Question	Scheme	Marks	AOs
2(a)	(i) $\frac{\mathrm{d}y}{\mathrm{d}x}=2x-2-12x^{-\frac{1}{2}}$	M1 A1	1.1b 1.1b
	(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2+6x^{-\frac{3}{2}}$	B1ft	1.1b
		(3)	
(b)	Substitutes $x=4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}=2\times4-2-12\times4^{-\frac{1}{2}}=...$	M1	1.1b
	Shows $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and states "hence there is a stationary point" oe	A1	2.1
		(2)	
(c)	Substitutes $x=4$ into their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2+6\times4^{-\frac{3}{2}}=(2.75)$	M1	1.1b
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2.75>0$ and states "hence minimum"	A1ft	2.2a
		(2)	
			(7 marks)

(a)(i)

M1: Differentiates to $\frac{\mathrm{d}y}{\mathrm{d}x}=Ax+B+Cx^{-\frac{1}{2}}$ **A1:** $\frac{\mathrm{d}y}{\mathrm{d}x}=2x-2-12x^{-\frac{1}{2}}$ (Coefficients may be unsimplified)

(a)(ii)

B1ft: Achieves a correct $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for their $\frac{\mathrm{d}y}{\mathrm{d}x}$ (Their $\frac{\mathrm{d}y}{\mathrm{d}x}$ must have a negative or fractional index)

(b)

M1: Substitutes $x=4$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and attempts to evaluate. There must be evidence $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=4}=...$

Alternatively substitutes $x=4$ into an equation resulting from $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ Eg. $\frac{36}{x}=(x-1)^2$ and equates

A1: There must be a reason and a minimal conclusion. Allow ✓, QED for a minimal conclusion

Shows $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and states "hence there is a stationary point" oe

Alt Shows that $x=4$ is a root of the resulting equation and states "hence there is a stationary point"
All aspects of the proof must be correct including a conclusion

(c)

M1: Substitutes $x=4$ into their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and calculates its value, or implies its sign by a statement such as when $x=4\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$. This must be seen in (c) and not labelled (b). Alternatively calculates the gradient of C either side of $x=4$ or calculates the value of y either side of $x=4$.

A1ft: For **a correct calculation**, **a valid reason** and **a correct conclusion**. Ignore additional work where candidate finds $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ left and right of $x=4$. Follow through on an incorrect $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ but it is dependent upon having a negative or fractional index. Ignore any references to the word convex. The nature of the turning point is "minimum".

Using the gradient look for correct calculations, a valid reason…. goes from negative to positive, and a correct conclusion …minimum.

Question	Scheme	Marks	AOs
5	$\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{(2\sin\theta+2\cos\theta)3\cos\theta-3\sin\theta(2\cos\theta-2\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$	M1 A1	1.1b 1.1b
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator or uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{...}{.........C\sin\theta\cos\theta}$	M1	3.1a
	Expands and uses $\sin^2\theta+\cos^2\theta=1$ the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{P}{Q+R\sin 2\theta}$	M1	2.1
	$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{3}{2+2\sin 2\theta}=\frac{3/2}{1+\sin 2\theta}$	A1	1.1b
	(5 marks)		

Notes:

M1: For choosing either the quotient, product rule or implicit differentiation and applying it to the given function. Look for the correct form of $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ (condone it being stated as $\frac{\mathrm{d}y}{\mathrm{d}x}$) but tolerate slips on the coefficients and also condone $\frac{\mathrm{d}(\sin\theta)}{\mathrm{d}\theta}=\pm\cos\theta$ and $\frac{\mathrm{d}(\cos\theta)}{\mathrm{d}\theta}=\pm\sin\theta$

For quotient rule look for $\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{(2\sin\theta+2\cos\theta)\times\pm...\cos\theta-3\sin\theta(\pm...\cos\theta\pm...\sin\theta)}{(2\sin\theta+2\cos\theta)^2}$

For product rule look for

$$\frac{\mathrm{d}y}{\mathrm{d}\theta}=(2\sin\theta+2\cos\theta)^{-1}\times\pm...\cos\theta\pm3\sin\theta\times(2\sin\theta+2\cos\theta)^{-2}\times(\pm...\cos\theta\pm...\sin\theta)$$

Implicit differentiation look for $(...\cos\theta\pm...\sin\theta)y+(2\sin\theta+2\cos\theta)\frac{\mathrm{d}y}{\mathrm{d}\theta}=...\cos\theta$

A1: A correct expression involving $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ condoning it appearing as $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta=\sin 2\theta$ in $\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{...}{.........C\sin\theta\cos\theta}$

M1: Expands and uses $\sin^2\theta+\cos^2\theta=1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta=\sin 2\theta$ in the denominator to reach an expression of the form $\frac{\mathrm{d}y}{\mathrm{d}\theta}=\frac{P}{Q+R\sin 2\theta}$.

A1: Fully correct proof with $A=\frac{3}{2}$ stated but allow for example $\frac{3/2}{1+\sin 2\theta}$

Allow recovery from missing brackets. Condone notation slips. This is not a given answer

Question	Scheme	Marks	AOs
11	$\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv A+\dfrac{B}{(x-3)}+\dfrac{C}{(1-2x)}$		
(a) **Way 1**	$1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using a correct identity	A1	1.1b
		(4)	
(a) **Way 2**	{long division gives} $\dfrac{1+11x-6x^2}{(x-3)(1-2x)} \equiv 3+\dfrac{-10x+10}{(x-3)(1-2x)}$		
	$-10x+10 \equiv B(1-2x)+C(x-3) \Rightarrow B=..., C=...$	M1	2.1
	$A=3$	B1	1.1b
	Uses substitution or compares terms to find either $B=...$ or $C=...$	M1	1.1b
	$B=4$ and $C=-2$ which have been found using $-10x+10 \equiv B(1-2x)+C(x-3)$	A1	1.1b
		(4)	
(b)	$\text{f}(x)=3+\dfrac{4}{(x-3)}-\dfrac{2}{(1-2x)}$ $\{=3+4(x-3)^{-1}-2(1-2x)^{-1}\}$; $x>3$		
	$\text{f}'(x)=-4(x-3)^{-2}-4(1-2x)^{-2}$ $\left\{=-\dfrac{4}{(x-3)^2}-\dfrac{4}{(1-2x)^2}\right\}$	M1	2.1
		A1ft	1.1b
	Correct $\text{f}'(x)$ **and** as $(x-3)^2>0$ and $(1-2x)^2>0$, then $\text{f}'(x)=-(+\text{ve})-(+\text{ve})<0$, so $\text{f}(x)$ is a decreasing function	A1	2.4
		(3)	
			(7 marks)

Notes for Question 11	
(a)	
M1:	**Way 1:** Uses a correct identity $1+11x-6x^2 \equiv A(1-2x)(x-3)+B(1-2x)+C(x-3)$ in a complete method to find values for *B* and *C*. **Note:** Allow one slip in copying $1+11x-6x^2$ **Way 2:** Uses a correct identity $-10x+10 \equiv B(1-2x)+C(x-3)$ (which has been found from long division) in a complete method to find values for *B* and *C*
B1:	$A=3$
M1:	Attempts to find the value of either *B* or *C* from their identity This can be achieved by ***either*** substituting values into their identity ***or*** by comparing coefficients and solving the resulting equations simultaneously
A1:	See scheme
Note:	**Way 1:** Comparing terms: $x^2: \ -6=-2A; \quad x: \ 11=7A-2B+C; \quad \text{constant}: \ 1=-3A+B-3C$ **Way 1:** Substituting: $x=3: \ -20=-5B \Rightarrow B=4$; $x=\dfrac{1}{2}: \ 5=-\dfrac{5}{2}C \Rightarrow C=-2$
Note:	**Way 2:** Comparing terms: $x: \ -10=-2B+C; \quad \text{constant}: \ 10=B-3C$ **Way 2:** Substituting: $x=3: \ -20=-5B \Rightarrow B=4$; $x=\dfrac{1}{2}: \ 5=-\dfrac{5}{2}C \Rightarrow C=-2$

Question	Scheme	Marks	AOs
3 (a)	Correct method used in attempting to differentiate $y=\dfrac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x+1)^2\times(10x+10)-(5x^2+10x)\times2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{(x+1)\times(10x+10)-(5x^2+10x)\times2}{(x+1)^3}=\dfrac{A}{(x+1)^3}$	M1	2.1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$ Follow through on their $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{A}{(x+1)^n},n=1,3$	B1ft	2.2a
		(1)	
			(5 marks)

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\dfrac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

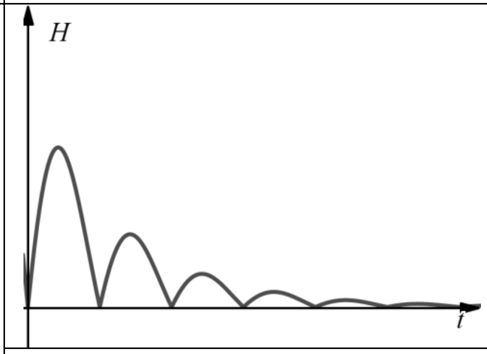
Condone slips but expect $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$ $(A,B,C,D>0)$ or

$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=\dfrac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{\left((x+1)^2\right)^2}$ $(A,B,C,D>0)$ using the quotient rule

or $\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow where they quote the correct formula, give values of *u* and *v,* but only have *v* rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Question	Scheme	Marks	AOs
12 (a)	$\mathrm{f}(x)=10\mathrm{e}^{-0.25x}\sin x$		
	$\Rightarrow \mathrm{f}'(x)=-2.5\mathrm{e}^{-0.25x}\sin x+10\mathrm{e}^{-0.25x}\cos x$ oe	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x)=0\Rightarrow -2.5\cancel{\mathrm{e}^{-0.25x}}\sin x+10\cancel{\mathrm{e}^{-0.25x}}\cos x=0$	M1	2.1
	$\frac{\sin x}{\cos x}=\frac{10}{2.5}\Rightarrow \tan x=4*$	A1*	1.1b
		(4)	
(b)	 "Correct" shape for 2 loops	M1	1.1b
	Fully correct with decreasing heights	A1	1.1b
		(2)	
(c)	Solves $\tan x=4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47)=\left\|10\mathrm{e}^{-0.25\times4.47}\sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .

So for example score expressions of the form $\pm ...\mathrm{e}^{-0.25x}\sin x \pm ...\mathrm{e}^{-0.25x}\cos x$ M1

Sight of $vdu-udv$ however is M0

A1: $\mathrm{f}'(x)=-2.5\mathrm{e}^{-0.25x}\sin x+10\mathrm{e}^{-0.25x}\cos x$ which may be unsimplified

M1: For clear reasoning in setting their $\mathrm{f}'(x)=0$, factorising/ cancelling out the $\mathrm{e}^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$

Do not allow candidates to substitute $x=\arctan 4$ into $\mathrm{f}'(x)$ to score this mark.

A1*: Shows the steps $\frac{\sin x}{\cos x}=\frac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x=4*$. $\frac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.

Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.

The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

Question	Scheme	Marks	AOs
9(a)	$f(x)=4\left(x^2-2\right)e^{-2x}$		
	Differentiates to $e^{-2x}\times 8x+4\left(x^2-2\right)\times -2e^{-2x}$	M1 A1	1.1b 1.1b
	$f'(x)=8e^{-2x}\left\{x-\left(x^2-2\right)\right\}=8\left(2+x-x^2\right)e^{-2x}$ *	A1*	2.1
		(3)	
(b)	States roots of $f'(x)=0$ $x=-1,2$	B1	1.1b
	Substitutes one x value to find a y value	M1	1.1b
	Stationary points are $\left(-1,-4e^2\right)$ and $\left(2,8e^{-4}\right)$	A1	1.1b
		(3)	
(c)	(i) Range $\left[-8e^2,\infty\right)$ o.e. such as $g(x)\geqslant -8e^2$	B1ft	2.5
	(ii) For • Either attempting to find $2f(0)-3=2\times -8-3=(-19)$ and identifying this as the **lower bound** • Or attempting to find $2\times "8e^{-4}"-3$ and identifying this as the **upper bound**	M1	3.1a
	Range $\left[-19,16e^{-4}-3\right]$	A1	1.1b
		(3)	
			(9 marks)
Notes:			

(a)

M1: Attempts the product rule and uses $e^{-2x}\to ke^{-2x},\quad k\neq 0$

If candidate states $u=4(x^2-2), v=e^{-2x}$ with $u'=....,v'=...e^{-2x}$ it can be implied by their $vu'+uv'$

If they just write down an answer without working award for $f'(x)=pxe^{-2x}\pm q\left(x^2-2\right)e^{-2x}$

They may multiply out first $f(x)=4x^2e^{-2x}-8e^{-2x}$. Apply in the same way condoning slips

Alternatively attempts the quotient rule on $f(x)=\frac{u}{v}=\frac{4\left(x^2-2\right)}{e^{2x}}$ with $v'=ke^{2x}$ and $f'(x)=\frac{vu'-uv'}{v^2}$

A1: A correct $f'(x)$ which may be unsimplified.

Via the quotient rule you can award for $f'(x)=\frac{8xe^{2x}-8\left(x^2-2\right)e^{2x}}{e^{4x}}$ o.e.

A1*: Proceeds correctly to given answer showing all necessary steps.

The $f'(x)$ or $\frac{dy}{dx}$ must be present at some point in the solution

This is a "show that" question and there must not be any errors. All bracketing must be correct.

