Question	Scheme		Marks	AOs
13 (a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178"\times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235"\log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
			(7 marks)	

Notes

(a)

M1: Establishes a link between $h = pm^q$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only “3” bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as “122” bpm is not equal to 119 bpm ✗

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

Question	Scheme	Marks	AOs
11 (a)	$35\,(\text{km}^2)$	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45\mathrm{e}^{14c} \Rightarrow 45\mathrm{e}^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \dfrac{1}{14}\ln\left(\dfrac{20}{45}\right) = \cdots -0.0579$	dM1	3.1b
	$A = 80 - 45\mathrm{e}^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
			(6 marks)

Notes

(a)

B1: Uses the equation of the model to find that $35\,(\text{km}^2)$ of the reserve was covered on 1st January 2005. Do not accept eg. 35 m^2

(b)

M1: Sets their $60 = 80 - 45\mathrm{e}^{14c} \Rightarrow A\mathrm{e}^{14c} = B$

A1: $45\mathrm{e}^{14c} = 20$ or equivalent.

dM1: A full and careful method using precise algebra, correct log laws and a knowledge that e^x and ln*x* are inverse functions and proceeds to a value for *c*.

A1: Gives a complete equation for the model $A = 80 - 45\mathrm{e}^{-0.0579t}$

(c)

B1: Gives a suitable interpretation (See scheme)

Question	Scheme		Marks	AOs
5 (a)	Identifies one of the two errors "You cannot use the subtraction law without dealing with the 2 first" " They undo the logs incorrectly. It should be $x=2^3=8$ "		B1	2.3
	Identifies both errors. See above.		B1	2.3
			(2)	
(b)	$\log_2\left(\dfrac{x^2}{\sqrt{x}}\right)=3$	$\dfrac{3}{2}\log_2(x)=3$	M1	1.1b
	$x^{\frac{3}{2}}=2^3$ or $\dfrac{x^2}{\sqrt{x}}=2^3$	$x=2^2$	M1	1.1b
	$x=\left(2^3\right)^{\frac{2}{3}}=4$	$x=4$	A1	1.1b
			(3)	
				(5 marks)

(a)
B1: States one of the two errors.
Error One: Either in words states 'They cannot use the subtraction law without dealing with the 2 first' or writes ' that line 2 should be $\log_2\left(\dfrac{x^2}{\sqrt{x}}\right)$ $(=3)$' If they rewrite line two it must be correct. Allow 'the coefficient of each log term is different so we cannot use the subtraction law'
Allow responses such as 'it must be $\log x^2$ before subtracting the logs'
Do not accept an incomplete response such as "the student ignored the 2". **There must be some reference to the subtraction law as well.**
Error Two: Either in words states 'They undo the log incorrectly' or writes that 'if $\log_2 x=3$ then $x=2^3=8$' If it is rewritten it must be correct. Eg $x=\log_2 9$ is B0
B1: States both of the two errors. (See above)

(b)
M1: Uses a correct method of combining the two log terms. Either uses both the power law and the subtraction law to reach a form $\log_2\left(\dfrac{x^2}{\sqrt{x}}\right)=3$ oe. Or uses both the power law and subtraction to reach $\dfrac{3}{2}\log_2(x)=3$
M1: Uses correct work to "undo" the log. Eg moves from $\log_2\left(Ax^n\right)=b\Rightarrow Ax^n=2^b$
This is independent of the previous mark so allow following earlier error.
A1: cso $x=4$ achieved with at least one intermediate step shown. Extra solutions would be A0
SC: If the "answer" rather than the "solution" is given score 1,0,0.

