OCR
Oxford Cambridge and RSA

# AS Level Mathematics B (MEI)

## H630/02 Pure Mathematics and Statistics
Question Paper

## Wednesday 23 May 2018 – Morning
**Time allowed: 1 hour 30 minutes**

**You must have:**
- Printed Answer Booklet

**You may use:**
- a scientific or graphical calculator

*7020509472*

**INSTRUCTIONS**
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION**
- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

# Formulae AS Level Mathematics B (MEI) (H630)

**Binomial series**

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_r a^{n-r} b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

**Differentiation from first principles**

$\mathrm{f}'(x) = \lim\limits_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

**Sample variance**

$s^2 = \dfrac{1}{n-1} S_{xx}$ where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

**The binomial distribution**

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of $X$ is $np$

**Kinematics**

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions

**1** Write down the value of

($A$) $\log_a (a^4)$, **[1]**

($B$) $\log_a \left(\frac{1}{a}\right)$. **[1]**

**2** Doug has a list of times taken by competitors in a 'fun run'. He has grouped the data and calculated the frequency densities in order to draw a histogram to represent the information. Some of the data are presented in Fig. 2.

| Time in minutes | 15 − | 20 − | 25 − | 35 − | 45 − 60 |
|---|---|---|---|---|---|
| Number of runners | 12 | 23 | 59 | 71 | |
| Frequency density | 2.4 | | 5.9 | 7.1 | 1.4 |

**Fig. 2**

**(i)** Write down the missing values in the copy of Fig. 2 in the Printed Answer Booklet. **[2]**

**(ii)** Doug labels the horizontal axis on the histogram 'time in minutes' and the vertical axis 'number of minutes per runner'. State which one of these labels is incorrect and write down a correct version. **[1]**

**3** $P$ and $Q$ are consecutive **odd** positive integers such that $P > Q$.

Prove that $P^2 - Q^2$ is a multiple of 8. **[3]**

**4** The probability distribution of the discrete random variable $X$ is given in Fig. 4.

| $r$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = r)$ | 0.2 | 0.15 | 0.3 | $k$ | 0.25 |

**Fig. 4**

**(i)** Find the value of $k$. **[2]**

$X_1$ and $X_2$ are two independent values of $X$.

**(ii)** Find $P(X_1 + X_2 = 6)$. **[3]**

**5** Find the set of values of $a$ for which the equation

$$ax^2 + 8x + 2 = 0$$

has no real roots. **[3]**

**6** Show that $\int_0^9 (3 + 4\sqrt{x})\,dx = 99$. **[4]**

**7** Rose and Emma each wear a device that records the number of steps they take in a day. All the results for a 7-day period are given in Fig. 7.

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Rose | 10014 | 11262 | 10149 | 9361 | 9708 | 9921 | 10369 |
| Emma | 9204 | 9913 | 8741 | 10015 | 10261 | 7391 | 10856 |

**Fig. 7**

The 7-day mean is the mean number of steps taken in the last 7 days. The 7-day mean for Rose is 10112.

**(i)** Calculate the 7-day mean for Emma. **[1]**

At the end of day 8 a new 7-day mean is calculated by including the number of steps taken on day 8 and omitting the number of steps taken on day 1. On day 8 Rose takes 10259 steps.

**(ii)** Determine the number of steps Emma must take on day 8 so that her 7-day mean at the end of day 8 is the same as for Rose. **[4]**

In fact, over a long period of time, the mean of the number of steps per day that Emma takes is 10341 and the standard deviation is 948.

**(iii)** Determine whether the number of steps Emma needs to take on day 8 so that her 7-day mean is the same as that for Rose in part **(ii)** is unusually high. **[3]**

**8** **In this question you must show detailed reasoning.**

The centre of a circle C is at the point $(-1, 3)$ and C passes through the point $(1, -1)$. The straight line L passes through the points $(1, 9)$ and $(4, 3)$. Show that L is a tangent to C. **[7]**

**9** **In this question you must show detailed reasoning.**

Research showed that in May 2017 62% of adults over 65 years of age in the UK used a certain online social media platform. Later in 2017 it was believed that this proportion had increased. In December 2017 a random sample of 59 adults over 65 years of age in the UK was collected. It was found that 46 of the 59 adults used this online social media platform.

Use a suitable hypothesis test to determine whether there is evidence at the 1% level to suggest that the proportion of adults over 65 in the UK who used this online social media platform had increased from May 2017 to December 2017. **[7]**

**10** **(i)** A curve has equation $y = 16x + \frac{1}{x^2}$. Find

($A$) $\frac{\mathrm{d}y}{\mathrm{d}x}$, **[2]**

($B$) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. **[2]**

**(ii)** Hence

- find the coordinates of the stationary point,
- determine the nature of the stationary point. **[5]**

**11** The pre-release material contains data concerning the death rate per thousand people and the birth rate per thousand people in all the countries of the world. The diagram in Fig. 11.1 was generated using a spreadsheet and summarises the birth rates for all the countries in Africa.