Allow a candidate to move from the **simplified** unfactorised answer of $f'(x)=8xe^{-2x}-8\left(x^2-2\right)e^{-2x}$

Question	Scheme	Marks	AOs
13(a)	$k = \mathrm{e}^2$ **or** $x \neq \mathrm{e}^2$	B1	2.2a
		(1)	
(b)	$g'(x) = \dfrac{(\ln x - 2)\times\frac{3}{x} - (3\ln x - 7)\times\frac{1}{x}}{(\ln x - 2)^2} = \dfrac{1}{x(\ln x - 2)^2}$ or $g'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(3 - (\ln(x) - 2)^{-1}\right) = (\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$ or $g'(x) = (\ln x - 2)^{-1}\times\dfrac{3}{x} - (3\ln x - 7)(\ln x - 2)^{-2}\times\dfrac{1}{x} = \dfrac{1}{x(\ln x - 2)^2}$	M1 A1	1.1b 2.1
	As $x > 0$ (or $1/x > 0$) **AND** $\ln x - 2$ is squared so $g'(x) > 0$	A1cso	2.4
		(3)	
(c)	Attempts to solve either $3\ln x - 7 \ldots 0$ or $\ln x - 2 \ldots 0$ or $3\ln a - 7 \ldots 0$ or $\ln a - 2 \ldots 0$ where … is "=" or ">" to reach a value for x or a but may be seen as an inequality e.g. $x > \ldots$ or $a > \ldots$	M1	3.1a
	$0 < a < \mathrm{e}^2, \quad a > \mathrm{e}^{\frac{7}{3}}$	A1	2.2a
		(2)	
			(6 marks)

Question Number	Scheme	Marks
1.	$\mathrm{f}(x)=\dfrac{2x}{x^2+3} \quad \Rightarrow \mathrm{f}'(x)=\dfrac{(x^2+3)2-2x\times 2x}{(x^2+3)^2}=\left(\dfrac{6-2x^2}{(x^2+3)^2}\right)$	M1A1
	$\mathrm{f}'(x)>0 \Rightarrow \dfrac{6-2x^2}{(x^2+3)^2}>0$	
	Critical values $6-2x^2=0 \Rightarrow x=\pm\sqrt{3}$	M1A1
	Inside region chosen $-\sqrt{3}<x<\sqrt{3}$	dM1A1
		(6 marks)

Notes

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\dfrac{2x}{x^2+3}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=2x, v=x^2+3, u'=.., v'=...$ followed by their $\dfrac{vu'-uv'}{v^2}$, then only accept answers of the form

$\dfrac{(x^2+3)A-2x\times Bx}{(x^2+3)^2}$ $A,B>0$. Condone invisible brackets for the M.

Alternatively applies the product rule with $u=2x, v=\left(x^2+3\right)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=2x, v=\left(x^2+3\right)^{-1}, u'=.., v'=...$ followed by their $vu'+uv'$, then only accept answers of the form $\left(x^2+3\right)^{-1}\times A\pm 2x\times\left(x^2+3\right)^{-2}\times Bx$.

Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of $\mathrm{f}'(x)$

Accept versions of $\mathrm{f}'(x)=\dfrac{(x^2+3)2-2x\times 2x}{(x^2+3)^2}$ for the quotient rule or

Versions of $\mathrm{f}'(x)=\left(x^2+3\right)^{-1}\times 2-2x\times\left(x^2+3\right)^{-2}\times 2x$ for use of the product rule.

M1 Setting their numerator of $\mathrm{f}'(x)=0$ or >0, and proceeding to find two critical values.

A1 Both critical values $\pm\sqrt{3}$ are found. Accept for this mark expressions like $x>\pm\sqrt{3}$ and ± 1.73

dM1 For choosing the inside region of their critical values.

The inequality (if seen) must have been of the correct form. Either $Ax^2....-B<0$, $C......-Dx^2>0$ or $x^2<C$ .It is dependent upon having set the numerator >0 or =0.

A1 Correct solution only. $-\sqrt{3}<x<\sqrt{3}$. Accept $\left(-\sqrt{3},\sqrt{3}\right)$ $x<\sqrt{3}$ and $x>-\sqrt{3}$

Do not accept $x<\sqrt{3}$ or $x>-\sqrt{3}$ or $-1.73<x<1.73$.

Do not accept a correct answer coming from an incorrect inequality. This would be dM0A0.

Condone a solution $x^2<3 \Rightarrow x<\pm\sqrt{3} \Rightarrow -\sqrt{3}<x<\sqrt{3}$

Do not accept a solution without seeing a correct $\mathrm{f}'(x)$ first. Note that this is a demand of the question.

Question Number	Scheme		Marks
	Apply quotient rule :	Or apply product rule to $y = \cos 2\theta(1+\sin 2\theta)^{-1}$	
3.	$\left\{\begin{matrix} u = \cos 2\theta & v = 1+\sin 2\theta \\ \frac{\mathrm{d}u}{\mathrm{d}\theta} = -2\sin 2\theta & \frac{\mathrm{d}v}{\mathrm{d}\theta} = 2\cos 2\theta \end{matrix}\right\}$	$\left\{\begin{matrix} u = \cos 2\theta & v = (1+\sin 2\theta)^{-1} \\ \frac{\mathrm{d}u}{\mathrm{d}\theta} = -2\sin 2\theta & \frac{\mathrm{d}v}{\mathrm{d}\theta} = -2\cos 2\theta(1+\sin 2\theta)^{-2} \end{matrix}\right\}$	
	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{-2\sin 2\theta(1+\sin 2\theta) - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$	$-2(1+\sin 2\theta)^{-1}\sin 2\theta - 2\cos^2 2\theta(1+\sin 2\theta)^{-2}$	M1 A1
	$= \frac{-2\sin 2\theta - 2\sin^2 2\theta - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$	$= (1+\sin 2\theta)^{-2}\{-2\sin 2\theta - 2\sin^2 2\theta - 2\cos^2 2\theta$	
	$= \frac{-2\sin 2\theta - 2}{(1+\sin 2\theta)^2}$	$= (1+\sin 2\theta)^{-2}\{-2\sin 2\theta - 2\}$	M1
	$= \frac{-2(1+\sin 2\theta)}{(1+\sin 2\theta)^2} = \frac{-2}{1+\sin 2\theta}$		A1 cso
			[4] **4**

Notes

M1: **Applies the Quotient rule**, a form of which appears in the formula book, to $\frac{\cos 2\theta}{1+\sin 2\theta}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u = \cos 2\theta,\ v = 1+\sin 2\theta,\ u' = ..,\ v' = ...$ followed by their $\frac{vu' - uv'}{v^2}$, then only accept answers of the form $\frac{(1+\sin 2\theta)A\sin 2\theta - \cos 2\theta \times (B\cos 2\theta)}{(1+\sin 2\theta)^2}$ where A and B are constant (could be 1) Condone "invisible" brackets for the M mark. If double angle formulae are used give marks for correct work.

Alternatively applies the product rule with $u = \cos 2\theta,\ v = (1+\sin 2\theta)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u = \cos 2\theta,\ v = (1+\sin 2\theta)^{-1},\ u' = ..,\ v' = ...$ followed by their $vu' + uv'$,

then only accept answers of the form $(1+\sin 2\theta)^{-1} \times A\sin 2\theta \pm \cos 2\theta \times (1+\sin 2\theta)^{-2} \times B\cos 2\theta$.

Condone "invisible brackets" for the M. If double angle formulae are used give marks for correct work.

A1: Any fully correct (unsimplified) form of $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ If double angle formulae are used give marks for correct work.

Accept versions of $\frac{\mathrm{d}y}{\mathrm{d}\theta} = \frac{-2\sin 2\theta(1+\sin 2\theta) - 2\cos^2 2\theta}{(1+\sin 2\theta)^2}$ for use of the quotient rule or versions of

$\frac{\mathrm{d}y}{\mathrm{d}\theta} = (1+\sin 2\theta)^{-1} \times -2\sin 2\theta + \cos 2\theta \times (-1) \times (1+\sin 2\theta)^{-2} \times 2\cos 2\theta$ for use of the product rule.

M1: **Applies** $\sin^2 2\theta + \cos^2 2\theta \equiv 1$ or $-2\sin^2 2\theta - 2\cos^2 2\theta \rightarrow -2$ **correctly to eliminate squared trig. terms from the numerator** to obtain an expression of the form $k\sin 2\theta + \lambda$ where k and λ are constants (including 1) If double angle formulae have been used give marks only if correct work leads to answer in correct form. (If in doubt, send to review)

A1: Need to see **factorisation of numerator then** answer, which is cso

so $\frac{-2}{1+\sin 2\theta}$ or $\frac{a}{1+\sin 2\theta}$ and $a = -2$, with no previous errors

Question Number	Scheme	Marks
8.	$\frac{\mathrm{d}V}{\mathrm{d}t}=250$	
	$\left\{V=\frac{4}{3}\pi r^3 \Rightarrow\right\}\ \frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$	B1
	$V=12\,000 \Rightarrow\ 12\,000=\frac{4}{3}\pi r^3\ \Rightarrow\ r=\sqrt[3]{\frac{9000}{\pi}}\ \left(=14.202480...\right)$	B1
	$\frac{\mathrm{d}r}{\mathrm{d}t}\ \left\{=\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}\right\}\ =\frac{1}{4\pi r^2}\times 250$	M1
	When $r=\sqrt[3]{\frac{9000}{\pi}}$, $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{250}{4\pi\left(\sqrt[3]{\frac{9000}{\pi}}\right)^2}$	dM1
	So, $\frac{\mathrm{d}r}{\mathrm{d}t}=0.0986283...\left(\mathrm{cm\,s^{-1}}\right)$ awrt 0.099	A1
		[5] **5**

Notes

B1: $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$. This may be stated or used and need not be simplified

Applies $12\,000=\frac{4}{3}\pi r^3$ and rearranges to find *r* using division then cube root with accurate algebra

May state $r=\sqrt[3]{\frac{3V}{4\pi}}$ then substitute $V=12000$ later which is equivalent. *r* does not need to be evaluated.

M1: **Uses** chain rule correctly so $\frac{1}{\left(\text{their }\frac{\mathrm{d}V}{\mathrm{d}r}\right)}\times 250$

dM1: Substitutes their *r* **correctly** into their equation for $\frac{\mathrm{d}r}{\mathrm{d}t}$ This depends on the previous method mark

A1: awrt 0.099 (Units may be ignored) If this answer is seen, then award A1 and isw.
Premature approximation usually results in all marks being earned prior to this one.

Question Number	Scheme	Marks
10. (a)	$A=B \Rightarrow \sin 2A = \underline{\underline{\sin\left(A+A\right)}} = \underline{\sin A\cos A+\cos A\sin A}$ *or* $\underline{\sin A\cos A+\sin A\cos A}$	M1
	Hence, $\underline{\underline{\sin 2A\ = 2\sin A\cos A}}$ (as required) *	A1 * **[2]**
	Way 1A:	
(b)	$\left\{y\ =\ln\left[\tan\left(\tfrac{1}{2}x\right)\right]\Rightarrow\right\}\ \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$ Way 1B $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$	M1 A1
	$=\frac{1}{2\tan\left(\frac{1}{2}x\right)\cos^2\left(\frac{1}{2}x\right)}=\frac{1}{\frac{2\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}\cdot\frac{\cos^2\left(\frac{1}{2}x\right)}{1}}$ $=\frac{1+\tan^2\left(\frac{1}{2}x\right)}{2\tan\left(\frac{1}{2}x\right)}==\frac{\cos^2\left(\frac{1}{2}x\right)+\sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}$	dM1
	$=\frac{1}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}=\frac{1}{\sin x}=\operatorname{cosec}x$ * $=\frac{1}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}=\frac{1}{\sin x}=\operatorname{cosec}x$ *	A1 * **[4]**
	Way 2: $\left\{y\ =\ln\left[\sin\left(\tfrac{1}{2}x\right)\right]-\ln\left[\cos\left(\tfrac{1}{2}x\right)\right]\Rightarrow\right\}\ \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{1}{2}\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)}-\frac{-\frac{1}{2}\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$	M1 A1
	$=\frac{\cos^2\left(\frac{1}{2}x\right)+\sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)};=\frac{1}{\sin x}\ =\operatorname{cosec}x$	M1;A1 [4]
	Way3: quotes $\int\operatorname{cosec}x\,\mathrm{d}x=\ \ln(\tan\left(\tfrac{1}{2}x\right))$	M1 A1
	(As differentiation is reverse of integration) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan\left(\tfrac{1}{2}x\right)\right]=\operatorname{cosec}x$	M1 A1 [4]
(c)	$\left\{y\ =\ln\left[\tan\left(\tfrac{1}{2}x\right)\right]-3\sin x\Rightarrow\right\}\ \frac{\mathrm{d}y}{\mathrm{d}x}=\ \operatorname{cosec}x-3\cos x$	B1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\right\}\ \ \operatorname{cosec}x-3\cos x=0\Rightarrow\frac{1}{\sin x}-3\cos x=0$	M1
	$\Rightarrow 1=3\sin x\cos x\ \Rightarrow 1=\tfrac{3}{2}(2\sin x\cos x)$ so $\sin 2x=k$, where $-1<k<1$ and $k\neq 0$	M1
	So $\sin 2x=\frac{2}{3}$	A1
	$\{\ 2x=\{0.729727...,\ 2.411864...\}\ \}$ So $x=\{0.364863...,\ 1.205932...\}$	A1 A1 **[6]** **12**
Way2 10 (c)	Method (Squaring Method) $\left\{y\ =\ln\left[\tan\left(\tfrac{1}{2}x\right)\right]-3\sin x\Rightarrow\right\}\ \frac{\mathrm{d}y}{\mathrm{d}x}=\ \operatorname{cosec}x-3\cos x$	**B1**
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow\right\}\ \ \operatorname{cosec}x-3\cos x=0\Rightarrow\frac{1}{\sin x}-3\cos x=0$	M1
	$\Rightarrow\ \frac{1}{1-\cos^2x}=9\cos^2x$ so $9\cos^4x-9\cos^2x+1=0$ or $9\sin^4x-9\sin^2x+1=0$	M1
	So $\cos^2x=0.873$ *or* 0.127 or $\sin^2x=0.873$ *or* 0.127	A1
	So $x=\{0.364863...,\ 1.205932...\}$	A1 A1 **[6]**