	For $p =$ awrt 63100 or $q =$ awrt 1.122	A1	1.1b
	For correct equations in p and q $p = 10^{4.8}$ and $q = 10^{0.05}$	dM1	3.1a
	For $p =$ awrt 63100 and $q =$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V = 63100 \times 1.122^{30}$ or $\log V = 0.05 \times 30 + 4.8$ leading to $V =$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: For a correct equation in p or q This is usually $p = 10^{4.8}$ or $q = 10^{0.05}$ but may be $\log q = 0.05$ or $\log p = 4.8$

A1: For $p =$ awrt 63100 or $q =$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p = 10^{4.8}$ and $q = 10^{0.05}$ but may be $\log q = 0.05$ and $\log p = 4.8$

A1: For $p =$ awrt 63100 and $q =$ awrt 1.122 Both these values implies M1 M1

……………………………………………………………………………………………

ALT I(a)

M1: Substitutes $t = 0$ and states that $\log p = 4.8$

A1: $p =$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p =$ awrt 63100 and $q =$ awrt 1.122

……………………………………………………………………………………………

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled "*p* is........... " and "*q* is ...."

(c)

M1: For substituting $t = 30$ into $V = pq^t$ using their values for p and q or substituting t =30 into $\log_{10} V = 0.05t + 4.8$ and proceeds to V

A1: For awrt either £1.99 million or £2.00 million. Condone the omission of the £ sign. Remember to isw after a correct answer

Question	Scheme	Marks	AOs
14 (a)	Attempts to complete the square $(x \pm 3)^2 + (y \pm 5)^2 = \ldots$	M1	1.1b

	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their *a*, *b* and *c* leading to values for *k* $"(10k-6)^2-36(1+k^2)...0" \to k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both *a* and *b* must have been expressions in *k*)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)

M1: Attempts $(x\pm3)^2+(y\pm5)^2=$ **..**

This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$

(i) A1: Centre $(3,-5)$

(ii) A1: Radius 5. Do not accept $\sqrt{25}$

Answers only (no working) scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value.
You may award for the correct $k<0$ but award if $k\leqslant 0$ or even with greater than symbols

M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $(x\pm3)^2+(y\pm5)^2=$ **...** to form an equation in just *x* and *k* . It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just *y* and *k*.

A1: Correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$ with the terms in *x* collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of *a*, *b* and *c* are used in $b^2-4ac...0$.

FYI The equation in *y* and *k* is $(1+k^2)y^2+(10k^2-6k)y+9k^2=0$ oe

M1: Attempts to find two critical values for *k* using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both *a* and *b* must have been expressions in *k*.
Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg.

$b^2-4ac=(k-6)^2-4\times(1+k^2)\times9>0 \Rightarrow -35k^2-12k>0 \Rightarrow k(35k+12)<0$

Question	Scheme	Marks	AOs
14 (a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt}=-3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T}=-500 \Rightarrow 3925e^{-0.25T}=500$ * cso	A1*	3.4
	(ii) $e^{-0.25T}=0.127... \Rightarrow -0.25T=\ln 0.127...$	M1	1.1b
	$T=8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)

B1: £18 000 There is no requirement to have the units

(b)(i)

M1: Award for making the link between gradient and rate of change.

Score for attempting to differentiate V to $\frac{dV}{dt}=ke^{-0.25t}$ An attempt at both sides are required.

For the left hand side you may condone attempts such as $\frac{dy}{dx}$

A1: Achieves $\frac{dV}{dt}=-3925e^{-0.25t}$ or $\frac{dV}{dt}=15700\times -0.25e^{-0.25t}$ with both sides correct

A1*: Sets $-3925e^{-0.25T}=-500$ oe and proceeds to $3925e^{-0.25T}=500$

This is a given answer and to achieve this mark, all aspects must be seen and be correct.

t must be changed to T at some point even if just at the end of their solution/proof

SC: Award SC 110 candidates who simply write

$-3925e^{-0.25T}=-500 \Rightarrow 3925e^{-0.25T}=500$ without any mention or reference to $\frac{dV}{dt}$

Or $15700\times -0.25e^{-0.25t}=-500 \Rightarrow 3925e^{-0.25T}=500$ without any mention or reference to $\frac{dV}{dt}$

