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Candidate surname

Other names

Centre Number

Candidate Number

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Pearson Edexcel Level 3 GCE**Tuesday 4 June 2024**

Afternoon (Time: 2 hours)

Paper
reference**9MA0/01****Mathematics****Advanced****PAPER 1: Pure Mathematics 1****You must have:**

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Question 1 continued

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(Total for Question 1 is 3 marks)



3. $f(x) = x + \tan\left(\frac{1}{2}x\right) \quad \pi < x < \frac{3\pi}{2}$

Given that the equation $f(x) = 0$ has a single root α

- (a) show that α lies in the interval $[3.6, 3.7]$ (2)
- (b) Find $f'(x)$ (2)
- (c) Using 3.7 as a first approximation for α , apply the Newton–Raphson method once to obtain a second approximation for α . Give your answer to 3 decimal places. (2)

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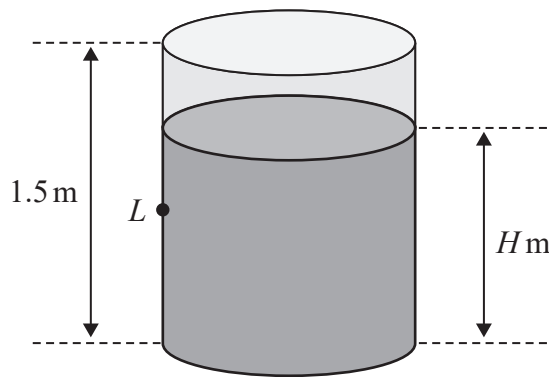


Diagram not drawn to scale.

Figure 2

Figure 2 shows a cylindrical tank of height 1.5 m.

Initially the tank is full of water.

The water starts to leak from a small hole, at a point L , in the side of the tank.

While the tank is leaking, the depth, H metres, of the water in the tank is modelled by the differential equation

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

where t hours is the time after the leak starts.

Using the model,

(a) show that

$$H = Ae^{-0.2t} + B$$

where A and B are constants to be found,

(3)

(b) find the time taken for the depth of the water to decrease to 1.2 m. Give your answer in hours and minutes, to the nearest minute.

(3)

In the long term, the water level in the tank falls to the same height as the hole.

(c) Find, according to the model, the height of the hole from the bottom of the tank.

(2)

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8. The functions f and g are defined by

$$f(x) = 4 - 3x^2 \quad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x - 9} \quad x \in \mathbb{R}, x \neq \frac{9}{2}$$

(a) Find $fg(2)$

(2)

(b) Find g^{-1}

(3)

(c) (i) Find $gf(x)$, giving your answer as a simplified fraction.

(ii) Deduce the range of $gf(x)$.

(3)

The function h is defined by

$$h(x) = 2x^2 - 6x + k \quad x \in \mathbb{R}$$

where k is a constant.

(d) Find the range of values of k for which the equation

$$f(x) = h(x)$$

has no real solutions.

(3)

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Question 8 continued

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(Total for Question 8 is 11 marks)



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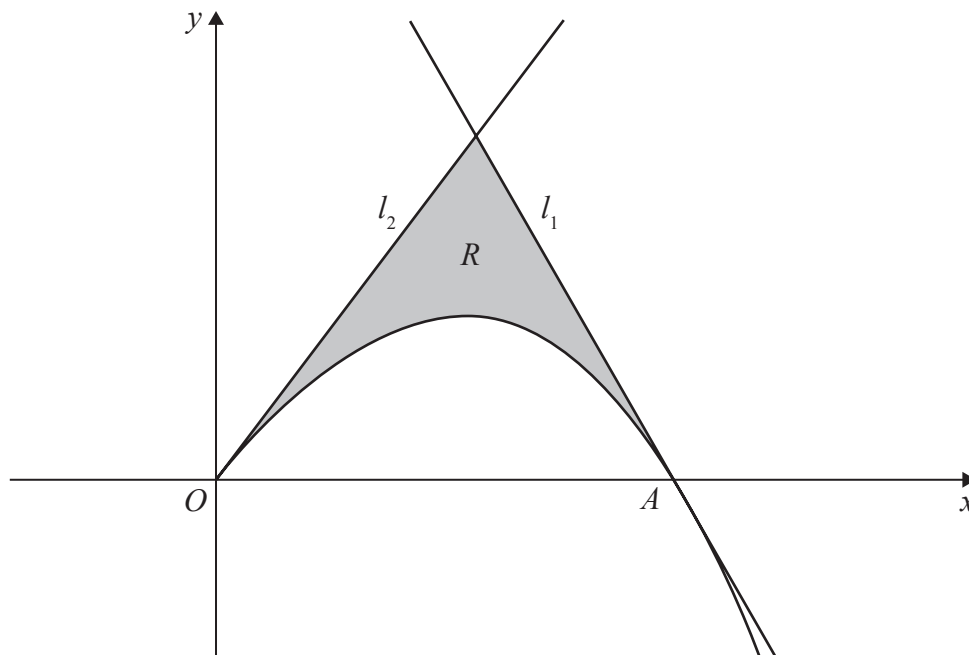