Way 3 10c)	"t" method $\left\{y = \ln\left[\tan\left(\tfrac{1}{2}x\right)\right] - 3\sin x \Rightarrow\right\}\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{cosec} x - 3\cos x$	B1
	$\left\{\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\ \operatorname{cosec} x - 3\cos x = 0 \Rightarrow \dfrac{1}{\sin x} - 3\cos x = 0$	M1
	$\Rightarrow \dfrac{1+t^2}{2t} - 3\dfrac{1-t^2}{1+t^2} = 0$ so $t^4 + 6t^3 + 2t^2 - 6t + 1 = 0$	M1
	$t = 0.1845$ or 0.6885	A1
	So $x = \{0.364863..., 1.205932...\}$	A1 A1
		[6]

Notes

(a) M1: This mark is for the underlined equation in either form

$\underline{\sin A\cos A + \cos A\sin A}$ *or* $\underline{\sin A\cos A + \sin A\cos A}$

A1: For this mark need to see :

sin2*A* at the start of the proof , or as part of a conclusion

sin(*A* + *A*) = at the start

$= \underline{\sin A\cos A + \cos A\sin A}$ *or* $\underline{\sin A\cos A + \sin A\cos A}$

= 2sin*A*cos*A* at the end

(b)M1: For expression of the form $\dfrac{\pm k\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$, where k is constant (could even be 1)

A1: Correct differentiation so $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)}$

Way 1A:

dM1: Use both $\tan\left(\frac{1}{2}x\right) = \dfrac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$ and $\sec^2\left(\frac{1}{2}x\right) = \dfrac{1}{\cos^2\left(\frac{1}{2}x\right)}$ in their differentiated expression. This may be implied.

This depends on **the** previous Method mark.

A1*: Simplify the fraction, use double angle formula, see $\dfrac{1}{\sin x}$ and obtain correct answer with completely correct work and no errors seen (NB Answer is given)

Way 1B

dM1: Use both $\sec^2\left(\frac{1}{2}x\right) = 1 + \tan^2\left(\frac{1}{2}x\right)$ and $\tan\left(\frac{1}{2}x\right) = \dfrac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

A1*: Simplify the fraction, use double angle formula, see $\dfrac{1}{\sin x}$ and obtain correct answer with completely correct work and no errors seen (NB Answer is given)

Way 2:

M1:Split into $\left\{y = \ln\left[\sin\left(\frac{1}{2}x\right)\right] - \ln\left[\cos\left(\frac{1}{2}x\right)\right] \Rightarrow\right\}$ then differentiate to give $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \dfrac{c\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

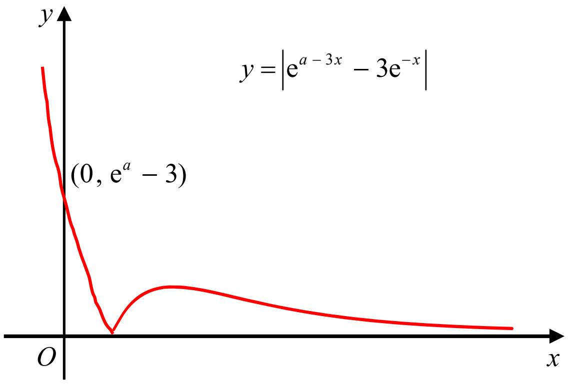
A1: Correct answer $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\frac{1}{2}\cos\left(\frac{1}{2}x\right)}{\sin\left(\frac{1}{2}x\right)} - \dfrac{-\frac{1}{2}\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)}$

M1: Obtain $= \dfrac{\cos^2\left(\frac{1}{2}x\right) + \sin^2\left(\frac{1}{2}x\right)}{2\sin\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right)}$ A1*: As before

Way 3:

Alternative method: This is rare, but is acceptable. Must be completely correct.

Quotes $\int \cos\mathrm{ec}x\,\mathrm{d}x = \ln(\tan\left(\frac{1}{2}x\right))$ and follows this by $\dfrac{\mathrm{d}}{\mathrm{d}x}\left[\tan\left(\frac{1}{2}x\right)\right] = \operatorname{cosec}x$ gets 4/4

Question Number	Scheme	Marks
11.		
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -3\mathrm{e}^{a-3x} + 3\mathrm{e}^{-x}$	M1 A1
	$-3\mathrm{e}^{a-3x} + 3\mathrm{e}^{-x} = 0 \Rightarrow \mathrm{e}^{-x} = \mathrm{e}^{a-3x} \Rightarrow -x = a - 3x \Rightarrow x =$	M1
	$x = \frac{1}{2}a$	A1
	So, $y_P = \mathrm{e}^{a-3\left(\frac{a}{2}\right)} - 3\mathrm{e}^{-\left(\frac{a}{2}\right)}$; $= -2\mathrm{e}^{-\frac{a}{2}}$	ddM1; A1
		[6]
	Mark parts (b) and (c) together.	
(b)	Method 1 $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow \mathrm{e}^{a-"2"x} = 3$ Method 2 $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow \mathrm{e}^{"2"x} = \frac{\mathrm{e}^{a}}{3}$ Method 3 $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow 3\mathrm{e}^{"2"x} = \mathrm{e}^{a}$	M1
	Method 1: $\Rightarrow a - "2"x = \ln 3$ Method 2: $"2"x = a - \ln 3$ Method 3: $\ln 3 + "2"x = a$	dM1
	$\Rightarrow x = \frac{a - \ln 3}{2}$ or equivalent e.g. $\frac{1}{2}\ln\left(\frac{e^{a}}{3}\right)$ or $-\ln\sqrt{\left(\frac{3}{e^{a}}\right)}$ etc	A1
		[3]
	Method 4 $0 = \mathrm{e}^{a-3x} - 3\mathrm{e}^{-x} \Rightarrow \mathrm{e}^{a-3x} = 3\mathrm{e}^{-x}$ and so $a - 3x = \ln 3 - x$	M1
	$"2"x = a - \ln 3$	dM1
	$\Rightarrow x = \frac{a - \ln 3}{2}$ o.e. e.g. $\frac{1}{2}\ln\left(\frac{e^{a}}{3}\right)$ or $-\ln\sqrt{\left(\frac{3}{e^{a}}\right)}$ etc	A1
		[3]
(c)		
	Shape	B1
	Cusp and behaviour for large x	B1
	$(0, \mathrm{e}^{a} - 3)$	B1
		[3] **12**

Question Number	Scheme	Marks
1	$y=7$ at point P	B1
	$y=\frac{3x-2}{(x-2)^2}\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)^2\times3-(3x-2)\times2(x-2)}{(x-2)^4}$	M1A1
	Sub $x=3$ into $\frac{\mathrm{d}y}{\mathrm{d}x}=(-11)$	M1
	$\frac{1}{11}=\frac{y-7}{x-3}\Rightarrow x-11y+74=0$ cso	M1A1 cso
		(6 marks)

B1 For seeing $y=7$ when $x=3$. This may be awarded if embedded within an equation.

M1 Application of Quotient rule. If the rule is quoted it must be correct.

It may be implied by their $u=3x-2, u'=.., v=(x-2)^2, v'=..$ followed by their $\frac{vu'-uv'}{v^2}$

If the rule is neither stated nor implied only accept expressions of the form $\frac{(x-2)^2\times A-(3x-2)\times B(x-2)}{\left((x-2)^2\right)^2}$ $A,B>0$ condoning missing brackets

Alternatively applies the Product rule to $(3x-2)(x-2)^{-2}$ If the rule is quoted it must be correct.

It may be implied by their u or $v=3x-2, u', v$ or $u=(x-2)^{-2}, v'$ followed by their $vu'+uv'$

If the rule is neither stated nor implied only accept expressions of the form $A(x-2)^{-2}\pm B(3x-2)(x-2)^{-3}$

If they use partial fractions expect to see $y=\frac{3x-2}{(x-2)^2}\Rightarrow y=\frac{P}{(x-2)}+\frac{Q}{(x-2)^2}(P=3,Q=4)\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\pm\frac{R}{(x-2)^2}\pm\frac{S}{(x-2)^3}$

You may also see implicit differentiation etc where the scheme is easily applied.

A1 A correct (unsimplified) form of the derivative.

Accept from the quotient rule versions equivalent to $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x-2)^2\times3-(3x-2)\times2(x-2)}{\left((x-2)^2\right)^2}$

Accept from the product rule versions equivalent to $\frac{\mathrm{d}y}{\mathrm{d}x}=3(x-2)^{-2}-2(3x-2)(x-2)^{-3}$

Accept from partial fractions $\frac{\mathrm{d}y}{\mathrm{d}x}=-3(x-2)^{-2}-8(x-2)^{-3}$

or $(x-2)^2\frac{\mathrm{d}y}{\mathrm{d}x}+y\times2(x-2)=3$ from implicit differentiation

FYI: Correct simplified expressions are $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3x^2+4x+4}{(x-2)^4}$ or $\frac{-3x-2}{(x-2)^3}$

M1 Sub $x=3$ into what they believe is their derivative to find a numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}$.

M1 Uses $x=3$ and their numerical value of y with their numerical $-\frac{\mathrm{d}x}{\mathrm{d}y}$ at $x=3$ to form an equation of a normal. If the form $y=mx+c$ is used then it must be a full method reaching a value for c.

A1 Correct solution only Accept $\pm A(x-11y+74)=0$ where $A\in\mathbb{N}$. from correct working.

Watch for correct answers coming from incorrect versions of $\frac{\mathrm{d}y}{\mathrm{d}x}$ with eg. $(x-2)^2$ on the denominator

Question Number	Scheme	Marks
6(i)	$x=\tan^2 4y\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=8\tan 4y\sec^2 4y$ oe	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\tan 4y\sec^2 4y},=\frac{1}{8\tan 4y(1+\tan^2 4y)}=\frac{1}{8\sqrt{x}(1+x)}=\frac{1}{8\left(x^{0.5}+x^{1.5}\right)}$	M1,M1A1
		(5)
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}t}=2,\quad V=x^3\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}x}=3x^2$	B1,B1
	Uses $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$	M1
	$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right\|_{x=4}=\frac{2}{3x^2}=\frac{1}{24}\left(\text{cm s}^{-1}\right)$	M1A1
		(5) **(10 marks)**

(i)

M1 Differentiates $\tan^2 4y$ to get an expression equivalent to the form $C\tan 4y\sec^2 4y$

You may see $\tan 4y\times A\sec^2 4y+\tan 4y\times B\sec^2 4y$ from the product rule or versions appearing from $\sqrt{x}=\tan 4y\Rightarrow Ax^{-0.5}\times...=B\sec^2 4y$ or

$Ax^{-0.5}=B\sec^2 4y\times...$ $x=\frac{\sin^2 4y}{\cos^2 4y}\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}y}=\frac{\cos^2 4y\times A\sin 4y\cos 4y-\sin^2 4y\times B\cos 4y\sin 4y}{\left(\cos^2 4y\right)^2}$

from the quotient rule

A1 Any fully correct answer, or equivalent, including the left hand side. $\frac{\mathrm{d}x}{\mathrm{d}y}=2\tan 4y\times 4\sec^2 4y$

Also accept the equivalent by implicit differentiation $1=8\tan 4y\sec^2 4y\frac{\mathrm{d}y}{\mathrm{d}x}$

M1 Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=1\Big/\frac{\mathrm{d}x}{\mathrm{d}y}$ Follow through on their $\frac{\mathrm{d}x}{\mathrm{d}y}$.

Condone issues with reciprocating the '8' but not the trigonometrical terms.

If implicit differentiation is used it is scored for writing $\frac{\mathrm{d}y}{\mathrm{d}x}$ as the subject.

M1 Uses $\sec^2 4y=1+\tan^2 4y$ where $x=\tan^2 4y$ to get their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of just x.

If they use other functions it is for using $\sin^2 4y=\frac{x}{1+x}$ and $\cos^2 4y=\frac{1}{1+x}$ where $x=\tan^2 4y$ to get their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of just x.

A1 Correct answer and solution. Accept $\frac{1}{8\left(x^{0.5}+x^{1.5}\right)},\frac{1}{8\left(x^{\frac{1}{2}}+x^{\frac{3}{2}}\right)}$ or A=8, p =0.5and q =1.5

Candidates do not have to explicitly state the values of A, p and q. Remember to isw after the sight of an acceptable answer.