(b)(ii)

M1: Proceeds from $e^{-0.25T}=A, A>0$ using ln's to $\pm 0.25T=..$

Alternatively takes lns first $3925e^{-0.25T}=500 \Rightarrow \ln 3925-0.25T=\ln 500 \Rightarrow \pm 0.25T=...$

but $3925e^{-0.25T}=500 \Rightarrow \ln 3925\times -0.25T=\ln 500 \Rightarrow \pm 0.25T=...$ is M0

A1: $T=$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t=$ awrt 8.24

Question	Scheme	Marks	AOs
8 (a)	Temperature = $83°\text{C}$	B1	3.4
		(1)	
(b)	$18+65e^{-\frac{t}{8}}=35 \Rightarrow 65e^{-\frac{t}{8}}=17$	M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$ $\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$	A1	1.1b
		(3)	
(c)	States a suitable reason • As $t\to\infty, \theta\to 18$ from above. • The minimum temperature is $18°\text{C}$	B1	2.4
		(1)	
(d)	$A+B=94$ or $A+Be^{-1}=50$	M1	3.4
	$A+B=94$ and $A+Be^{-1}=50$	A1	1.1b
	Full method to find at least a value for A	dM1	2.1
	Deduces $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4	A1	2.2a
		(4)	
			(9 marks)

Notes

(a)

B1: Uses the model to state that the temperature $=83°\text{C}$ Requires units as well

(b)

M1: Uses the information and proceeds to $Pe^{\pm\frac{t}{8}}=Q$ condoning slips

dM1: A full method using correct log laws and a knowledge that e^{x} and ln$x$ are inverse functions. This cannot be scored from unsolvable equations, e.g $P>0, Q<0$. Condone one error in their solution.

A1: $t=$ awrt 10.7

(c)

B1: States a suitable reason with minimal conclusion

- As $t\to\infty, \theta\to 18$ from above.
- The minimum temperature is $18°\text{C}$ (so it cannot drop to $15°\text{C}$)
- Substitutes $\theta=15$ (or a value between 15 and 18) into $18+65e^{-\frac{t}{8}}=15$ (may be impied by $15-18=-3$ or similar) and makes a statement that $e^{-\frac{t}{8}}$ cannot be less than zero or else that $\ln(-ve)$ is undefined and hence $\theta\neq 15$. All calculations must be correct
- (According to the model) the room temperature is $18°\text{C}$ (so cannot fall below this)

Question	Scheme		Marks	AOs
12 (a)	$\log_{10} V = 0.072t + 2.379$ $\Rightarrow V = 10^{0.072t+2.379}$ $\Rightarrow V = 10^{0.072t} \times 10^{2.379}$	$V = ab^t$ $\Rightarrow \log_{10} V = \log_{10} a + \log_{10} b^t$ $\Rightarrow \log_{10} V = \log_{10} a + t\log_{10} b$	B1	2.1
	States either $a = 10^{2.379}$ or $b = 10^{0.072}$	States either $\log_{10} a = 2.379$ or $\log_{10} b = 0.072$	M1	1.1b
	$a = 239$ or $b = 1.18$	$a = 239$ or $b = 1.18$	A1	1.1b
	Either $V = 239 \times 1.18^t$ or imply by $a = 239, b = 1.18$		A1	1.1b
			(4)	
(b)	The value of ab is the (total) number of views of the advert **1 day after** it went live.		B1	3.4
			(1)	
(c)	Substitutes $t = 20$ in either equation and finds V Eg $V = 239 \times 1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	
				(7 marks)

(a) **Condone** $\log_{10}$ **written** $\log$ **or** $\lg$ **written throughout the question**

B1: Scored for showing that $\log_{10} V = 0.072t + 2.379$ can be written in the form $V = ab^t$ or vice versa