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \quad x \geq 0$$

The curve crosses the x -axis at the point A .

(a) Verify that the x coordinate of A is 4

(1)

The line l_1 is the tangent to the curve at A .

(b) Use calculus to show that an equation of line l_1 is

$$12x + y = 48$$

(3)

The line l_2 has equation $y = 8x$

The region R , shown shaded in Figure 3, is bounded by the curve, the line l_1 and the line l_2

(c) Use algebraic integration to find the exact area of R .

(5)

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Question 10 continued

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(Total for Question 10 is 9 marks)



11.

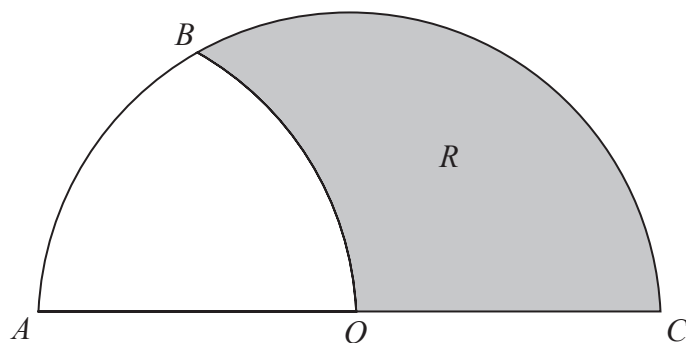


Figure 4

Figure 4 shows the design of a badge.

The shape $ABCOA$ is a semicircle with centre O and diameter 10 cm.

OB is the arc of a circle with centre A and radius 5 cm.

The region R , shown shaded in Figure 4, is bounded by the arc OB , the arc BC and the line OC .

Find the exact area of R .

Give your answer in the form $(a\sqrt{3} + b\pi)\text{cm}^2$, where a and b are rational numbers.

(4)

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12. (a) Express $140 \cos \theta - 480 \sin \theta$ in the form $K \cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90^\circ$

State the value of K and give the value of α , in degrees, to 2 decimal places.

(3)

A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, R , is modelled by the equation

$$R = A + 140 \cos(30t)^\circ - 480 \sin(30t)^\circ$$

where t months is the time after the start of the year and A is a constant.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(b) (i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.

(2)

The actual number of rabbits in the wood is at its minimum value in the middle of April.

(c) Use this information to comment on the model for the number of rabbits.

(2)

The number of foxes, F , in the wood during the same year is modelled by the equation

$$F = 100 + 70 \sin(30t + 70)^\circ$$

The number of foxes is at its minimum value after T months.

(d) Find, according to the models, the number of **rabbits** in the wood at time T months.

(4)

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13. (a) Given that a is a positive constant, use the substitution $x = a \sin^2 \theta$ to show that

$$\int_0^a x^{\frac{1}{2}} \sqrt{a-x} \, dx = \frac{1}{2} a^2 \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta \quad (4)$$

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}} \sqrt{a-x} \, dx = k\pi a^2$$

where k is a constant to be found.

(4)

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Question 13 continued

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(Total for Question 13 is 8 marks)



15. (i) Show that $k^2 - 4k + 5$ is positive for all real values of k .

(2)

(ii) A student was asked to prove by contradiction that

“There are no positive integers x and y such that $(3x + 2y)(2x - 5y) = 28$ ”

The start of the student’s proof is shown below.

Assume that positive integers x and y exist such that
 $(3x + 2y)(2x - 5y) = 28$

If $3x + 2y = 14$ and $2x - 5y = 2$

$$\left. \begin{array}{l} 3x + 2y = 14 \\ 2x - 5y = 2 \end{array} \right\} \Rightarrow x = \frac{74}{19}, y = \frac{22}{19} \text{ Not integers}$$

Show the calculations and statements needed to complete the proof.

(4)

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