Question Number	Scheme	Marks
8 (a)	$1000 < V \leq 23000$	B1,B1 **(2)**
(b)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 18000 \times -0.2\mathrm{e}^{-0.2t} + 4000 \times -0.1\mathrm{e}^{-0.1t}$	M1
	$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right\|_{t=10} = 18000 \times -0.2\mathrm{e}^{-2} + 4000 \times -0.1\mathrm{e}^{-1} = \text{awrt}(-)634$	M1A1 **(3)**
(c)	$15000 = 18000\mathrm{e}^{-0.2t} + 4000\mathrm{e}^{-0.1t} + 1000$ $0 = 9\mathrm{e}^{-0.2t} + 2\mathrm{e}^{-0.1t} - 7$	
	$0 = (9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$	M1A1
	$9\mathrm{e}^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\frac{9}{7}\right)$ oe	dM1A1
		(4) **(9 marks)**

(a)

B1 Accept either boundary: $V < 23000$ or $V \leq 23000$ or $V_{\max}\,23000$ for the upper boundary and $V > 1000$ or $V \geq 1000$ or $V_{\min}\,1000$ for the lower boundary. Answers like $V \geq 23000$ are B0

B1 Completely correct solution.
Accept $1000 < V \leq 23000$, $1000 < Range$ or $y \leq 23000$, $(1000,23000]$, $V > 1000$ *and* $V \leq 23000$

(b)

M1 Score for a $\frac{\mathrm{d}V}{\mathrm{d}t} = A\mathrm{e}^{-0.2t} + B\mathrm{e}^{-0.1t}$, where $A \neq 18000, B \neq 4000$

M1 Sub $t = 10$ into a $\frac{\mathrm{d}V}{\mathrm{d}t}$ of the form $A\mathrm{e}^{-0.2t} + B\mathrm{e}^{-0.1t}$ where $A \neq 18000, B \neq 4000$

Condone substitution of $t = 10$ into a $\frac{\mathrm{d}V}{\mathrm{d}t}$ of the form $A\mathrm{e}^{-0.2t} + B\mathrm{e}^{-0.1t} + 1000$ $A \neq 18000, B \neq 4000$

A1 Correct solution and answer only. Accept ± 634 following correct $\frac{\mathrm{d}V}{\mathrm{d}t} = -3600\mathrm{e}^{-0.2t} - 400\mathrm{e}^{-0.1t}$

Watch for students who sub $t = 10$ into their V first and then differentiate. This is 0,0,0.

Watch for students who achieve +634 following $\frac{\mathrm{d}V}{\mathrm{d}t} = 3600\mathrm{e}^{-0.2t} + 400\mathrm{e}^{-0.1t}$. This is 1,1,0

A correct answer with no working can score all marks.

(c)

M1 Setting up 3TQ in $\mathrm{e}^{\pm 0.1t}$ AND correct attempt to factorise or solve by the formula.

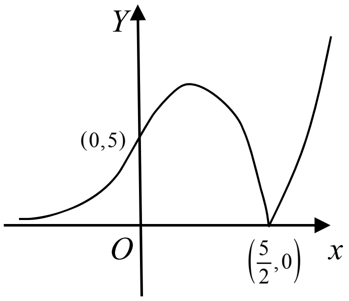
For this to be scored the $\mathrm{e}^{\pm 0.2t}$ term must be the x^2 term.

A1 Correct factors $(9\mathrm{e}^{-0.1t} - 7)(\mathrm{e}^{-0.1t} + 1)$ or $(7\mathrm{e}^{0.1t} - 9)(\mathrm{e}^{0.1t} + 1)$ or a root $\mathrm{e}^{-0.1t} = \frac{7}{9}$

dM1 Dependent upon the previous M1.
This is scored for setting the $a\mathrm{e}^{\pm 0.1t} - b = 0$ and proceeding using correct ln work to $t = ...$

A1 $t = 10\ln\left(\frac{9}{7}\right)$. Accept alternatives such as $t = \frac{1}{0.1}\ln\left(\frac{9}{7}\right), \frac{1}{-0.1}\ln\left(\frac{7}{9}\right), -10\ln\left(\frac{7}{9}\right)$

If any extra solutions are given withhold this mark.

Question Number	Scheme	Marks
3 (a)	$f'(x) = e^{x} \times 2 + (2x-5)e^{x}$	M1A1
	$f'(x) = 0 \Rightarrow (2x-3)e^{x} = 0 \Rightarrow x = \frac{3}{2}$	M1A1
	{Coordinates of *A*= $\left(\frac{3}{2}, -2e^{\frac{3}{2}}\right)$} obtains $y = -2e^{\frac{3}{2}}$	A1ft
		(5)
(b)	$-2e^{\frac{3}{2}} < k < 0$	M1A1
		(2)
(c)	 Shape including cusp	B1
	$\left(\frac{5}{2},0\right)$ only	B1
	$(0,5)$ only	B1
		(3)
		(10 marks)

Question Number	Scheme	Notes	Marks
4 (a)	$\begin{array}{r} x^2+5 \\ x^2+x-12\overline{)x^4+x^3-7x^2+8x-48} \\ \underline{x^4+x^3-12x^2} \\ 5x^2+8x-48 \\ \underline{5x^2+5x-60} \\ 3x+12 \end{array}$ M1: Divides $x^4+x^3-7x^2+8x-48$ by x^2+x-12 to get a quadratic quotient and a remainder of the form $\alpha x+\beta$ where α and β are not both zero A1: Correct quotient and remainder		M1A1
	$\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv x^2+5+\frac{3(x+4) \text{ or } 3x+12}{(x+4)(x-3)}$ Writes their answer as $\frac{x^4+x^3-7x^2+8x-48}{x^2+x-12} \equiv \text{Their Quotient}+\frac{\text{Their Remainder}}{(x+4)(x-3)}$		M1
	$\equiv x^2+5+\frac{3}{(x-3)}$ or states $A = 5$, $B = 3$		A1
			(4)

(b)	$g'(x)=2x-\frac{3}{(x-3)^2}$	M1: $x^2+A+\frac{B}{x-3}\rightarrow 2x\pm\frac{B}{(x-3)^2}$ A1: $x^2+A+\frac{B}{x-3}\rightarrow 2x-\frac{B}{(x-3)^2}$ Follow through their B or the letter B or a made up B.	M1A1ft
	Special Case: If they write g(x) as $x^2+5+\frac{3x+12}{(x-3)}$ and correctly attempt to differentiate as $2x$ + the quotient rule on $\frac{3x+12}{(x-3)}$ then the M mark is available but **not** the A1ft. It must be the correct quotient rule and the numerator must be a linear expression.		
	$g'(4)=2\times4-\frac{3}{(4-3)^2}(=5)$	Substitutes $x=4$ into their derivative	M1
	Uses $m=g'(4)=(5)$ with $(4,g(4))=(4,24)$ to form eqn of tangent		
	$y-24=5(x-4)$	Correct method of finding an equation of the tangent. The gradient must be $g'(4)$ and the point must be an attempt on $(4,g(4))$	M1
	$y=5x+4$	Cso. This mark may be withheld for an incorrect "A" earlier or any incorrect work leading to a correct gradient.	A1
			(5)
			(9 marks)
	Alternative to part (b) for first 3 marks		
	$g'(x)=\frac{(x^2+x-12)(4x^3+3x^2-14x+8)-(x^4+x^3-7x^2+8x-48)(2x+1)}{(x^2+x-12)^2}$ M1: Correct use of the quotient rule – there must be evidence of the application of $\frac{vu'-uv'}{v^2}$ or this formula quoted and attempted. A1: Correct derivative		M1A1
	$g'(4)=\frac{8\times256-192\times9}{8^2}(=5)$	Substitutes $x=4$ into their derivative	M1

Qu	Scheme	Marks
6.(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^2 \times \frac{3}{3x} + \ln(3x) \times 10x$	M1 A1
		(2)
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin x + \cos x)^2}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin^2 x + \cos^2 x) + (2\sin x \cos x)} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{1 + \sin 2x}$	B1, B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}$ *	A1 *
		(4)
		(6 marks)

(i)

M1: Applies the Product rule to $y = 5x^2 \ln 3x$

Expect $\frac{\mathrm{d}y}{\mathrm{d}x} = Ax + Bx\ln(3x)$ for this mark (A, B positive constant)

A1: cao- need not be simplified

(ii)

M1: Applies the Quotient rule, a form of which appears in the formula book, to $y = \frac{x}{\sin x + \cos x}$

Expect $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(\sin x + \cos x)1 - x(\pm\cos x \pm \sin x)}{(\sin x + \cos x)^2}$ for M1

Condone invisible brackets for the M and an attempted incorrect 'squared' term on the denominator

Eg $\sin^2 x + \cos^2 x$

B1: Denominator should be expanded to $\sin^2 x + \cos^2 x + ...$ and $(\sin^2 x + \cos^2 x) \to 1$

B1: Denominator should be expanded to $... + k\sin x \cos x$ and $(k\sin x \cos x) \to \frac{k}{2}\sin 2x \cdot$

For example sight of $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x = 1 + \sin 2x$ without the intermediate line on the Denominator is B0 B1

A1: cso – answer is given. This mark is withheld if there is poor notation $\cos x \leftrightarrow \cos$ $\sin^2 x \leftrightarrow \sin x^2$

If the only error is the omission of $(\sin^2 x + \cos^2 x) \to 1$ then this final A1* can be awarded.

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Use of product rule or implicit differentiation needs to be applied correctly with possible sign errors differentiating functions for M1, then other marks as before. If quoted the product rule must be correct

Product rule $\frac{\mathrm{d}y}{\mathrm{d}x} = (\sin x + \cos x)^{-1} \times 1 \pm x \times (\sin x + \cos x)^{-2}(\pm\cos x \pm \sin x)$

Implicit differentiation $(\sin x + \cos x)y = x \Rightarrow (\sin x + \cos x)\frac{\mathrm{d}y}{\mathrm{d}x} + y(\pm\cos x \pm \sin x) = 1$

To score the B's under this method there must have been an attempt to write $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a single fraction

Question	Scheme	Marks	AOs
9(a)	Either $3y^2 \to Ay\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2xy \to 2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$	M1	2.1
	$2x-2x\frac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$	A1	1.1b
	$(6y-2x)\frac{\mathrm{d}y}{\mathrm{d}x}=2y-2x$	M1	2.1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2y-2x}{6y-2x}=\frac{y-x}{3y-x}$ *	A1*	1.1b
		(4)	
(b)	$\left(\text{At } P \text{ and } Q\ \frac{\mathrm{d}y}{\mathrm{d}x}\to\infty\Rightarrow\right)$ Deduces that $3y-x=0$	M1	2.2a
	Solves $y=\frac{1}{3}x$ and $x^2-2xy+3y^2=50$ simultaneously	M1	3.1a
	$\Rightarrow x=(\pm)5\sqrt{3}$ OR $\Rightarrow y=(\pm)\frac{5}{3}\sqrt{3}$	A1	1.1b
	Using $y=\frac{1}{3}x \Rightarrow x=..$ AND $y=..$	dM1	1.1b
	$P=\left(-5\sqrt{3},-\frac{5}{3}\sqrt{3}\right)$	A1	2.2a
		(5)	
(c)	Explains that you need to solve $y=x$ and $x^2-2xy+3y^2=50$ simultaneously and choose the positive solution	B1ft	2.4
		(1)	
			(10 marks)

Notes:

(a)

M1: For selecting the appropriate method of differentiating either $3y^2 \to Ay\frac{\mathrm{d}y}{\mathrm{d}x}$ or $2xy \to 2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$

It may be quite difficult awarding it for the product rule but condone $-2xy \to -2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$ unless you see evidence that they have used the incorrect law vu'-uv'

A1: Fully correct derivative $2x-2x\frac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$

Allow attempts where candidates write $2x\mathrm{d}x-2x\mathrm{d}y-2y\mathrm{d}x+6y\mathrm{d}y=0$

but watch for students who write $\frac{\mathrm{d}y}{\mathrm{d}x}=2x-2x\frac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\frac{\mathrm{d}y}{\mathrm{d}x}$ This, on its own, is A0 unless you are convinced that this is just their notation. Eg $\frac{\mathrm{d}y}{\mathrm{d}x}=2x-2x\frac{\mathrm{d}y}{\mathrm{d}x}-2y+6y\frac{\mathrm{d}y}{\mathrm{d}x}=0$

Question	Scheme	Marks	AOs
14 (a)	Attempts to differentiate $x=4\sin 2y$ and inverts $\frac{\mathrm{d}x}{\mathrm{d}y}=8\cos 2y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8}$	A1	1.1b
		(2)	
(b)	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b
	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of y and, using both $\sin^2 2y+\cos^2 2y=1$ and $x=4\sin 2y$ in an attempt to write $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ as a function of x Allow for $\frac{\mathrm{d}y}{\mathrm{d}x}=k\frac{1}{\cos 2y}=..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}=8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
		(7 marks)	

(a)

M1: Attempts to differentiate $x=4\sin 2y$ and inverts.