Either starts with $\log_{10} V = 0.072t + 2.379$ (may be implied) and **shows lines**

$$V = 10^{0.072t+2.379} \text{ and } V = 10^{0.072t} \times 10^{2.379}$$

Or starts with $V = ab^t$ (implied) and **shows the lines**

$$\log_{10} V = \log_{10} a + \log_{10} b^t \text{ and } \log_{10} V = \log_{10} a + t\log_{10} b$$

M1: For a correct equation in a or a correct equation in b

A1: Finds either constant. Allow $a = \text{awrt } 240$ or $b = \text{awrt } 1.2$ following a correct method

A1: Correct solution: Look for $V = 239 \times 1.18^t$ or $a = 239, b = 1.18$

Note that this is NOT awrt

(b)

B1: See scheme. Condone not seeing total. Do not allow number of views at the start or similar.

(c)

M1: Substitutes $t = 20$ in either their $V = 239 \times 1.18^t$ or $\log_{10} V = 0.072t + 2.379$ and uses a correct method to find V

A1: Awrt 6500 or 6600

Question	Scheme	Marks	AOs
14(a)	$\log_{10} P = mt + c$	M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$	A1	1.1b
		(2)	
(b)	**Way 1:** As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$ **Way 2:** As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	Way 1: $\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$ **Way 2:** $a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	So a = 100 000 **or** b = 1.0116	A1	1.1b
	Both a = 100 000 **and** b = 1.0116 (awrt 1.01)	A1	1.1b
		(4)	
(c)(i)	The initial population	B1	3.4
(c)(ii)	The proportional increase of population each year	B1	3.4
		(2)	
(d)(i)	300000 to nearest hundred thousand	B1	3.4
(d)(ii)	Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give $t =$	M1	3.4
	60.2 years to 3sf	A1ft	1.1b
		(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth	B2	3.5b
		(2)	

Question Number	Scheme	Marks
7. (i)	Use of power rule so $\log(x+a)^2 = \log 16a^6$ or $2\log(x+a) = 2\log 4a^3$ or $\log(x+a) = \log(16a^6)^{\frac{1}{2}}$	M1
	Removes logs and square roots, **or** halves then removes logs to give $(x+a) = 4a^3$ Or $x^2 + 2ax + a^2 - 16a^6 = 0$ followed by factorisation or formula to give $x = \sqrt{16a^6} - a$	M1
	$(x=)\ 4a^3 - a$ (depends on previous M's and must be this expression or equivalent)	A1cao **(3)**
(ii) Way 1	$\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ — Applies quotient law of logarithms	M1
	$\dfrac{(9y+b)}{(2y-b)} = 3^2$ — Uses $\log_3 3^2 = 2$	M1
	$(9y+b) = 9(2y-b) \Rightarrow y =$ — Multiplies across and makes y the subject	M1
	$y = \frac{10}{9}b$	A1cso **(4)**
Way 2	Or: $\log_3(9y+b) = \log_3 9 + \log_3(2y-b)$ — 2nd M mark	M1
	$\log_3(9y+b) = \log_3 9(2y-b)$ — 1st M mark	M1
	$(9y+b) = 9(2y-b) \Rightarrow y = \frac{10}{9}b$ — Multiplies across and makes y the subject	M1 A1cso **(4)** **[7]**

