

Mark Scheme (Results)

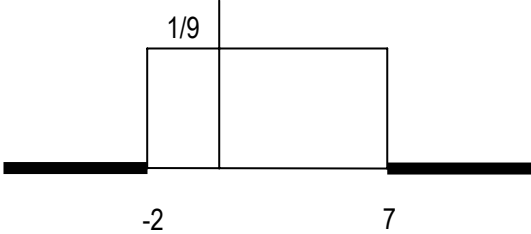
January 2009

GCE

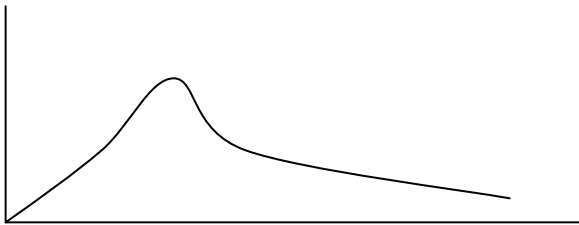
GCE Mathematics (6684/01)

**January 2009
6684 Statistics S2
Mark Scheme**

Question Number	Scheme	Marks
1	The random variable X is the number of daisies in a square. Poisson(3)	B1
(a)	$1 - P(X \leq 2) = 1 - 0.4232 \quad 1 - e^{-3}(1 + 3 + \frac{3^2}{2!})$ $= 0.5768$	M1 A1 (3)
(b)	$P(X \leq 6) - P(X \leq 4) = 0.9665 - 0.8153 \quad e^{-3} \left(\frac{3^5}{5!} + \frac{3^6}{6!} \right)$ $= 0.1512$	M1 A1 (2)
(c)	$\mu = 3.69$ $\text{Var}(X) = \frac{1386}{80} - \left(\frac{295}{80} \right)^2$ $= 3.73/3.72/3.71 \quad \text{accept } s^2 = 3.77$	B1 M1 A1 (3)
(d)	For a Poisson model, Mean = Variance; For these data $3.69 \approx 3.73$ \Rightarrow Poisson model	B1 (1)
(e)	$\frac{e^{-3.6875} 3.6875^4}{4!} = 0.193$ <p style="text-align: right;">allow their mean or var Awrt 0.193 or 0.194</p>	M1 A1 ft (2)

Question Number	Scheme	Marks
2	<p>(a) $f(x) = \begin{cases} \frac{1}{9} & -2 \leq x \leq 7 \\ 0 & \text{otherwise} \end{cases}$</p> <p>(b) </p> <p>(c) $E(X) = \underline{2.5}$ $\text{Var}(X) = \frac{1}{12}(7+2)^2$ or $\underline{6.75}$ both</p> <p>$E(X^2) = \text{Var}(X) + E(X)^2$</p> <p>$= 6.75 + 2.5^2$</p> <p>$= 13$</p> <p>alternative</p> <p>$\int_{-2}^7 x^2 f(x) dx = \left[\frac{x^3}{27} \right]_{-2}^7$ attempt to integrate and use limits of -2 and 7</p> <p>$= 13$</p> <p>(d) $P(-0.2 < X < 0.6) = \frac{1}{9} \times 0.8$</p> <p>$= \frac{4}{45}$ or 0.0889 Or equiv awrt 0.089</p>	<p>B1 B1 (2)</p> <p>B1 B1 (2)</p> <p>B1 M1 A1 (3)</p> <p>B1 M1 A1 M1 A1 (2)</p>

Question Number	Scheme	Marks
3	<p>(a) $X \sim B(20, 0.3)$</p> <p>$P(X \leq 2) = 0.0355$</p> <p>$P(X \geq 11) = 1 - 0.9829 = 0.0171$</p> <p>Critical region is $(X \leq 2) \cup (X \geq 11)$</p> <p>(b) Significance level = $0.0355 + 0.0171, = 0.0526$ or 5.26%</p> <p>(c) Insufficient evidence to reject H_0 Or sufficient evidence to accept H_0 /not significant $x = 3$ (or the value) is not in the critical region or $0.1071 > 0.025$</p> <p>Do not allow inconsistent comments</p>	<p>M1</p> <p>A1 A1 (3)</p> <p>M1 A1 (2)</p> <p>B1 ft</p> <p>B1 ft (2)</p>

Question Number	Scheme	Marks
4	<p>(a) $\int_0^{10} kt dt = 1$ or Area of triangle = 1</p> <p>$\left[\frac{kt^2}{2} \right]_0^{10} = 1$ or $10 \times 0.5 \times 10k = 1$ or linear equation in k</p> <p>$50k = 1$</p> <p>$k = \frac{1}{50}$ cso</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p>
	<p>(b) $\int_6^{10} kt dt = \left[\frac{kt^2}{2} \right]_6^{10}$</p> <p>$= \frac{16}{25}$</p>	<p>M1</p> <p>A1</p> <p>(2)</p>
	<p>(c) $E(T) = \int_0^{10} kt^2 dt = \left[\frac{kt^3}{3} \right]_0^{10}$</p> <p>$= 6\frac{2}{3}$</p>	<p>M1</p> <p>A1</p>
	<p>$\text{Var}(T) = \int_0^{10} kt^3 dt - \left(6\frac{2}{3}\right)^2 = \left[\frac{kt^4}{4} \right]_0^{10} - \left(6\frac{2}{3}\right)^2$</p> <p>$= 50 - \left(6\frac{2}{3}\right)^2$</p> <p>$= 5\frac{5}{9}$</p>	<p>M1;M1dep</p> <p>A1</p> <p>(5)</p>
	<p>(d) 10</p>	<p>B1</p> <p>(1)</p>
<p>(e)</p> 	<p>B1</p> <p>(1)</p>	

