AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Paper 2

Wednesday 13 June 2018 Morning Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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TOTAL	

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Section A

Answer **all** questions in the spaces provided.

1 Which of these statements is correct?

Tick **one** box.

[1 mark]

$x = 2 \Rightarrow x^2 = 4$ ☐

$x^2 = 4 \Rightarrow x = 2$ ☐

$x^2 = 4 \Leftrightarrow x = 2$ ☐

$x^2 = 4 \Rightarrow x = -2$ ☐

2 Find the coefficient of x^2 in the expansion of $(1 + 2x)^7$

Circle your answer.

[1 mark]

42 4 21 84

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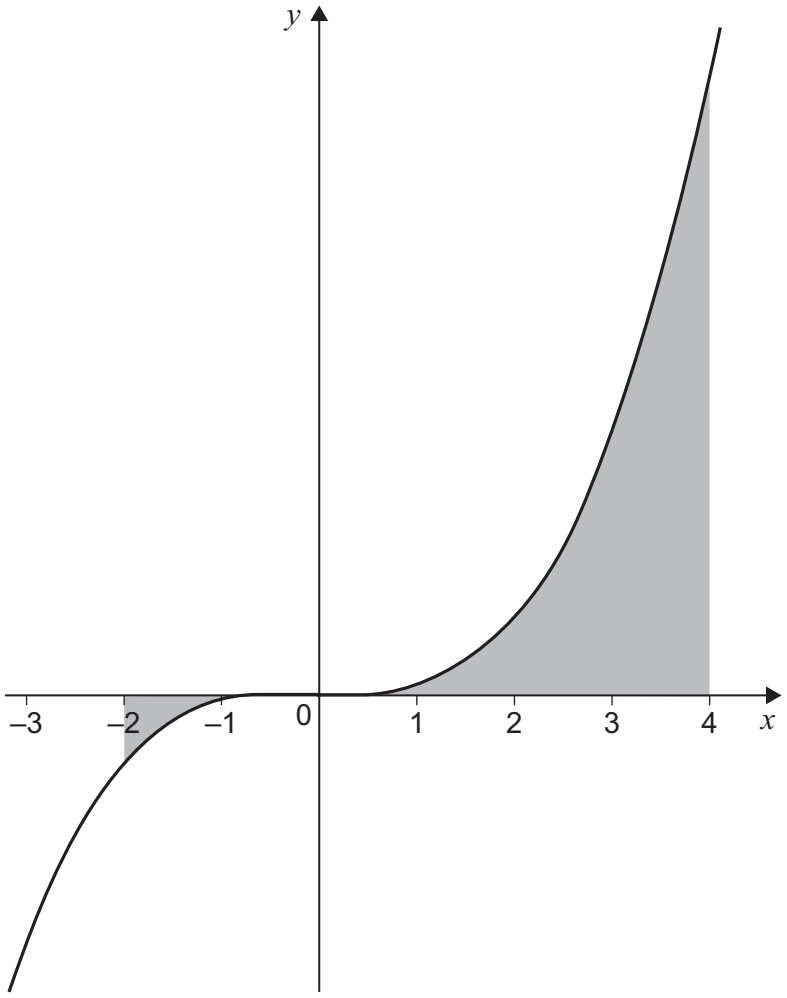
3 The graph of $y = x^3$ is shown.



Find the total shaded area.

Circle your answer.

[1 mark]

-68 60 68 128

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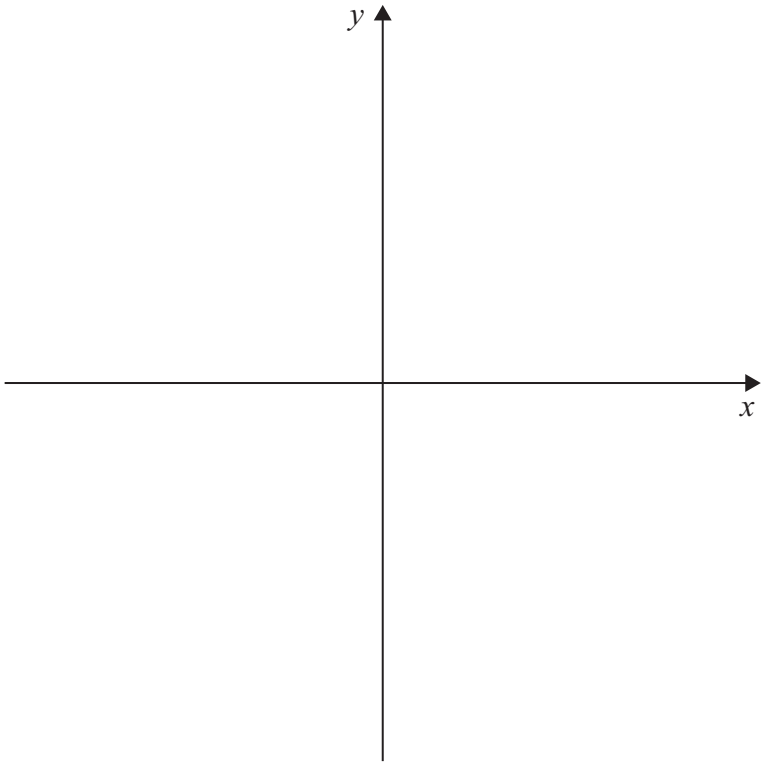
4 A curve, *C*, has equation $y = x^2 - 6x + k$, where k is a constant.