	Notes
(i)	1st M1: Applies power law of logarithms correctly to one side of the equation M1: Correct log work in correct order. If they square and obtain a quadratic the algebra should be correct. The marks is for $x+a = \sqrt{16a^6}$ isw so allow $x+a = \pm\ 4a^3$ for Method mark. Also allow $x+a = 4a^4$ or $x+a = \pm\ 4a^{5.5}$ or even $x+a = 16a^3$ as there is evidence of attempted square root. May see the correct $x+a = 10^{(\log 4 + 3\log a)}$ *so* $x = -a + 10^{(\log 4 + 3\log a)}$ which gains M1A0 unless followed by the answer in the scheme. A1: Do not allow $x = \pm\ 4a^3 - a$ for accuracy mark. You may see the factorised $a(2a+1)(2a-1)$ o.e.
(ii)	M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term in y M1: Uses $\log_3 3^2 = 2$ 3rd M1: Obtains **correct** linear equation in y usually the one in the scheme and attempts $y =$ A1cso: $y = \frac{10}{9}b$ or correct equivalent after **completely correct** work. **Special case:** $\dfrac{\log_3(9y+b)}{\log_3(2y-b)} = 2$ is M0 unless clearly crossed out and replaced by the correct $\log_3 \dfrac{(9y+b)}{(2y-b)} = 2$ Candidates may then write $\dfrac{(9y+b)}{(2y-b)} = 3^2$ and proceed to the *correct* answer – allow M0M1M1A0 as the answer requires a completely correct solution.

Question Number	Scheme	Marks
8(i)	Two Ways of answering the question are given in part (i)	
Way 1	$\log_3\left(\dfrac{3b+1}{a-2}\right) = -1$ or $\log_3\left(\dfrac{a-2}{3b+1}\right) = 1$ — Applying the subtraction law of logarithms	M1
	$\dfrac{3b+1}{a-2} = 3^{-1}\left\{=\dfrac{1}{3}\right\}$ or $\left(\dfrac{a-2}{3b+1}\right) = 3$ — Making a correct connection between log base 3 and 3 to a power.	M1
	$\{9b+3=a-2 \Rightarrow\}\ b=\dfrac{1}{9}a-\dfrac{5}{9}$ — $b=\dfrac{1}{9}a-\dfrac{5}{9}$ or $b=\dfrac{a-5}{9}$	A1 oe
		[3]
	In **Way 2** a correct connection between log base 3 and "3 to a power" is used before applying the subtraction or addition law of logs	
(i) **Way 2**	Either $\log_3(3b+1)-\log_3(a-2) = -\log_3 3$ or $\log_3(3b+1)+\log_3 3 = \log_3(a-2)$	2nd M1
	$\log_3(3b+1) = \log_3(a-2) - \log_3 3 = \log_3\left(\dfrac{a-2}{3}\right)$ or $\log_3 3(3b+1) = \log_3(a-2)$	1st M1
	$\{3b+1=\dfrac{a-2}{3}\}\ b=\dfrac{1}{9}a-\dfrac{5}{9}$	A1
		[3]
	Five Ways of answering the question are given in part (ii)	
(ii) **Way 1** **See also common approach below in notes**	$32(2^{2x})-7(2^x)=0$ — Deals with power 5 correctly giving $\times 32$	M1
	So, $2^x = \dfrac{7}{32}$ — $2^x=\dfrac{7}{32}$ or $y=\dfrac{7}{32}$ or awrt 0.219	A1 oe
	$x\log 2 = \log\left(\dfrac{7}{32}\right)$ or $x=\dfrac{\log\left(\frac{7}{32}\right)}{\log 2}$ or $x=\log_2\left(\dfrac{7}{32}\right)$ — A valid method for solving $2^x=\frac{7}{32}$ Or $2^x=k$ to achieve $x=\ldots$	dM1
	$x=-2.192645\ldots$ — awrt -2.19	A1
		[4]
	Begins with $2^{2x+5}=7(2^x)$ (for Way 2 and Way 3) (see notes below)	
(ii) **Way 2**	$(2x+5)\log 2 = \log 7 + x\log 2$ — Correct application of **either** the power law **or** addition law of logarithms	M1
	Correct result after applying the power **and** addition laws of logarithms.	A1
	$2x\log 2 + 5\log 2 = \log 7 + x\log 2$ $\Rightarrow\ x=\dfrac{\log 7 - 5\log 2}{\log 2}$ — Multiplies out, collects x terms to achieve $x=\ldots$	dM1
	$x=-2.192645\ldots$ — awrt -2.19	A1
		[4]
(ii) **Way 3**	$2x+5=\log_2 7 + x$ — Evidence of $\log_2$ and either $2^{2x+5}\to 2x+5$ or $7(2^x)\to \log_2 7+\log_2(2^x)$	M1
	$2x+5=\log_2 7 + x$ oe.	A1
	$2x - x = \log_2 7 - 5$ $\Rightarrow\ x=\log_2 7 - 5$ — Collects x terms to achieve $x=\ldots$	dM1
	$x=-2.192645\ldots$ — awrt -2.19	A1
		[4]

