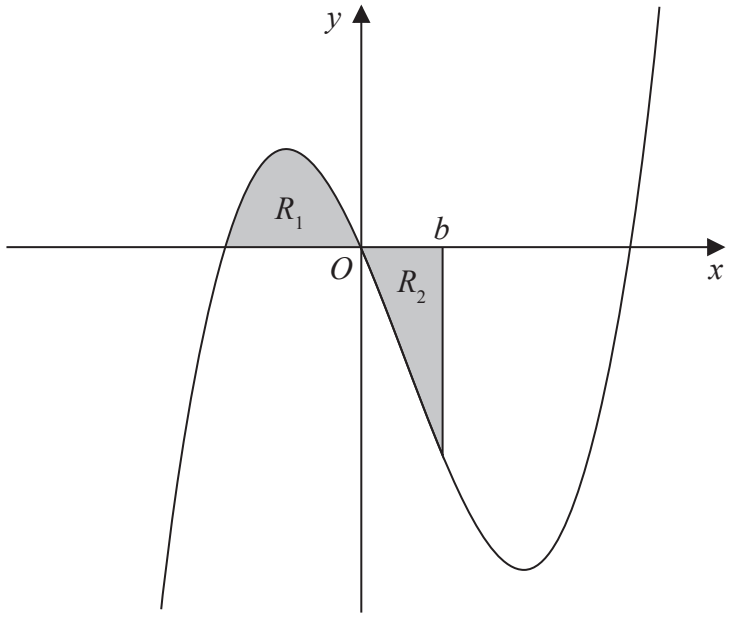

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

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13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

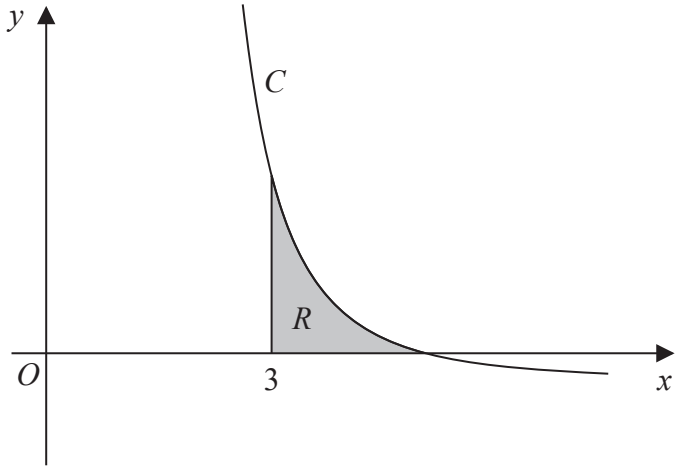

Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a \ln 2 + b \ln 3$, where a and b are rational constants to be found.

(8)

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5.

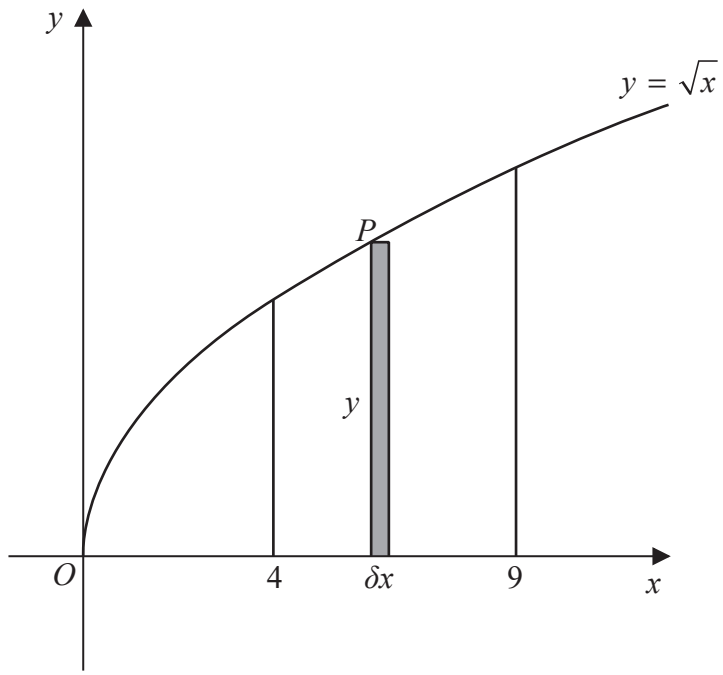

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{\delta x \to 0} \sum_{x=4}^{9} \sqrt{x}\, \delta x$$

(3)

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10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6)

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6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \qquad x \in \mathbb{R} \;\; x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2}\,\mathrm{d}x$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

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8. A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$

(6)

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3. Given that

$$4x^3 + 2x^2 + 17x + 8 \equiv (Ax + B)(x^2 + 4) + Cx + D$$

(a) find the values of the constants A, B, C and D.

(4)

(b) Hence find

$$\int_1^4 \frac{4x^3 + 2x^2 + 17x + 8}{x^2 + 4}\,\mathrm{d}x$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(6)

Leave blank

4. Find

(a) $\int (2x + 3)^{12} \, dx$

(2)

(b) $\int \frac{5x}{4x^2 + 1} \, dx$

(2)

Leave blank

4. Use the substitution $x = 2\sin\theta$ to find the exact value of

$$\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

(7)

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5. (i) Find the x coordinate of each point on the curve $y = \dfrac{x}{x+1}$, $x \neq -1$, at which the gradient is $\dfrac{1}{4}$

(4)

(ii) Given that

$$\int_{a}^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7 \qquad a > 0$$

find the exact value of the constant a.

(4)

Leave blank

8. $$\mathrm{f}(\theta) = 9\cos^2\theta + \sin^2\theta$$

(a) Show that $\mathrm{f}(\theta) = a + b\cos 2\theta$, where a and b are integers which should be found.

(3)

(b) Using your answer to part (a) and integration by parts, find the exact value of

$$\int_0^{\frac{\pi}{2}} \theta^2 \mathrm{f}(\theta)\,\mathrm{d}\theta$$

(6)

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5. Use integration by parts to find the exact value of

$$\int_0^2 x2^x \, dx$$

Write your answer as a single simplified fraction.

(6)

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2. Use integration by parts to find the exact value of $\int_1^{e} \frac{\ln x}{x^2}\,dx$

Write your answer in the form $a + \frac{b}{e}$, where a and b are integers.

(6)

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5. (i) Find

$$\int \left((3x + 5)^9 + e^{5x} \right) dx$$

(3)

(ii) Given that b is a constant greater than 2, and

$$\int_2^b \frac{x}{x^2 + 5} dx = \ln\left(\sqrt{6}\right)$$

use integration to find the value of b.

(5)

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8. Use partial fractions, and integration, to find the exact value of $\displaystyle\int_3^4 \frac{2x^2 - 3}{x(x-1)}\,\mathrm{d}x$

Write your answer in the form $a + \ln b$, where a is an integer and b is a rational constant.

(8)

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6. (i) Find

$$\int x e^{4x} \, dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3} \, dx, \quad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)