The equation $x^2 - 6x + k = 0$ has two distinct positive roots.

4 (a) Sketch *C* on the axes below.

[2 marks]



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4 (b) Find the range of possible values for k.

Fully justify your answer.

[4 marks]

Turn over for the next question

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5 Prove that 23 is a prime number.

[2 marks]

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6 Find the coordinates of the stationary point of the curve with equation

$$(x + y - 2)^2 = e^y - 1$$

[7 marks]

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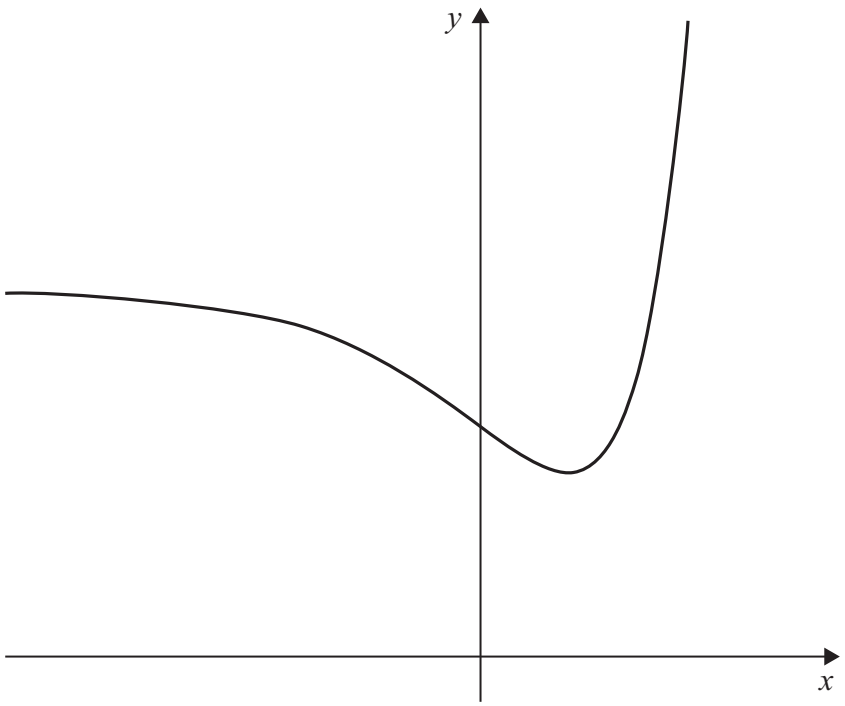
7 A function f has domain $\mathbb{R}$ and range $\{y \in \mathbb{R} : y \geq \mathrm{e}\}$

The graph of $y = \mathrm{f}(x)$ is shown.



The gradient of the curve at the point (x, y) is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = (x - 1)\mathrm{e}^x$

Find an expression for $\mathrm{f}(x)$.

Fully justify your answer.

[8 marks]

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Turn over for the next question

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8 (a) Determine a sequence of transformations which maps the graph of $y = \sin x$ onto the graph of $y = \sqrt{3}\sin x - 3\cos x + 4$

Fully justify your answer.

[7 marks]

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8 (b) (i) Show that the least value of $\frac{1}{\sqrt{3}\sin x - 3\cos x + 4}$ is $\frac{2-\sqrt{3}}{2}$

[2 marks]

8 (b) (ii) Find the greatest value of $\frac{1}{\sqrt{3}\sin x - 3\cos x + 4}$

[1 mark]

Turn over for the next question

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9 A market trader notices that daily sales are dependent on two variables:

number of hours, t, after the stall opens

total sales, x, in pounds since the stall opened.