Question Number	Scheme	Marks
9(i)	$2\log_{10}(x-2)-\log_{10}(x+5)=0 \Rightarrow \log_{10}(x-2)^2=\log_{10}(x+5)$	M1
	$\Rightarrow (x-2)^2=(x+5)$	M1
	$\Rightarrow x^2-5x-1=0$	A1
	$x=\dfrac{5\pm\sqrt{29}}{2} \Rightarrow x=\dfrac{5+\sqrt{29}}{2}$ only	M1,A1
		(5)
(ii)	$\log_p(4y+1)-\log_p(2y-2)=1 \Rightarrow \log_p\left(\dfrac{4y+1}{2y-2}\right)=\log_p p$	M1, M1
	$\Rightarrow \left(\dfrac{4y+1}{2y-2}\right)=p$	A1
	$\Rightarrow 4y+1=2py-2p \Rightarrow y=\dfrac{1+2p}{2p-4}$	M1A1
		(5) **(10 marks)**

(i)

M1 Use of the power law of logs

M1 For 'undoing' the logs by either setting $\log_{10}...=\log_{10}...$ or using the subtraction law and $0=\log_{10}1$

A1 A correct simplified quadratic $x^2-5x-1=0$

M1 A correct attempt to find a solution to a 3TQ of equivalent difficulty (ie no factors). Allow formula, completing the square and use of a calculator giving exact or decimal answers

A1 cso $\dfrac{5+\sqrt{29}}{2}$ or exact simplified equivalent without extra answers.

(ii)

M1 Use of subtraction (or addition) law of logs

M1 For using $1=\log_p p$ or equivalent in an attempt to get an equation not involving logs.

$\log_p(4y+1)-\log_p(2y-2)=1 \Rightarrow (4y+1)-(2y-2)=p$ implies this and scores M0 M1.

A1 A correct equation in p and y not involving logs. Accept $\left(\dfrac{4y+1}{2y-2}\right)=p^1$

M1 Score for an attempt to change the subject. This must include cross multiplication, collection of terms in y, followed by factorisation of the y term.

A1 cso $y=\dfrac{1+2p}{2p-4}$ or equivalent such as $y=\dfrac{-1-2p}{4-2p}$

Special cases in (i): Case 1 Allow the subtraction law either way around as the rhs of the equation will be 1

Case 2 $\log_{10}\dfrac{(x-2)^2}{(x+5)}=0 \Rightarrow \dfrac{(x-2)^2}{(x+5)}=0 \Rightarrow (x-2)^2=(x+5) \Rightarrow x^2-5x-1=0$

$\Rightarrow x=\dfrac{5+\sqrt{29}}{2}$ only will be awarded M1 M0 A1 M1 A0

Special cases in (ii): $\log_p(4y+1)-\log_p(2y-2)=1 \Rightarrow \dfrac{\log_p(4y+1)}{\log_p(2y-2)}=\log_p p \Rightarrow \left(\dfrac{4y+1}{2y-2}\right)=p^1$

$\Rightarrow 4y+1=2py-2p \Rightarrow y=\dfrac{1+2p}{2p-4}$ will be awarded M0 M1A1 M1 A0|