Allow for $\frac{\mathrm{d}x}{\mathrm{d}y}=k\cos 2y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{k\cos 2y}$ or $1=k\cos 2y\frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates $x=4\sin 2y \rightarrow y=\ldots\arcsin\left(\frac{x}{4}\right) \rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x=8\sin y\cos y \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\pm 8\sin^2 y \pm 8\cos^2 y$ before inverting

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{\mathrm{d}x}{\mathrm{d}y}=8\cos 2x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\cos 2x}$ This is M0 A0

Question	Scheme	Marks	AOs
15 (a)	$x^2 \tan y = 9 \Rightarrow 2x\tan y + x^2 \sec^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1	3.1a 1.1b
	Full method to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + \mathrm{f}(x)$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2x \times \frac{9}{x^2}}{x^2\left(1 + \frac{81}{x^4}\right)} = \frac{-18x}{x^4 + 81}$ *	A1*	2.1
		(4)	
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-18x}{x^4 + 81}$ $\Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{-18 \times \left(x^4 + 81\right) - \left(-18x\right)\left(4x^3\right)}{\left(x^4 + 81\right)^2} = \frac{54\left(x^4 - 27\right)}{\left(x^4 + 81\right)^2}$ o.e.	M1 A1	1.1b 1.1b
	States that when $x < \sqrt[4]{27} \Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} < 0$ when $x = \sqrt[4]{27} \Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0$ **AND** when $x > \sqrt[4]{27} \Rightarrow \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} > 0$ giving a point of inflection when $x = \sqrt[4]{27}$	A1	2.4
		(3)	
			(7 marks)
Notes:			

(a)

M1: Attempts to differentiate $\tan y$ implicitly. Eg. $\tan y \to \sec^2 y \frac{\mathrm{d}y}{\mathrm{d}x}$ or $\cot y \to -\mathrm{cosec}^2 y \frac{\mathrm{d}y}{\mathrm{d}x}$

You may well see an attempt $\tan y = \frac{9}{x^2} \Rightarrow \sec^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = ...$

When a candidate writes $x^2 \tan y = 9 \Rightarrow x = 3\tan^{-\frac{1}{2}} y$ the mark is scored for $\tan^{-\frac{1}{2}} y \to ...\tan^{-\frac{3}{2}} y \sec^2 y$

A1: Correct differentiation $2x\tan y + x^2 \sec^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Allow also $\sec^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{18}{x^3}$ or $2x = -9\mathrm{cosec}^2 y \frac{\mathrm{d}y}{\mathrm{d}x}$ amongst others

M1: Full method to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x using $\sec^2 y = 1 + \tan^2 y = 1 + \mathrm{f}(x)$

A1*: Proceeds correctly to the given answer of $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-18x}{x^4 + 81}$

Question Number	Scheme	Marks
5 (a)	Sets $y=2^x$ and takes ln of both sides to get $\ln y = x\ln 2$	M1
	Differentiates wrt x to get $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=\ln 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=..$	dM1
	Rearranges to achieve $\frac{\mathrm{d}y}{\mathrm{d}x}=2^x\ln 2$ cao	A1*
		(3)
(b)	Differentiates wrt x $\quad \underline{\underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}}}+\underline{6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}=4\times 2^x\ln 2$ oe	$\underline{\text{M1}}$, $\underline{\underline{\text{B1}}}$, A1
	Substitutes (2, 0) **AND** rearranges to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow 2+12\frac{\mathrm{d}y}{\mathrm{d}x}=16\ln 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{16\ln 2-2}{12} \quad (=0.758)$	M1
	Find equation of tangent using (2, 0) and their numerical $\frac{\mathrm{d}y}{\mathrm{d}x}$	dM1
	$y=\frac{(16\ln 2-2)(x-2)}{12}$ oe Accept $y=0.76x-1.52$	A1
		(6)
		(9 marks)
Alt 1 **5 (a)**	Writes $2^x = \mathrm{e}^{x\ln 2}$	M1
	Differentiates wrt x to get $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x\ln 2}\right)=\mathrm{e}^{x\ln 2}\ln 2 = 2^x\ln 2$ cao	dM1 A1*
		(3)
Alt 2 **5 (a)**	Sets $y=2^x$ and takes ln₂ of both sides to get $\ln_2 y = x \Rightarrow \frac{\ln y}{\ln 2}=x \Rightarrow \ln y = x\ln 2$	M1
		(3)

Question Number	Scheme	Marks
2.	$\underline{3x^2} - \underline{\underline{\left(3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}\right)}} \underline{- 1 + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}} = \underline{0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 - 3y - 1}{3x - 3y^2}\right\}$ *not necessarily required.*	
	At $(2, -1)$, $\mathrm{m}(\mathbf{T}) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3(2)^2 - 3(-1) - 1}{3(2) - 3(-1)^2}$ $\left\{= \frac{14}{3}\right\}$	M1
	T: $y - -1 = \frac{14}{3}(x - 2)$	dM1
	T: $14x - 3y - 31 = 0$ or equivalent	A1
		[6] **6**

Notes

1st M1: Differentiates implicitly to include either $\pm ky^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **or** $\pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$.

(Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right)$ at start and omission of = 0 at end.)

1st A1: $x^3 \to \underline{3x^2}$ **and** $-x + y^3 - 11 \to \underline{-1 + 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}}$ (so the -11 should have gone) **and** = 0 needed **here or implied by further work**. Ignore $\left(\frac{\mathrm{d}y}{\mathrm{d}x} =\right)$ at start.

2nd M1: An attempt to apply the product rule: $-3xy \to -\left(3y + 3x\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ or $\pm 3y \pm 3x\frac{\mathrm{d}y}{\mathrm{d}x}$ o.e.

3rd M1: Correct method to collect **two (not three)** dy/dx terms and to evaluate the gradient at $x = 2$ $y = -1$ (This stage may imply the earlier "=0")

4th dM1: This is dependent on all previous method marks

Uses line equation with their $\frac{14}{3}$. May use $y = \frac{14}{3}x + c$ and attempt to evaluate c by substituting $x = 2$ and $y = -1$. (May be implied by correct answer)

2nd A1: Any positive or negative whole number multiple of $14x - 3y - 31 = 0$ is acceptable. Must have = 0.

N.B. If anyone attempts the question using $\frac{\mathrm{d}x}{\mathrm{d}y}$ instead of $\frac{\mathrm{d}y}{\mathrm{d}x}$, please send to review

Question Number	Scheme	Marks
1. (a)	$\xcancel{\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)} = 8x\underline{-2y\frac{\mathrm{d}y}{\mathrm{d}x}}+\underline{\underline{2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y}}=0$	$\underline{\text{M1}}\underline{\underline{\text{B1}}}$A1
	Either Way 1: Sets $\frac{\mathrm{d}y}{\mathrm{d}x}=2$ in each term in their differentiated expression $\Rightarrow 8x-4y+4x+2y=0, \Rightarrow y-6x=0$ *	dM1 ddM1,A1*
	Or Way 2: Obtains $\frac{\mathrm{d}y}{\mathrm{d}x}=\left(\frac{8x+2y}{2y-2x}\right)$ (ft their differentiated expression) $\frac{8x+2y}{2y-2x}=2$, so $y-6x=0$*	dM1 ddM1,A1*
		(6)
(b)	Put $y=6x$ or $x=\frac{y}{6}$ into $4x^2-y^2+2xy+5=0$ and obtains $Ay^2=B$ or $Ax^2=B$ where A and B are constants $x=\pm\frac{1}{2}$ **or** $y=\pm3$ **or** $\left(\frac{1}{2},3\right)$ **or** $\left(-\frac{1}{2},-3\right)$ both $\left(\frac{1}{2},3\right)$ and $\left(-\frac{1}{2},-3\right)$ and no extra solutions	M1 A1 A1 **(3)** **(9 marks)**

Question Number	Scheme	Marks
3	Differentiates wrt x $\quad 3^x \ln 3 + \underline{\underline{6\frac{\mathrm{d}y}{\mathrm{d}x}}} = \underline{\tfrac{3}{2}y^2 + 3xy\frac{\mathrm{d}y}{\mathrm{d}x}}$	B1 $\underline{\underline{\text{B1}}}$, $\underline{\text{M1, A1}}$
	Substitutes (2, 3) **AND** rearranges to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ $\Rightarrow 9\ln 3 + 6\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2} + 18\frac{\mathrm{d}y}{\mathrm{d}x} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9\ln 3 - \frac{27}{2}}{12} = \frac{6\ln 3 - 9}{8}, = \frac{-9 + \ln 729}{8}$	M1 A1, A1
		(7) **(7 marks)**

B1 Differentiates $3^x \to 3^x \ln 3$ or $\mathrm{e}^{x\ln 3} \to \mathrm{e}^{x\ln 3}\ln 3$

B1 Differentiates $6y \to 6\frac{\mathrm{d}y}{\mathrm{d}x}$

M1 Uses the product rule to differentiate $\frac{3}{2}xy^2$. Evidence could be sight of $\underline{\tfrac{3}{2}y^2 + kxy\frac{\mathrm{d}y}{\mathrm{d}x}}$

If the rule is quoted it must be correct. It could be implied by u=.., u'=.., v =.., v'=.. followed by their vu'+uv'. For this M to be scored y^2 must differentiate to $ky\frac{\mathrm{d}y}{\mathrm{d}x}$, it cannot differentiate to $2y$.

A1 A completely correct differential of $\frac{3}{2}xy^2$. It need not be simplified.

M1 Substitutes $x = 2,\ y = 3$ into their expression containing a derivative to find a 'numerical' value for $\frac{\mathrm{d}y}{\mathrm{d}x}$

The candidate may well have attempted to change the subject. Do not penalise accuracy errors on this method mark

A1 Any correct numerical answer in the form $\frac{p\ln q - r}{s}$ where p, q, r and s are constants e.g. $\frac{9\ln 3 - \frac{27}{2}}{12}$

A1 Exact answer. Accept either $\frac{-9 + \ln 729}{8}$ or $\frac{\ln 729 - 9}{8}$

……

Note: There may be candidates who multiply by 2 first and start with $2\times 3^x + 12y = 3xy^2$

This is perfectly acceptable and the mark scheme can be applied in a similar way.

……

Question Number	Scheme	Notes	Marks
2	$\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x} = 4x\cos x + 4\sin x$	Applies product rule to $4x\sin x$ to give $\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x} = \pm 4x\cos x + 4\sin x$	M1
	$\frac{\mathrm{d}\left(\pi y^2\right)}{\mathrm{d}y} = 2\pi y\frac{\mathrm{d}y}{\mathrm{d}x}$	Applies chain rule to πy^2 to give $\frac{\mathrm{d}\left(\pi y^2\right)}{\mathrm{d}y} = Ay\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$4x\sin x = \pi y^2 + 2x \Rightarrow 4x\cos x + 4\sin x = 2\pi y\frac{\mathrm{d}y}{\mathrm{d}x} + 2$ Fully correct differentiation. oe Accept $4x\cos x\mathrm{d}x + 4\sin x\mathrm{d}x = 2\pi y\mathrm{d}y + 2\mathrm{d}x$		A1
	For the differentiation ignore any spurious $"\frac{\mathrm{d}y}{\mathrm{d}x} = "$		
	Alternative for first 3 marks using explicit differentiation: $y = \left(\frac{1}{\sqrt{\pi}}\right)(4x\sin x - 2x)^{\frac{1}{2}}$		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{1}{2\sqrt{\pi}}\right)(4x\sin x - 2x)^{-\frac{1}{2}}(4x\cos x + 4\sin x - 2)$ M1: $\frac{\mathrm{d}(4x\sin x)}{\mathrm{d}x} = \pm 4x\cos x + 4\sin x$ (as before) M1: $(4x\sin x - 2x)^{\frac{1}{2}} \to k(4x\sin x - 2x)^{-\frac{1}{2}}$		M1 M1
	Allow omission of π and sign errors when rearranging for the M marks		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2\sqrt{\pi}}(4x\sin x - 2x)^{-\frac{1}{2}}(4x\cos x + 4\sin x - 2)$ oe		A1
	$x = \frac{\pi}{2}, y = 1$ $\Rightarrow 4 = 2\pi\frac{\mathrm{d}y}{\mathrm{d}x} + 2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \ldots\left(\frac{1}{\pi}\right)$	Uses $x = \frac{\pi}{2}$ and $y = 1$ to obtain a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (may be implied). For implicit differentiation, there must be a dy/dx and there must be x's and y's. Explicit differentiation just requires use of $x = \frac{\pi}{2}$.	M1
	$y - 1 = "-\pi"\left(x - \frac{\pi}{2}\right)$ or $y = "-\pi"x + c \Rightarrow c = 1 + \frac{\pi^2}{2}$ Uses normal gradient $-1\Big/\frac{\mathrm{d}y}{\mathrm{d}x}$ and $x = \frac{\pi}{2}, y = 1$ to find equation of normal. Must use $-1\Big/\left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}\right)$ and $x = \frac{\pi}{2}$ and $y = 1$ must be correctly placed. If using $y = mx + c$ must reach as far as $c = \ldots$		M1
	$y - 1 = -\pi\left(x - \frac{\pi}{2}\right)$ oe	Allow 3sf or more decimal equivalent answers e.g. $y = -3.14x + 5.93$, $y - 1 = -3.14(x - 1.57)$ etc.	A1cso
			(6 marks)

Qu	Scheme	Marks
1	Differentiate wrt x $\quad \underline{3x^2} + \underline{\underline{6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}} + \underline{3y^2\frac{\mathrm{d}y}{\mathrm{d}x}} = \underline{0}$	M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	Substitutes (1, 3) **AND** rearranges to get $\frac{\mathrm{d}y}{\mathrm{d}x}\left(=-\frac{7}{10}\right)$	M1
	$(y-3) = -\frac{7}{10}(x-1)$ $\quad$ so $7x + 10y - 37 = 0$	M1A1 **(6)** **(6 marks)**

M1 : Differentiates implicitly to include either $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **or** $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **term**

Accept $3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ appearing as $3x^2y'$ or $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as $3y^2y'$

A1: Differentiates $y^3 \to 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ **and** $x^3 \to 3x^2$ **and** $37 \to 0$

B1: Uses the product rule to differentiate $3x^2y$ giving $\underline{\underline{6xy + 3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}}$

Do not penalise students who write $3x^2\mathrm{d}x + 6xy\mathrm{d}x + 3x^2\mathrm{d}y + 3y^2\mathrm{d}y = 0$

M1: Substitutes $x = 1,\ y = 3$ into their expression (correctly each at least once) to find a 'numerical' value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ (may be incorrect). Note that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-3x^2 - 6xy}{3x^2 + 3y^2}$

M1: Use of $(y-3) = m(x-1)$ where m is their numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}$

Alternatively uses $y = mx + c$ with $(1,3)$ and their m as far as c =..