The trader models the rate of sales as directly proportional to $\frac{8-t}{x}$

After two hours the rate of sales is £72 per hour and total sales are £336

9 (a) Show that

$$x\frac{\mathrm{d}x}{\mathrm{d}t} = 4032(8-t)$$

[3 marks]

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9 (b) Hence, show that

$$x^2 = 4032t(16 - t)$$

[3 marks]

Question 9 continues on the next page

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9 (c) The stall opens at 09.30.

9 (c) (i) The trader closes the stall when the rate of sales falls below £24 per hour.

Using the results in parts **(a)** and **(b)**, calculate the earliest time that the trader closes the stall.

[6 marks]

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9 (c) (ii) Explain why the model used by the trader is not valid at 09.30.

[2 marks]

Turn over for Section B

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Section B

Answer **all** questions in the spaces provided.

10 A garden snail moves in a straight line from rest to $1.28\,\text{cm}\,\text{s}^{-1}$, with a constant acceleration in 1.8 seconds.

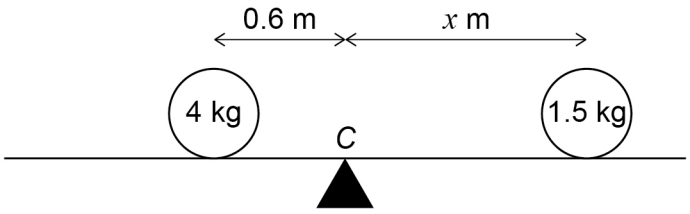
Find the acceleration of the snail.

Circle your answer.

[1 mark]

$2.30\,\text{m}\,\text{s}^{-2}$ $0.71\,\text{m}\,\text{s}^{-2}$ $0.0071\,\text{m}\,\text{s}^{-2}$ $0.023\,\text{m}\,\text{s}^{-2}$

11 A uniform rod, *AB*, has length 4 metres.
The rod is resting on a support at its midpoint *C*.
A particle of mass 4 kg is placed 0.6 metres to the left of *C*.
Another particle of mass 1.5 kg is placed x metres to the right of *C*, as shown.



The rod is balanced in equilibrium at *C*.

Find x.

Circle your answer.

[1 mark]

1.8 m 1.5 m 1.75 m 1.6 m

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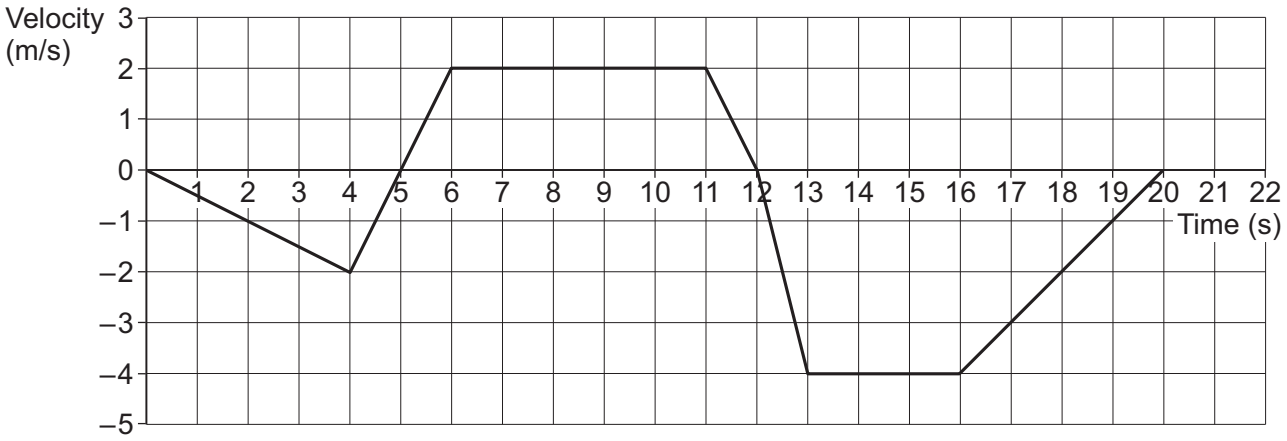
12 The graph below shows the velocity of an object moving in a straight line over a 20 second journey.



12 (a) Find the maximum magnitude of the acceleration of the object.

[1 mark]

12 (b) The object is at its starting position at times 0, t_1 and t_2 seconds.

