9. Show that

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

1. In an arithmetic series

- the first term is 16
- the 21st term is 24

(a) Find the common difference of the series.

(2)

(b) Hence find the sum of the first 500 terms of the series.

(2)

3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

4. (i) Show that $\sum_{r=1}^{16}\left(3 + 5r + 2^r\right) = 131\,798$

(4)

(ii) A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{u_n}, \quad u_1 = \frac{2}{3}$$

Find the exact value of $\sum_{r=1}^{100} u_r$

(3)

8. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

13. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

Leave blank

5. A sequence is defined by

$$u_1 = 3$$

$$u_{n+1} = 2 - \frac{4}{u_n}, \quad n \geqslant 1$$

Find the exact values of

(a) u_2, u_3 and u_4

(3)

(b) u_{61}

(1)

(c) $\sum_{i=1}^{99} u_i$

(3)

Leave blank

11. The first three terms of an arithmetic series are 60, $4p$ and $2p - 6$ respectively.

(a) Show that $p = 9$

(2)

(b) Find the value of the 20th term of this series.

(3)

(c) Prove that the sum of the first n terms of this series is given by the expression

$$12n\,(6 - n)$$

(3)

Leave blank

9. (i) Find the value of $\sum_{r=1}^{20}(3+5r)$

(3)

(ii) Given that $\sum_{r=0}^{\infty}\frac{a}{4^r}=16$, find the value of the constant a.

(4)

Leave blank

5. (a) Prove that the sum of the first n terms of an arithmetic series is given by the formula

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term of the series and d is the common difference between the terms.

(4)

(b) Find the sum of the integers which are divisible by 7 and lie between 1 and 500

(3)

Leave blank

8. A sequence is defined by

$$u_1 = k$$
$$u_{n+1} = 3u_n - 12, \quad n \geqslant 1$$

where k is a constant.

(a) Write down fully simplified expressions for u_2, u_3 and u_4 in terms of k.

(4)

Given that $u_4 = 15$

(b) find the value of k,

(2)

(c) find $\sum_{i=1}^{4} u_i$, giving an exact numerical answer.

(3)

Leave blank

10. A sequence is defined by

$$u_1 = 4$$

$$u_{n+1} = \frac{2u_n}{3}, \qquad n \geqslant 1$$

(a) Find the exact values of u_2, u_3 and u_4

(2)

(b) Find the value of u_{20}, giving your answer to 3 significant figures.

(2)

(c) Evaluate

$$12 - \sum_{i=1}^{16} u_i$$

giving your answer to 3 significant figures.

(3)

(d) Explain why $\sum_{i=1}^{N} u_i < 12$ for all positive integer values of N.

(1)

Leave blank

5. (i) $$U_{n+1} = \frac{U_n}{U_n - 3}, \quad n \geqslant 1$$

Given $U_1 = 4$, find

(a) U_2

(1)

(b) $\sum_{n=1}^{100} U_n$

(2)

(ii) Given

$$\sum_{r=1}^{n} (100 - 3r) < 0$$

find the least value of the positive integer n.

(3)

Leave blank

6. A sequence is defined by

$$u_1 = 36$$
$$u_{n+1} = \frac{2}{3}u_n, \quad n \geqslant 1$$

(a) Find the exact simplified values of u_2, u_3 and u_4

(2)

(b) Write down the common ratio of the sequence.

(1)

(c) Find, giving your answer to 4 significant figures, the value of u_{11}

(2)

(d) Find the exact value of $\sum_{i=1}^{6} u_i$

(2)

(e) Find the value of $\sum_{i=1}^{\infty} u_i$

(2)

Leave blank

4. An arithmetic series has first term a and common difference d.

Given that the sum of the first 9 terms is 54

(a) show that

$$a + 4d = 6$$

(2)

Given also that the 8th term is half the 7th term,

(b) find the values of a and d.

(4)

Leave blank

9. The first three terms of a geometric sequence are

$$7k - 5, \quad 5k - 7, \quad 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \frac{9}{11}$

(2)

For this value of k,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

Leave blank

5. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162

Find

(a) the common ratio,

(4)

(b) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290

(4)

Leave blank

6. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$

The sum to infinity of the series is S_∞

(a) Find the value of S_∞

(2)

The sum to N terms of the series is S_N

(b) Find, to 1 decimal place, the value of S_{12}

(2)

(c) Find the smallest value of N, for which

$$S_\infty - S_N < 0.5$$

(4)

Leave blank

5. The first three terms of a geometric series are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a **positive** constant.

(a) Show that $11p^2 - 10p - 225 = 0$

(4)

(b) Hence show that $p = 5$

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer.

(3)