A1: Accept integer multiples of the answer i.e. $7kx + 10ky - 37k = 0$ for example $21x + 30y - 111 = 0$

Note: If the gradient $-\frac{7}{10}$ just appears (from a graphical calculator) only M3 may be awarded

Question Number	Scheme	Marks
1	$3x^2+2xy-2y^2+4=0 \Rightarrow 6x+\overline{\overline{2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y}}\underline{-4y\frac{\mathrm{d}y}{\mathrm{d}x}}=0$	$\overline{\overline{\text{B1}}}$ $\underline{\text{M1}}$ A1
	Sets $x=2, y=4 \Rightarrow 12+4\frac{\mathrm{d}y}{\mathrm{d}x}+8-16\frac{\mathrm{d}y}{\mathrm{d}x}=0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3}$	M1
	Uses $x=2, y=4$ and their $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5}{3} \Rightarrow (y-4)=\frac{5}{3}(x-2)$	M1
	$5x-3y+2=0$	A1
		(6 marks)

B1: $2xy$ differentiated correctly to give $2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y$ or any equivalent correct expression.

M1: Attempts to apply the chain rule to $-2y^2$ to give an expression of the form $Ay\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: Fully correct differentiation of $3x^2-2y^2+4$ to give $6x-4y\frac{\mathrm{d}y}{\mathrm{d}x}$ and "= 0" which may be implied by subsequent work. "= 0" may also be implied if the candidate rearranges the given equation first.

Allow the candidate to start with $\frac{\mathrm{d}y}{\mathrm{d}x}=\ldots$ for all the above marks but if **this** $\frac{\mathrm{d}y}{\mathrm{d}x}$ is used to find the gradient, the next mark would be withheld as the two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms must come from the $2xy$ and $2y^2$ terms – see below.

Note: If $6x\mathrm{d}x+2x\mathrm{d}y+2y\mathrm{d}x-4y\mathrm{d}y=0 \Rightarrow 6x+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$ is seen, score B1 for $2x\mathrm{d}y+2y\mathrm{d}x$ and M1 for $6x\mathrm{d}x-4y\mathrm{d}y=0$ then A1 for $6x+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2y-4y\frac{\mathrm{d}y}{\mathrm{d}x}=0$

M1: Substitutes $x=2$ and $y=4$ and attempts to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ (this may be implied e.g. they may rearrange their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find $-\frac{\mathrm{d}x}{\mathrm{d}y}$ and then substitute). This is not formally dependent on the first M but is dependent upon them having two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms in their derivative. One coming from $2xy$ and one coming from $2y^2$.

M1: Uses $x=2$ and $y=4$ and their numerical value of $\frac{\mathrm{d}y}{\mathrm{d}x}\left(=\frac{20}{12}=\frac{5}{3}\right)$ to find an equation of a <u>tangent</u> (not a normal). If $y = mx + c$ is used they much reach as far as finding a value for c.

A1: Accept $5x-3y+2=0$ or any integer multiple of this equation.

Question Number	Scheme	Marks
2.(a)	$y^3+x^2y-6x=0 \Rightarrow \underline{3y^2\frac{\mathrm{d}y}{\mathrm{d}x}} \overline{\overline{+x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy}}-6=0$	$\overline{\overline{\text{B1}}}$ $\underline{\text{M1}}$ A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6-2xy}{x^2+3y^2}$	M1A1
		(5)
(b)	$6-2xy=0 \Rightarrow y=\frac{3}{x}$	M1
	Substitute $y=\frac{3}{x}$ into $y^3+x^2y-6x=0 \Rightarrow \frac{27}{x^3}+\frac{3x^2}{x}-6x=0$	**d**M1
	$\Rightarrow x^4=9$	**dd**M1A1
	Points $\left(\sqrt{3},\sqrt{3}\right)\left(-\sqrt{3},-\sqrt{3}\right)$	A1A1
		(6)
		(11 marks)
Alt(b)	$6-2xy=0 \Rightarrow x=\frac{3}{y}$	M1
	Substitute $x=\frac{3}{y}$ into $y^3+x^2y-6x=0 \Rightarrow y^3+\frac{9}{y^2}y-6\times\frac{3}{y}=0$	**d**M1
	$\Rightarrow y^4=9$	**dd**M1A1
	Points $\left(\sqrt{3},\sqrt{3}\right)\left(-\sqrt{3},-\sqrt{3}\right)$	A1A1
		(6)

(a)

B1: Applies the product rule to x^2y to obtain $x^2\frac{\mathrm{d}y}{\mathrm{d}x}+2xy$

M1: Applies the chain rule to y^3 to obtain $Ay^2\frac{\mathrm{d}y}{\mathrm{d}x}$

A1: $y^3-6x=0 \Rightarrow 3y^2\frac{\mathrm{d}y}{\mathrm{d}x}-6=0$. i.e. y^3 differentiated correctly **and** $-6x \to -6$ **and** "= 0" seen or implied.

M1: Attempts to make $\frac{\mathrm{d}y}{\mathrm{d}x}$ the subject. This is dependent upon them having two $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms in their derivative. One coming from their differentiation of x^2y and the other from their differentiation of y^3

A1: Accept $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6-2xy}{x^2+3y^2}$ or equivalent.

Ignore a spurious "$\frac{\mathrm{d}y}{\mathrm{d}x}=$" at the start but see the note above regarding where the $\frac{\mathrm{d}y}{\mathrm{d}x}$'s must come from for the second method mark.

<u>If the candidate differentiates with respect to y, the same scheme can be applied:</u>

B1: $x^2y \to x^2+2xy\frac{\mathrm{d}x}{\mathrm{d}y}$. M1: $-6x \to A\frac{\mathrm{d}x}{\mathrm{d}y}$ A1: $y^3-6x=0 \Rightarrow 3y^2-6\frac{\mathrm{d}x}{\mathrm{d}y}=0$

M1: Attempts to make $\frac{\mathrm{d}x}{\mathrm{d}y}$ the subject. This is dependent upon them having two $\frac{\mathrm{d}x}{\mathrm{d}y}$ terms in their derivative. One coming from their differentiation of x^2y and the other from their differentiation of $-6x$

Question Number	Scheme	Notes	Marks
4.	$4x^2 - y^3 - 4xy + 2^y = 0$		
(a) **Way 1**	$\left\{\cancel{\frac{dy}{dx}} \cancel{=}\right\} \underline{8x - 3y^2\frac{dy}{dx}} \underline{\underline{- 4y - 4x\frac{dy}{dx}}} + \overline{\overline{2^y \ln 2}}\frac{dy}{dx} \underline{= 0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$ $\overline{\overline{\text{B1}}}$
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4\ln 2\frac{dy}{dx} = 0$	**dependent on the first M mark**	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$		
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent		A1 **cso**
	NOTE: You can recover work for part (a) in part (b)		**[6]**
(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	Applying $m_N = \frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ • $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	Using a numerical m_N (¹ m_T), either $y - 4 = m_N(x - -2)$ **and** sets $x = 0$ in their normal equation **or** $4 = (\text{their } m_N)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16}, \text{ so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$		
	y (or c) $= \frac{13}{2} - \ln 2$	$\frac{104}{16} - \ln 2$ **or** $\frac{13}{2} - \ln 2$ **or** $-\ln 2 + \frac{13}{2}$	A1 **cso isw**
	Note: Allow exact equivalents in the form $p - \ln 2$ for the final A mark		**[3]**
			9
(a) **Way 2**	$\left\{\cancel{\frac{dx}{dy}} \cancel{=}\right\} \underline{8x\frac{dx}{dy} - 3y^2} \underline{\underline{- 4y\frac{dx}{dy} - 4x}} + \overline{\overline{2^y \ln 2}} \underline{= 0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{M1}}}$ $\overline{\overline{\text{B1}}}$
	$8(-2)\frac{dx}{dy} - 3(4)^2 - 4(4)\frac{dx}{dy} - 4(-2) + 2^4\ln 2 = 0$	**dependent on the first M mark**	dM1
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent		A1 **cso**
	Note: You must be clear that Way 2 is being applied before you use this scheme		**[6]**

		Question 4 Notes
4. (a)	**Note**	**For the first four marks**
		Writing down ***from no working*** • $\frac{dy}{dx} = \frac{4y-8x}{-3y^2-4x+2^y\ln 2}$ or $\frac{8x-4y}{3y^2+4x-2^y\ln 2}$ scores M1A1M1B1 • $\frac{dy}{dx} = \frac{8x-4y}{-3y^2-4x+2^y\ln 2}$ or $\frac{4y-8x}{3y^2+4x-2^y\ln 2}$ scores M1A0M1B1
		Writing $8x\,dx - 3y^2\,dy - 4y\,dx - 4x\,dy + 2^y\ln 2\,dy = 0$ scores M1A1M1B1

Question Number	Scheme	Notes	Marks
3.	$2x^2y+2x+4y-\cos(\pi y)=17$		
(a) **Way 1**	$\left\{\cancel{\frac{dx}{dx}}\times\right\}\ \underline{\underline{\left(4xy+2x^2\frac{dy}{dx}\right)}}\ \underline{+2+4\frac{dy}{dx}+\pi\sin(\pi y)\frac{dy}{dx}=0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$\frac{dy}{dx}\left(2x^2+4+\pi\sin(\pi y)\right)+4xy+2=0$		dM1
	$\left\{\frac{dy}{dx}=\right\}\frac{-4xy-2}{2x^2+4+\pi\sin(\pi y)}$ or $\frac{4xy+2}{-2x^2-4-\pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
(b)	At $\left(3,\frac{1}{2}\right)$, $m_T=\frac{dy}{dx}=\frac{-4(3)(\frac{1}{2})-2}{2(3)^2+4+\pi\sin\left(\frac{1}{2}\pi\right)}$ $\left\{=\frac{-8}{22+\pi}\right\}$	Substituting $x=3$ & $y=\frac{1}{2}$ into an equation involving $\frac{dy}{dx}$	M1
	$m_N=\frac{22+\pi}{8}$	Applying $m_N=\frac{-1}{m_T}$ to find a numerical m_N **Can be implied by later working**	M1
	• $y-\frac{1}{2}=\left(\frac{22+\pi}{8}\right)(x-3)$ • $\frac{1}{2}=\left(\frac{22+\pi}{8}\right)(3)+c\Rightarrow c=\frac{1}{2}-\frac{66+3\pi}{8}$ $\Rightarrow y=\left(\frac{22+\pi}{8}\right)x+\frac{1}{2}-\frac{66+3\pi}{8}$ Cuts x-axis $\Rightarrow y=0$ $\Rightarrow -\frac{1}{2}=\left(\frac{22+\pi}{8}\right)(x-3)$	$y-\frac{1}{2}=m_N(x-3)$ or $y=m_Nx+c$ where $\frac{1}{2}=(\text{their } m_N)3+c$ with a numerical m_N $(\neq m_T)$ where m_N is in terms of π and sets $y=0$ in their normal equation.	dM1
	So, $\left\{x=\frac{-4}{22+\pi}+3\Rightarrow\right\}$ $x=\frac{3\pi+62}{\pi+22}$	$\frac{3\pi+62}{\pi+22}$ or $\frac{6\pi+124}{2\pi+44}$ or $\frac{62+3\pi}{22+\pi}$	A1 o.e.
			[4]
			9
(a) **Way 2**	$\left\{\cancel{\frac{dx}{dy}}\times\right\}\ \underline{\underline{\left(4xy\frac{dx}{dy}+2x^2\right)}}\ \underline{+2\frac{dx}{dy}+4+\pi\sin(\pi y)=0}$		M1 $\underline{\text{A1}}$ $\underline{\underline{\text{B1}}}$
	$\frac{dx}{dy}(4xy+2)+2x^2+4+\pi\sin(\pi y)=0$		dM1
	$\frac{dy}{dx}=\frac{-4xy-2}{2x^2+4+\pi\sin(\pi y)}$ or $\frac{4xy+2}{-2x^2-4-\pi\sin(\pi y)}$	Correct answer or equivalent	A1 **cso**
			[5]
	Question 3 Notes		
3. (a)	**Note** Writing down ***from no working*** • $\frac{dy}{dx}=\frac{-4xy-2}{2x^2+4+\pi\sin(\pi y)}$ or $\frac{4xy+2}{-2x^2-4-\pi\sin(\pi y)}$ scores M1A1B1M1A1 • $\frac{dy}{dx}=\frac{4xy+2}{2x^2+4+\pi\sin(\pi y)}$ scores M1A0B1M1A0		
	Note Few candidates will write $4xy\,dx+2x^2dy+2\,dx+4dy+\pi\sin(\pi y)dy=0$ leading to $\frac{dy}{dx}=\frac{-4xy-2}{2x^2+4+\pi\sin(\pi y)}$ or equivalent. This should get full marks.		

Question Number	Scheme		Marks
2.	$3^{x-1}+xy-y^2+5=0$		
		$3^{x-1}\to 3^{x-1}\ln 3$	B1 oe
	$\left\{\cancel{\frac{dy}{dx}}\cancel{\times}\right\}$ (ignore) $3^{x-1}\ln 3+\left(y+x\frac{dy}{dx}\right)-2y\frac{dy}{dx}=0$	Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$.	M1*
		$xy\to +y+x\frac{dy}{dx}$	B1
		$...+y+x\frac{dy}{dx}-2y\frac{dy}{dx}=0$	A1
	$\{(1,3)\Rightarrow\}\ 3^{(1-1)}\ln 3+3+(1)\frac{dy}{dx}-2(3)\frac{dy}{dx}=0$	Substitutes $x=1,\ y=3$ into their differentiated equation or expression.	dM1*
	$\ln 3+3+\frac{dy}{dx}-6\frac{dy}{dx}=0\ \Rightarrow\ 3+\ln 3=5\frac{dy}{dx}$		
	$\frac{dy}{dx}=\frac{3+\ln 3}{5}$		dM1*
	$\frac{dy}{dx}=\frac{1}{5}\left(\ln e^3+\ln 3\right)=\frac{1}{5}\ln\left(3e^3\right)$	**Uses** $3=\ln e^3$ to achieve $\frac{dy}{dx}=\frac{1}{5}\ln\left(3e^3\right)$	A1 **cso**
			[7] **7**

Notes for Question 2

B1: Correct differentiation of 3^{x-1}. I.e. $3^{x-1}\to 3^{x-1}\ln 3$ or $3^{x-1}=\frac{1}{3}\left(3^x\right)\to\frac{1}{3}\left(3^x\right)\ln 3$

or $3^{x-1}=e^{(x-1)\ln 3}\to \ln 3\,e^{(x-1)\ln 3}$ or $3^{x-1}=\frac{1}{3}\left(3^x\right)=\frac{1}{3}e^{x\ln 3}\to\frac{1}{3}(\ln 3)e^{x\ln 3}$

M1: Differentiates implicitly to include either $\pm\lambda x\frac{dy}{dx}$ or $\pm ky\frac{dy}{dx}$. (Ignore $\left(\frac{dy}{dx}=\right)$).