Find t_1 and t_2

[4 marks]

Turn over ▶

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13 **In this question use $g = 9.8\,\text{m}\,\text{s}^{-2}$**

A boy attempts to move a wooden crate of mass 20 kg along horizontal ground. The coefficient of friction between the crate and the ground is 0.85

13 (a) The boy applies a horizontal force of 150 N. Show that the crate remains stationary.

[3 marks]

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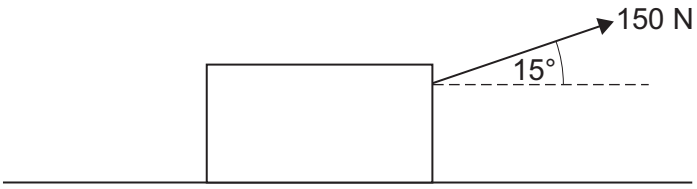
13 (b) Instead, the boy uses a handle to pull the crate forward. He exerts a force of 150 N, at an angle of 15° above the horizontal, as shown in the diagram.



Determine whether the crate remains stationary.

Fully justify your answer.

[5 marks]

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14 A quadrilateral has vertices A, B, C and D with position vectors given by

$$\overrightarrow{OA} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}, \overrightarrow{OC} = \begin{bmatrix} 0 \\ 7 \\ 6 \end{bmatrix} \text{ and } \overrightarrow{OD} = \begin{bmatrix} 4 \\ 10 \\ 0 \end{bmatrix}$$

14 (a) Write down the vector $\overrightarrow{AB}$

[1 mark]

14 (b) Show that $ABCD$ is a parallelogram, but not a rhombus.

[5 marks]

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15 A driver is road-testing two minibuses, A and B, for a taxi company.

The performance of each minibus along a straight track is compared.

A flag is dropped to indicate the start of the test.

Each minibus starts from rest.

The acceleration in $\mathrm{m\,s^{-2}}$ of each minibus is modelled as a function of time, t seconds, after the flag is dropped:

$$\text{The acceleration of A} = 0.138\,t^2$$
$$\text{The acceleration of B} = 0.024\,t^3$$

15 (a) Find the time taken for A to travel 100 metres.

Give your answer to four significant figures.

[4 marks]

Question 15 continues on the next page

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15 (b) The company decides to buy the minibus which travels 100 metres in the shortest time.

Determine which minibus should be bought.

[4 marks]

15 (c) The models assume that both minibuses start moving immediately when $t = 0$

In light of this, explain why the company may, in reality, make the wrong decision.

[1 mark]

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16 **In this question use $g = 9.81\,\text{m}\,\text{s}^{-2}$**

A particle is projected with an initial speed u, at an angle of 35° above the horizontal.

It lands at a point 10 metres vertically below its starting position.

The particle takes 1.5 seconds to reach the highest point of its trajectory.

16 (a) Find u.

[3 marks]

16 (b) Find the total time that the particle is in flight.

[3 marks]

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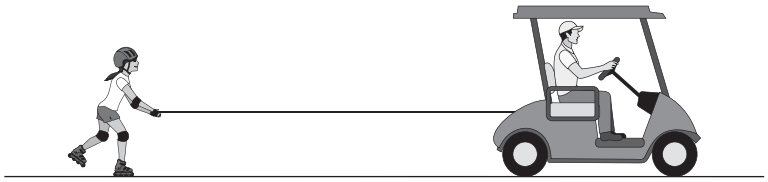
17 A buggy is pulling a roller-skater, in a straight line along a horizontal road, by means of a connecting rope as shown in the diagram.



The combined mass of the buggy and driver is 410 kg
A driving force of 300 N and a total resistance force of 140 N act on the buggy.

The mass of the roller-skater is 72 kg
A total resistance force of R newtons acts on the roller-skater.

The buggy and the roller-skater have an acceleration of $0.2\,\text{m}\,\text{s}^{-2}$

17 (a) (i) Find R.

[3 marks]

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17 (a) (ii) Find the tension in the rope.

[3 marks]

17 (b) State a necessary assumption that you have made.

[1 mark]

Question 17 continues on the next page

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17 (c) The roller-skater releases the rope at a point A, when she reaches a speed of $6\,m\,s^{-1}$

She continues to move forward, experiencing the same resistance force.

The driver notices a change in motion of the buggy, and brings it to rest at a distance of 20 m from A.

17 (c) (i) Determine whether the roller-skater will stop before reaching the stationary buggy.

Fully justify your answer.

[5 marks]

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17 (c) (ii) Explain the change in motion that the driver noticed.

[2 marks]

END OF QUESTIONS

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