Question Number	Scheme		Marks
5 (a)(i)	$\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=y-2$	M1: $\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9$ or $\log_3\left(\frac{x}{9}\right)=\log_3 x+\log_3\frac{1}{9}$ Correct use of the subtraction rule or addition rule. Ignore the presence or absence of a base and any spurious "= 0"	M1A1
		A1: $y-2$	
	An answer left as $\log_3 3^{y-2}$ scores M1A0		
	Note that $\log_3\left(\frac{x}{9}\right)=\log_3 x-\log_3 9=y-\log_3 9$ scores M1A0		
(ii)	$\log_3\sqrt{x}=\log_3 x^{\frac{1}{2}}=\frac{1}{2}\log_3 x=\frac{1}{2}y$	$\frac{1}{2}y$ or equivalent	B1
			(3)
(b)	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\Rightarrow 2(y-2)-\frac{1}{2}y=2$ Uses their answers from part (a) to create a **linear** equation in y (condone poor use of brackets e.g. $2(y-2)=2y-2$ and also the slip $(y-2)-\frac{1}{2}y=2$ for this mark)		M1
	$\Rightarrow y=4$	Correct value for y.	A1
	Note that arriving at $(y-2)^2-\frac{1}{2}y=2$ above scores M0 (not linear) but does have a solution $y = 4$ so look out for $y = 4$ not being derived correctly.		
	$\log_3 x=4\Rightarrow x=3^4$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1
			(4)
			(7 marks)
Alt 1 (b)	$2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=\log_3\left(\frac{(x/9)^2}{\sqrt{x}}\right)$ or $2\log_3\left(\frac{x}{9}\right)-\log_3\sqrt{x}=2\log_3 x-2\log_3 9-\log_3\sqrt{x}=\log_3\frac{x^2}{\sqrt{x}}+\ldots$ Combines **two log terms in *x* correctly** to obtain a single log term		M1
	$\log_3\left(\frac{(x/9)^2}{\sqrt{x}}\right)=2$ or $\log_3\left(\frac{x^2}{\sqrt{x}}\right)=6$	Correct equation	A1
	$\left(\frac{(x/9)^2}{\sqrt{x}}\right)=3^2$ or $\left(\frac{x^2}{\sqrt{x}}\right)=3^6$	Correct method for undoing log. **Dependent on the first M**	**d**M1
	$\Rightarrow x=81$	cao	A1

Question	Scheme	Marks
2.	(a) $2x\log 7 = \log 14$ or $x\log 49 = \log 14$ or $2x = \log_7 14$	M1
	$x = \dfrac{\log 14}{2\log 7}$ = awrt 0.678	M1A1
		(3)
	(b) $3x+1 = 5^{-2}$	M1
	So $x = -\dfrac{8}{25}$ or -0.32	A1
		(2)
		5 marks
	Notes	

(a)
M1: Uses logs and brings down x correctly
M1: Makes x the subject correctly. This must follow a method that did involve taking logs
A1: Accept awrt 0.678 (N.B. Correct answer with no working implies two previous marks)

(b)
M1: Uses powers correctly to undo log. Accept $3x+1=5^{-2}$ or equivalent such as $3x+1=0.04$
A1: Correct answer (Correct answer implies method mark). Accept -0.320

Question Number	Scheme	Marks
13 (a)	$2\log_2 y = 5 - \log_2 x \Rightarrow \log_2 y^2 = 5 - \log_2 x$	M1
	$\Rightarrow \log_2 y^2 = \log_2 32 - \log_2 x \Rightarrow \log_2 y^2 = \log_2\left(\dfrac{32}{x}\right)$	
	$\Rightarrow y^2 = \dfrac{32}{x}$	M1A1
		(3)
(b)	$\log_x y = -3 \Rightarrow y = x^{-3}$	M1
	Sub $y = x^{-3}$ into $y^2 = \dfrac{32}{x} \Rightarrow x^{-6} = \dfrac{32}{x} \Rightarrow x^5 = \dfrac{1}{32} \Rightarrow x = \dfrac{1}{2}$	M1A1
	Sub $x = \dfrac{1}{2}$ into either eqn $\Rightarrow y = 8$	M1A1
		(5) **(8 marks)**
Alt (b)	Sub $y^2 = \dfrac{32}{x}$ into $\log_x y = -3 \Rightarrow \log_x \sqrt{\dfrac{32}{x}} = -3$	2nd M1
	$\Rightarrow \sqrt{\dfrac{32}{x}} = x^{-3}$	1st M1
	$\Rightarrow x^5 = \dfrac{1}{32} \Rightarrow x = \dfrac{1}{2}$	A1