B1: $xy\to +y+x\frac{dy}{dx}$

1st A1: $...+y+x\frac{dy}{dx}-2y\frac{dy}{dx}=0$ **Note:** The 1st A0 follows from an award of the 2nd B0.

Note: The $"=0"$ can be implied by rearrangement of their equation.

ie: $3^{x-1}\ln 3+y+x\frac{dy}{dx}-2y\frac{dy}{dx}$ leading to $3^{x-1}\ln 3+y=2y\frac{dy}{dx}-x\frac{dy}{dx}$ will get A1 (implied).

2nd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.
Substitutes $x=1,\ y=3$ into their differentiated equation or expression. Allow one slip.

3rd M1: **Note:** This method mark is dependent upon the 1st M1* mark being awarded.

Candidate has two differentiated terms in $\frac{dy}{dx}$ and rearranges to make $\frac{dy}{dx}$ the subject.

Note: It is possible to gain the 3rd M1 mark before the 2nd M1 mark.

Eg: Candidate may write $\frac{dy}{dx}=\frac{y+3^{x-1}\ln 3}{2y-x}$ before substituting in $x=1$ and $y=3$

2nd A1: **cso.** Uses $3=\ln e^3$ to achieve $\frac{dy}{dx}=\frac{1}{5}\ln\left(3e^3\right)$, $\left(=\frac{1}{\lambda}\ln\left(\mu e^3\right),\ \lambda=5\text{ and }\mu=3\right)$

Note: $3=\ln e^3$ needs to be seen in their proof.

Notes for Question 2 Continued

Question	Scheme	Marks	AOs
13 (a)	States or uses $6=\pi r^2 h+\frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow\ h=\frac{6}{\pi r^2}-\frac{2}{3}r,\ \ \pi h=\frac{6}{r^2}-\frac{2}{3}\pi r,\ \ \pi rh=\frac{6}{r}-\frac{2}{3}\pi r^2,\ \ rh=\frac{6}{\pi r}-\frac{2}{3}r^2$		
	$A=\pi r^2+2\pi rh+2\pi r^2\ \{\Rightarrow A=3\pi r^2+2\pi rh\}$		
	$A=2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$	M1 A1	3.1a 1.1b
	$A=3\pi r^2+\frac{12}{r}-\frac{4}{3}\pi r^2\ \Rightarrow\ A=\frac{12}{r}+\frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A=12r^{-1}+\frac{5}{3}\pi r^2\Rightarrow\right\}\ \frac{\mathrm{d}A}{\mathrm{d}r}=-12r^{-2}+\frac{10}{3}\pi r$	M1 A1	3.4 1.1b
	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r}=0\Rightarrow\right\}\ -\frac{12}{r^2}+\frac{10}{3}\pi r=0\ \Rightarrow\ -36+10\pi r^3=0\Rightarrow r^{\pm 3}=...\left\{=\frac{18}{5\pi}\right\}$	M1	2.1
	$r=1.046447736...\Rightarrow r=$ 1.05 (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for *r*	**(4)**	
(c)	$A_{\min}=\frac{12}{(1.046...)}+\frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{\min}=17.20...\ \Rightarrow\}\ A=17\ (\mathrm{m}^2)$ or $A=\text{awrt } 17\ (\mathrm{m}^2)$	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 13

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h=...$ or $\pi h=...$ or $\pi rh=...$ or $rh=...$, where $'...'=\mathrm{f}(r)$ into an expression for the surface area which is of the form $A=\lambda\pi r^2+\mu\pi rh;\ \lambda,\ \mu\neq 0$
A1:	Obtains correct simplified or un-simplified $\{A=\}\ 2\pi r^2+2\pi r\left(\frac{6}{\pi r^2}-\frac{2}{3}r\right)+\pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A=\frac{12}{r}+\frac{5}{3}\pi r^2$
Note:	Condone the lack of $A=...$ or $S=...$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r}+\mu r^2$ to give $\alpha r^{-2}+\beta r$; $\lambda,\mu,\alpha,\beta\neq 0$
A1:	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r}=\right\}\ -12r^{-2}+\frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ and rearranges to give $r^{\pm 3}=k,\ k\neq 0$ (**Note:** *k* can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36+10\pi r^3=0\ \rightarrow r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r=$ awrt 1.05 (ignoring units) or $r=$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r=$ 1.05 (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for *r*. E.g. $r=\left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r=\left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Question Number	Scheme	Marks
14(a)	Area of triangle = $'\frac{1}{2}ab\sin C' = \frac{1}{2}\times 2x\times 2x\times\sin 60 = \sqrt{3}x^2$	M1
	$S = 2\times\sqrt{3}x^2 + 3\times 2xl = 2x^2\sqrt{3} + 6xl$	dM1A1*
		(3)
(b)	$960 = 2x^2\sqrt{3} + 6xl \Rightarrow l = \frac{960 - 2x^2\sqrt{3}}{6x}$	M1A1
	$V = x^2\sqrt{3}\, l$	B1
	Substitute $l = \frac{960 - 2x^2\sqrt{3}}{6x}$ into $V = x^2\sqrt{3}\, l$	
	$\Rightarrow V = x^2\sqrt{3}\times\left(\frac{960 - 2x^2\sqrt{3}}{6x}\right) = 160x\sqrt{3} - x^3$	dM1A1*
		(5)
(c)	$\frac{dV}{dx} = 160\sqrt{3} - 3x^2 = 0$	M1A1
	$\Rightarrow x =$ awrt 9.6	A1
	$\Rightarrow V = 160\times 9.611\times\sqrt{3} - 9.611^3 = 1776$	dM1 A1
		(5)
(d)	$\frac{d^2V}{dx^2} = -6x < 0 \Rightarrow$ Maximum	M1A1
		(2)
		(15 marks)

Question Number	Scheme	Marks
16. (a)	$\pi R^2H+\frac{2}{3}\pi R^3=800\pi$ so $H=\frac{800}{R^2}-\frac{2}{3}R$ *	M1 A1*
(b)	$A=\pi R^2+2\pi RH+2\pi R^2$	B1
	$A=3\pi R^2+2\pi R\left(\frac{800}{R^2}-\frac{2}{3}R\right)$ so $A=\frac{5\pi R^2}{3}+\frac{1600\pi}{R}$ *	M1 A1 *
		[5]
(c)	Find $\frac{\mathrm{d}A}{\mathrm{d}R}=\frac{10}{3}\pi R-\frac{1600\pi}{R^2}$	M1 A1
	Put derivative equal to zero and obtain $R^3=480$	dM1 A1
	So $R=7.83$	**A1**
		[5]
(d)	Consider $\frac{\mathrm{d}^2A}{\mathrm{d}R^2}=\frac{10\pi}{3}+3200\pi R^{-3}>0$ so minimum	M1A1
		[2]
(e)	$H=$ awrt 7.83	B1
		[1]
		13 marks

(a)

M1 Sets up volume equation with $800\pi=\pi R^2H+\frac{2}{3}\pi R^3$ and attempts to make H the subject. Condone 800 instead of 800π. Accept for this mark lower case letters $800\pi=\pi r^2H+\frac{2}{3}\pi r^3$ and a lack of consistency in lettering.

A1* This is a show that question and there must be an intermediate line showing (or implying) a division of πr^2 / πR^2. Lettering must be correct and consistent from the point where you see $800\pi=$ ..... .
Examples of an intermediate line are;

$$800\pi=\pi R^2H+\frac{2}{3}\pi R^3\Rightarrow H=\frac{800\pi-\frac{2}{3}\pi R^3}{\pi R^2}\Rightarrow H=\frac{800}{R^2}-\frac{2}{3}R$$

$$800\pi=\pi R^2H+\frac{2}{3}\pi R^3\Rightarrow\frac{800}{R^2}=H+\frac{2}{3}R\Rightarrow H=\frac{800}{R^2}-\frac{2}{3}R$$

(b)

B1 A correct expression for the surface area containing three separate correct elements
Allow either $A=\pi R^2+2\pi RH+2\pi R^2$ or $A=\pi R^2+2\pi RH+\frac{4\pi R^2}{2}$
Allow lower case lettering for this mark

M1 Score for replacing $H=\frac{800}{R^2}-\frac{2}{3}R$ in their expression for A which must be of the form,
$A=B\pi R^2+C\pi RH,\quad B,C\in\mathbb{N}$,condoning missing brackets.

A1* This is a show that question and all aspects must be correct. Lettering in (b) must be consistent and correct from the point at which $\frac{800}{R^2}-\frac{2}{3}R$ is substituted. Do not, however, withhold a second mark for using lower case letters if it has been withheld in part (a) for mixed lettering.
Accept $A=2\pi R^2+\pi R^2+2\pi R\left(\frac{800}{R^2}-\frac{2}{3}R\right)\Rightarrow A=\frac{5\pi R^2}{3}+\frac{1600\pi}{R}$ with little or no evidence

Question Number	Scheme	Marks
15(a)	Uses Volume = 60 000 $\qquad 60000=\pi r^2 h \Rightarrow h=\dfrac{60000}{\pi r^2}$	M1
	Subs in $S=\pi r^2+2\pi rh \Rightarrow S=\pi r^2+2\pi r\times\dfrac{60000}{\pi r^2}$	M1
	$\Rightarrow S=\pi r^2+\dfrac{120000}{r}$	A1*
		(3)
(b)	$\dfrac{\mathrm{d}S}{\mathrm{d}r}=2\pi r-\dfrac{120000}{r^2}$	M1A1
	$\Rightarrow \dfrac{\mathrm{d}S}{\mathrm{d}r}=0 \Rightarrow r^3=\dfrac{120000}{2\pi} \Rightarrow r = awrt\ 27\left(\text{cm}\right)$	dM1A1
	$\Rightarrow S=\pi\times"26.7"^2+\dfrac{120000}{"26.7"}=awrt\ 6730\left(\text{cm}^2\right)$	dM1 A1
		(6)
(c)	$\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2}=2\pi+\left.\dfrac{240000}{r^3}\right\vert_{r=26.7}=awrt\,19>0\Rightarrow\text{Minimum}$	M1A1
		(2) **(11 marks)**

(a)

M1 Uses $60000=\pi r^2 h \Rightarrow h=..$ Alternatively uses $60000=\pi r^2 h \Rightarrow \pi rh=..$
Condone errors on the number of zeros but the formula must be correct

M1 Score for the attempt to substitute any $h=..$ or $\pi rh=..$ from a dimensionally correct formula for V $\left(\text{Eg. } 60000=\dfrac{1}{3}\pi r^2 h \Rightarrow h=..\right)$ into $S=k\pi r^2+2\pi rh$ where $k=1$ or 2 to get S in terms of r
Allow if S is called something else such as A.

A1* Completes proof with no errors (or omissions) $S=\pi r^2+\dfrac{120000}{r}$.
Allow from $S=\pi r^2+\dfrac{2V}{r}$ if quoted. S = must be somewhere in the proof

Question	Scheme	Marks
15 (a)	$200 = \pi r^2 + \pi rh + 2\,rh$	M1 A1
	$(h =)\dfrac{200-\pi r^2}{\pi r + 2r}$ or $(rh =)\dfrac{200-\pi r^2}{\pi + 2}$	dM1
	$V = \dfrac{1}{2}\pi r^2 h =$	M1
	$\Rightarrow V = \dfrac{\pi r^2(200-\pi r^2)}{2(2r+\pi r)} = \dfrac{\pi r(200-\pi r^2)}{4+2\pi}$ *	A1 cso * **[5]**
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}r} = \dfrac{200\pi - 3\pi^2 r^2}{4+2\pi}$ Accept awrt $\dfrac{\mathrm{d}V}{\mathrm{d}r} = 61.1 - 2.9r^2$	M1 A1
	$\dfrac{200\pi - 3\pi^2 r^2}{4+2\pi} = 0$ or $200\pi - 3\pi^2 r^2 = 0$ leading to $r^2 =$	dM1
	$r = \sqrt{\dfrac{200}{3\pi}}$ or answers which round to 4.6	dM1 A1
	$V = 188$	B1 **[6]**
(c)	$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = \dfrac{-6\pi^2 r}{4+2\pi}$, and sign considered Accept $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = \text{awrt} -5.8r$	M1
	$\left.\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}\right\|_{r=..} = -27 < 0$ and therefore maximum	A1 **[2]**
		13 marks

Question Number	Scheme	Marks
9. (a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$ $\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360} \times \pi x^2$ simplified or un-simplified	M1
	$\frac{\pi x^2}{3}$	A1
		[2]
	Parts (b) and (c) may be marked together	
(b)	$\{A=\}\ \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$ Attempt to sum 3 areas (at least one correct)	M1
	Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}\,x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}\,x}{8} - \frac{\pi x}{6}$ $\Rightarrow \underline{y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)}$ * Correct proof.	A1 *
		[3]
(c)	$\{P=\}\ x + x\theta + y + 2x + y \ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$ Correct expression in x and y for their θ measured in rads	B1ft
	$\ldots 2y = \ + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$ Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$	
	$\Rightarrow \underline{P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)}$ * Correct proof.	A1 *
		[3]
	Parts (d) and (e) should be marked together	
(d)	$\frac{\mathrm{d}P}{\mathrm{d}x} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$ $\frac{1000}{x} \rightarrow \frac{\pm\lambda}{x^2}$	M1
	Correct differentiation (need not be simplified).	A1;
	Their $P' = 0$	M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ (= 16.63392808...) $\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..$ (m) awrt 120	A1
		[5]
(e)	Finds P'' **and** considers sign.	M1
	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{2000}{x^3} > 0 \Rightarrow$ Minimum $\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
		[2]
		15

Question Number	Scheme	Marks
9. (a)	Either: (Cost of polishing top and bottom (two circles) is)$3\times 2\pi r^2$ **or** (Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{\mathrm{d}C}{\mathrm{d}r}=\right\}$ $12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ **so** r^k = value where $k=\pm 2,\pm 3,\pm 4$	dM1
	Use **cube** root to obtain $r=\left(\mathit{their}\,\frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow r = 3, and thus C =	ddM1
	Then C = awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right\}$ $12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1) **[10]**

Notes

(a) **B1:** States $3\times 2\pi r^2$ or states $2\times 2\pi rh$

B1ft: Obtains a **correct** expression for h in terms of r (ft only follows misread of V)

M1: Substitutes their expression for h into **area or cost** expression of form Ar^2+Brh

A1*: Had correct expression for C and achieves **given** answer in part (a) including "C =" or "Cost=" and **no errors seen** such as C = area expression without multiples of (£)3 and (£)2 at any point. Cost and area must be perfectly distinguished at all stages for this A mark.