Question Number	Scheme	Marks
5	<p>(a) X represents the number of defective components.</p> $P(X = 1) = (0.99)^9 (0.01) \times 10 = 0.0914$ <p>(b) $P(X \geq 2) = 1 - P(X \leq 1)$ $= 1 - (p)^{10} - (a)$ $= 0.0043$</p> <p>(c) $X \sim \text{Po}(2.5)$</p> $P(1 \leq X \leq 4) = P(X \leq 4) - P(X = 0)$ $= 0.8912 - 0.0821$ $= 0.809$ <p>Normal distribution used. B1 for mean only</p> <hr/> <p>Special case for parts a and b If they use 0.1 do not treat as misread as it makes it easier. (a) M1 A0 if they have 0.3874 (b) M1 A1ft A0 they will get 0.2639 (c) Could get B1 B0 M1 A0</p> <hr/> <p>For any other values of p which are in the table do not use misread. Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c) B1 B0 M1 A0</p>	<p>M1A1 (2)</p> <p>M1 A1✓ A1 (3)</p> <p>B1B1 M1 A1 (4)</p>

Question Number	Scheme	Marks
<p>6 (a)(i)</p> <p>(ii)</p> <p>(b)</p> <p>(c)</p>	$H_0 : \lambda = 7 \quad H_1 : \lambda > 7$	B1
	$X = \text{number of visits. } X \sim \text{Po}(7)$	B1
	$P(X \geq 10) = 1 - P(X \leq 9) = 0.1695$	M1
	$1 - P(X \leq 10) = 0.0985$ $1 - P(X \leq 9) = 0.1695$ CR $X \geq 11$	A1
	$0.1695 > 0.10$, CR $X \geq 11$ Not significant or it is not in the critical region or do not reject H_0 The rate of visits on a Saturday is not greater/ is unchanged	M1 A1 no ft
	$X = 11$	B1
	(The visits occur) randomly/ independently or singly or constant rate	B1 (7) (1)
	$[H_0 : \lambda = 7 \quad H_1 : \lambda > 7 \quad (\text{or } H_0 : \lambda = 14 \quad H_1 : \lambda > 14)]$	
	$X \sim N;(14,14)$	B1;B1
	$P(X \geq 20) = P\left(z \geq \frac{19.5 - 14}{\sqrt{14}}\right)$ $= P(z \geq 1.47)$ $= 0.0708 \quad \text{or } z = 1.2816$	+/- 0.5, stand M1 M1 A1dep both M A1dep 2 nd M (6)

Question Number	Scheme	Marks
7	<p>(a) $F(x_0) = \int_1^x -\frac{2}{9}x + \frac{8}{9} dx = \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right]_1^x$ $= \left[-\frac{1}{9}x^2 + \frac{8}{9}x\right] - \left[-\frac{1}{9} + \frac{8}{9}\right]$ $= -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}$</p> <p>(b) $F(x) = \begin{cases} 0 & x < 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$</p> <p>(c) $F(x) = 0.75$; or $F(2.5) = -\frac{1}{9} \times 2.5^2 + \frac{8}{9} \times 2.5 - \frac{7}{9}$</p> <p>$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.75$</p> <p>$4x^2 - 32x + 55 = 0$</p> <p>$-x^2 + 8x - 13.75 = 0$ $x = 2.5$ $= 0.75$ cso</p> <p>and $F(x) = 0.25$</p> <p>$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.25$</p> <p>$-x^2 + 8x - 7 = 2.25$</p> <p>$-x^2 + 8x - 9.25 = 0$ quadratic 3 terms = 0</p> <p>$x = \frac{-8 \pm \sqrt{8^2 - 4 \times -1 \times -9.25}}{2 \times -1}$</p> <p>$x = 1.40$</p> <p>(d) $Q_3 - Q_2 > Q_2 - Q_1$ Or mode = 1 and mode < median Or mean = 2 and median < mode Sketch of pdf here or be referred to if in a different part of the question Box plot with Q_1, Q_2, Q_3 values marked on Positive skew</p>	<p>M1A1</p> <p>A1</p> <p>(3)</p> <p>B1B1f</p> <p>(2)</p> <p>M1;</p> <p>A1</p> <p>M1</p> <p>M1 dep M1 dep</p> <p>A1</p> <p>(6)</p> <p>M1</p> <p>A1</p> <p>(2)</p>