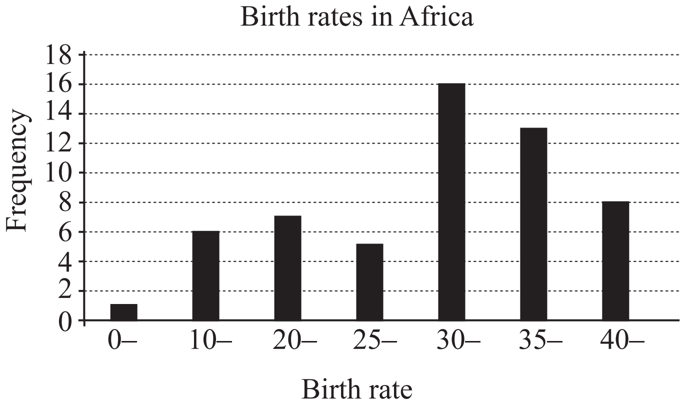


**Fig. 11.1**

**(i)** Identify **two** respects in which the presentation of the data is incorrect. **[2]**

Fig. 11.2 shows a scatter diagram of death rate, $y$, against birth rate, $x$, for a sample of 55 countries, all of which are in Africa. A line of best fit has also been drawn.

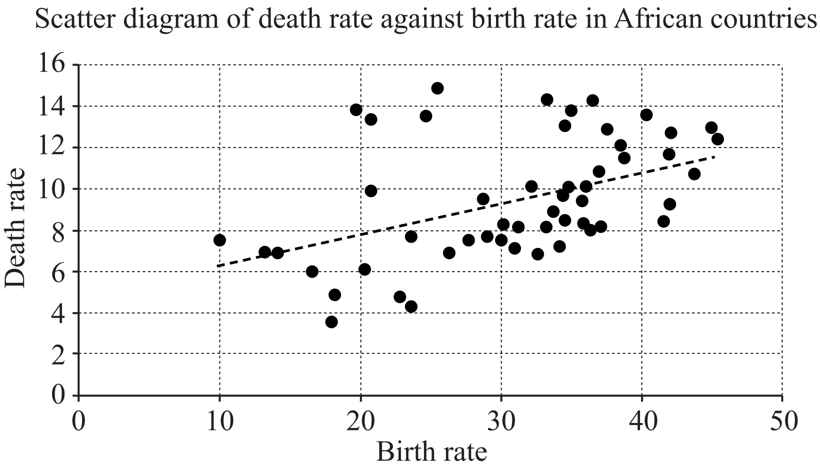


**Fig. 11.2**

The equation of the line of best fit is $y = 0.15x + 4.72$.

**(ii)** (*A*) What does the diagram suggest about the relationship between death rate and birth rate? **[1]**

(*B*) The birth rate in Togo is recorded as 34.13 per thousand, but the data on death rate has been lost. Use the equation of the line of best fit to estimate the death rate in Togo. **[1]**

(*C*) Explain why it would not be sensible to use the equation of the line of best fit to estimate the death rate in a country where the birth rate is 5.5 per thousand. **[1]**

(*D*) Explain why it would not be sensible to use the equation of the line of best fit to estimate the death rate in a Caribbean country where the birth rate is known. **[1]**

(*E*) Explain why it is unlikely that the sample is random. **[1]**

Including Togo there were 56 items available for selection.

**(iii)** Describe how a sample of size 14 from this data could be generated for further analysis using systematic sampling. **[2]**

**12** In an experiment 500 fruit flies were released into a controlled environment. After 10 days there were 650 fruit flies present.

Munirah believes that $N$, the number of fruit flies present at time $t$ days after the fruit flies are released, will increase at the rate of 4.4% per day. She proposes that the situation is modelled by the formula $N = Ak^t$.

**(i)** Write down the values of $A$ and $k$. **[2]**

**(ii)** Determine whether the model is consistent with the value of $N$ at $t = 10$. **[2]**

**(iii)** What does the model suggest about the number of fruit flies in the long run? **[1]**

Subsequently it is found that for large values of $t$ the number of fruit flies in the controlled environment oscillates about 750. It is also found that as $t$ increases the oscillations decrease in magnitude.

Munirah proposes a second model in the light of this new information.

$$N = 750 - 250 \times e^{-0.092t}.$$

**(iv)** Identify three ways in which this second model is consistent with the known data. **[3]**

**(v)** ($A$) Identify one feature which is not accounted for by the second model. **[1]**

($B$) Give an example of a mathematical function which needs to be incorporated in the model to account for this feature. **[1]**

**END OF QUESTION PAPER**