N.B. Candidates using Curved Surface Area = $\dfrac{2V}{r}$ - please send to review

(b) **M1:** Attempts to differentiate as evidenced by at least one term differentiated correctly

A1ft: Correct derivative – allow $12\pi r-300\pi r^{-2}$ then isw if the power is misinterpreted (ft only for misread)

dM1: Sets their $\dfrac{\mathrm{d}C}{\mathrm{d}r}$ to 0, and obtains r^k = value where k = 2, 3 or 4 (needs correct collection of powers of r from their original derivative expression – allow errors dividing by 12π)

ddM1: Uses **cube** root to find r **or** see r = awrt 3 as evidence of cube root and substitutes into correct expression for C to obtain value for C

A1: Accept awrt 483 or 484

(c) **B1ft: Finds** correct expression for $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}$ and deduces value of $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}>0$ so minimum (r may have been wrong)

OR checks gradient to left and right of 2.92 and shows gradient goes from negative to zero to positive so minimum

OR checks value of C to left and right of 2.92 and shows that $C>483$ so deduces minimum (i.e. uses shape of graph) Only ft on misread of V for each ft mark (see below)

N..B. Some candidates have **misread** the volume as 75 instead of 75π. PTO for marking instruction.

Question Number	Scheme		Marks
10. (a)	$\frac{1}{2}(9x + 6x)4x$ or $2x \times 15x$ or $\left(\frac{1}{2}4x \times (9x - 6x) + 6x \times 4x\right)$ or $6x^2 + 24x^2$ or $\left(9x \times 4x - \frac{1}{2}4x \times (9x - 6x)\right)$ or $36x^2 - 6x^2$	M1: Correct attempt at the area of a trapezium. Note that $30x^2$ on its own or $30x^2$ from incorrect work e.g. $5x \times 6x$ is M0. If there is a clear intention to find the area of the trapezium correctly allow the M1 but the A1 can be withheld if there are any slips.	M1A1**cso**
	$\Rightarrow 30x^2y = 9600 \Rightarrow y = \frac{9600}{30x^2} \Rightarrow y = \frac{320}{x^2}$ *	A1: Correct proof with at least one intermediate step and no errors seen. **“ y =” is required.**	
			[2]
(b)	$(S =)\ \frac{1}{2}(9x + 6x)4x + \frac{1}{2}(9x + 6x)4x + 6xy + 9xy + 5xy + 4xy$		M1A1
	M1: An attempt to find the area of **six** faces of the prism. The 2 trapezia may be combined as $(9x + 6x)4x$ or $60x^2$ and the 4 other faces may be combined as $24xy$ but all six faces must be included. There must be attempt at the areas of two trapezia that are dimensionally correct. A1: Correct expression in any form. Allow just $(S =)\ 60x^2 + 24xy$ for M1A1		
	$y = \frac{320}{x^2} \Rightarrow (S =)\ 30x^2 + 30x^2 + 24x\left(\frac{320}{x^2}\right)$		M1
	Substitutes $y = \frac{320}{x^2}$ into their expression for S (may be done earlier). S should have at least one x^2 term and one xy term but there may be other terms which may be dimensionally incorrect.		
	So, $(S =)\ 60x^2 + \frac{7680}{x}$ *	Correct solution only. “S = “ is **not** required here.	A1* **cso**
			[4]

10(c)	$\frac{\mathrm{d}S}{\mathrm{d}x} = 120x - 7680x^{-2}$ $\left\{= 120x - \frac{7680}{x^2}\right\}$	M1: Either $60x^2 \to 120x$ or $\frac{7680}{x} \to \frac{\pm\lambda}{x^2}$	M1
		A1: Correct differentiation (need not be simplified).	A1 aef
	$120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$	M1: $S' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value" Setting their $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$ and "candidate's ft ***correct*** power of x = a value". **The power of *x* must be consistent with their differentiation**. If inequalities are used this mark cannot be gained until candidate states value of x or S from their x without inequalities. $S' = 0$ can be implied by $120x = \frac{7680}{x^2}$. Some may spot that $x = 4$ gives $S' = 0$ and provided they clearly show $S'(4) = 0$ allow this mark as long as S' is correct. (If S' is incorrect this method is allowed if their derivative is clearly zero for their value of x)	M1A1**cso**
		A1: $x = 4$ **only** ($x^3 = 64 \Rightarrow x = \pm 4$ scores A0) Note that the value of x is not explicitly required so the use of $x = \sqrt[3]{64}$ to give $S = 2880$ would imply this mark.	
	Note some candidates stop here and do not go on to find *S* – maximum mark is 4/6		
	$\{x = 4,\}$ $S = 60(4)^2 + \frac{7680}{4} = 2880 \ (\text{cm}^2)$	Substitute candidate's value of $x\,(\neq 0)$ into a formula for S. **Dependent on both previous M marks.**	**dd**M1
		2880 **cso (Must come from correct work)**	A1 **cao and cso**
			[6]

10(d)	$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow$ Minimum	M1: Attempt S'' $\left(x^n \to x^{n-1}\right)$ **and** considers sign. This mark requires an attempt at the second derivative and **some consideration of its sign**. There does not necessarily need to be any substitution. An attempt to solve $S''=0$ is M0 A1: $120+\frac{15360}{x^3}$ and >0 **and** conclusion. Requires a **correct** second derivative of $120+\frac{15360}{x^3}$ (need not be simplified) **and** a valid reason (e.g. > 0), **and** conclusion. Only follow through a correct second derivative i.e. x may be incorrect **but must be positive** and/or S'' may have been evaluated incorrectly.	M1A1ft
	A correct S'' followed by $S''("4")="360"$ therefore minimum would score no marks in (d) **A correct S'' followed by $S''("4")="360"$ which is positive therefore minimum would score both marks**		
			[2]
	Note parts (c) and (d) can be marked together.		
			Total 14

Question number	Scheme	Marks
8 (a)	$(h=)\dfrac{60}{\pi x^2}$ or **equivalent exact** (not decimal) expression e.g. $(h=)60\div\pi x^2$	B1 (1)
(b)	$(A=)2\pi x^2+2\pi xh$ or $(A=)2\pi r^2+2\pi rh$ or $(A=)2\pi r^2+\pi dh$ may not be simplified and may appear on separate lines	B1
	Either $(A)=2\pi x^2+2\pi x\left(\dfrac{60}{\pi x^2}\right)$ or As $\pi xh=\dfrac{60}{x}$ then $(A=)2\pi x^2+2\left(\dfrac{60}{x}\right)$	M1
	$A=2\pi x^2+\left(\dfrac{120}{x}\right)$ *	A1 cso (3)
(c)	$(\dfrac{\mathrm{d}A}{\mathrm{d}x})=4\pi x-\dfrac{120}{x^2}$ or $=4\pi x-120x^{-2}$	M1 A1
	$4\pi x-\dfrac{120}{x^2}=0$ implies $x^3=$ (Use of >0 or <0 is M0 then M0A0)	M1
	$x=\sqrt[3]{\dfrac{120}{4\pi}}$ or answers which round to 2.12 (-2.12 is A0)	dM1 A1 (5)
(d)	$A=2\pi(2.12)^2+\dfrac{120}{2.12}$, $=85$ (only ft $x=2$ or 2.1 – both give 85)	M1, A1 (2)
(e)	**Either** $\dfrac{d^2A}{dx^2}=4\pi+\dfrac{240}{x^3}$ and sign considered (May appear in (c)) **Or** *(method 2)* considers gradient to left and right of their 2.12 (e.g at 2 and 2.5) **Or** (*method 3*) considers value of A either side	M1
	which is >0 and therefore minimum (most substitute 2.12 but it is not essential to see a substitution) (may appear in (c)) Finds numerical values for gradients and observes gradients go from negative to zero to positive so concludes minimum **OR** finds numerical values of A , observing greater than minimum value and draws conclusion	A1 (2)
		13 marks
Notes	(a) **B1**: This expression must be correct and in part (a) $\dfrac{60}{\pi r^2}$ is B0 (b) **B1:** Accept any equivalent correct form – may be on two or more lines. **M1 :** substitute their expression for h in terms of x into Area formula **of the form** kx^2+cxh **A1**: There should have been no errors in part (b) in obtaining this printed answer (c) **M1:** At least one power of x decreased by 1 **A1** accept any equivalent correct answer **M1:** Setting $\dfrac{\mathrm{d}A}{\mathrm{d}x}$=0 and finding a value for x^3 (x^3 = may be implied by answer). Allow $\dfrac{dy}{dx}=0$ **dM1:** Using **cube root** to find x **A1 :** For any equivalent correct answer (need 3sf or more) Correct answer implies previous M mark (d) **M1 :** Substitute the (+ve) x value found in (c) into equation for A and evaluate . **A1** is for **85 only** (e) **M1**: Complete method, usually one of the three listed in the scheme. For first method $A''(x)$ must be attempted and sign considered **A1**: Clear statements and conclusion. (numerical substitution of x is not necessary in first method shown, and x or calculation could be wrong but $A''(x)$ **must be correct** . Must **not** see 85 substituted)	

Question number	Scheme	Marks
8 (a)	$kr^2+cxy=4$ or $kr^2+c[(x+y)^2-x^2-y^2]=4$	M1
	$\frac{1}{4}\pi x^2+2xy=4$	A1
	$y=\frac{4-\frac{1}{4}\pi x^2}{2x}=\frac{16-\pi x^2}{8x}$ *	B1 cso (3)
(b)	$P=2x+cy+k\pi r$ where c = 2 or 4 and k = ¼ or ½	M1
	$P=\frac{\pi x}{2}+2x+4\left(\frac{4-\frac{1}{4}\pi x^2}{2x}\right)$ or $P=\frac{\pi x}{2}+2x+4\left(\frac{16-\pi x^2}{8x}\right)$ o.e.	A1
	$P=\frac{\pi x}{2}+2x+\frac{8}{x}-\frac{\pi x}{2}$ so $P=\frac{8}{x}+2x$ *	A1 (3)
(c)	$\left(\frac{\mathrm{d}P}{\mathrm{d}x}=\right)-\frac{8}{x^2}+2$	M1 A1
	$-\frac{8}{x^2}+2=0\Rightarrow x^2=..$	M1
	and so $x=2$ o.e. (ignore extra answer x = -2)	A1
	$P=4+4=8$ (m)	B1 (5)
(d)	$y=\frac{4-\pi}{4}$, (and so width) $=21$ (cm)	M1, A1 (2) **13**
Notes	(a) **M1**: Putting sum of one or two xy terms and one kr^2 term equal to 4 (**k and c may be wrong**) **A1:** For any correct form of this equation with x for radius (may be unsimplified) **B1 :** Making y the subject of their formula to give this printed answer with no errors (b) **M1** : Uses Perimeter formula of the form $2x+cy+k\pi r$ where c = 2 or 4 and k = ¼ or ½ **A1:** Correct unsimplified formula with y substituted as shown, i.e. $c=4$, k = ½ , $r=x$ and $y=\frac{16-\pi x^2}{8x}$ or $y=\left(\frac{4-\frac{1}{4}\pi x^2}{2x}\right)$ **A1:** obtains printed answer with at least one line of correct simplification or expansion before giving printed answer or stating result has been shown or equivalent (c) **M1:** At least one power of x decreased by 1 (Allow $2x$ becomes 2) **A1:** accept any equivalent correct answer **M1:** Setting $\frac{\mathrm{d}P}{\mathrm{d}x}$=0 and finding a value for correct power of x for candidate **A1** : For x = 2. (This mark may be given for equivalent and may be implied by correct P) **B1:** 8 (cao) N.B. This may be awarded if seen in part (d) (d) **M1 :** Substitute x value found in (c) into equation for y from (a) (or substitute x and P into equation for P from (b)) and evaluate (may see 0.2146 and correct answer implies M1 or need to see substitution if x value was wrong.) **A1** is for 21 or 21cm or 0.21m as this is to nearest cm	