(a)

M1 Uses one correct log law.
Eg Uses the index law and writes $2\log_2 y = \log_2 y^2$.
Alternatively writes 5 as $\log_2 32$. This may well come from $\log_2 ... = 5 \Rightarrow ... = 32$
Note that $2\log_2 y + \log_2 x = 2\log_2 xy$ is M0

M1 Uses two correct log laws
Award for $\log_2 y^2 = \log_2(32) - \log_2 x$
or $\log_2 x + \log_2 y^2 = 5 \Rightarrow \log_2 xy^2 = 5$

A1 Proceeds correctly to $y^2 = \dfrac{32}{x}$

(b)

M1 Undoes the log in the second equation $\log_x y = -3 \Rightarrow y = x^{-3}$
This may well appear later in the solution

M1 Combines both equations to form a single equation in one variable.

A1 $x = \dfrac{1}{2}$ or $y = 8$. Condone a solution $y = \pm 8$ for this mark

M1 Substitutes their $x = \dfrac{1}{2}$ into an equation to find y.
Alternatively substitutes their $y = 8$ into an equation to find x

A1 $x = \dfrac{1}{2}$ and $y = 8$ only. Note $x = \dfrac{1}{2}$ and $y = \pm 8$ is A0

SC. If a candidate uses $y = \dfrac{k}{x}$ with $\log_x y = -3$ this can potentially score M1: Undoing logs, M0: as combining the equations has been made easier, A0: M1: If they substitute their x to find y and vice versa followed by A0: scoring 10010

Question Number	Scheme	Marks
14(i)	$\log_a x + \log_a 3 = \log_a 27 - 1$ so $\log_a \frac{3x}{27} = -1$ Or $\log_a x + \log_a 3 = \log_a 27 - \log_a a$ so $\log_a 3x = \log_a \frac{27}{a}$ Or $\log_a x + 1 = \log_a 27 - \log_a 3 = \log_a 9$ so $\log_a ax = \log_a 9$	M1 A1
	$\frac{3x}{27} = a^{-1}$	M1
	$x = 9a^{-1}$ or $\frac{9}{a}$	A1
		[4]
(ii)	$x^2 - 7x + 12 = 0$ and attempt to solve to give $x = \ldots$ or $\log_5 y = \ldots$ (implied by correct answers)	M1
	x (or $\log_5 y$) $=$ 3 **and** 4	A1
	$y = 5^3$ **or** 5^4	dM1
	y = 125 **and** 625	A1
		[4]
		8 marks
	Notes	
(i)	**M1**: Uses sum or difference of logs correctly e.g. $\log x + \log 3 = \log 3x$ or $\log 27 - \log 3 = \log 9$ or $\log 27 - \log x = \log \frac{27}{x}$ etc. or writes 1 as $\log_a a$ **A1:** Uses **two** rules correctly to obtain correct log equation **M1:** Removes logs correctly to obtain an equation connecting x and a **A1:** Correct simplified answer Note that some candidates interpret $\log_a 27 - 1$ as $\log_a (27-1)$. This can score a maximum of 1 out of 4 if they have $\log x + \log 3 = \log 3x$ Note that $\log_a x + \log_a 3 = \log_a 27 - 1$ so $\frac{\log_a 3x}{\log_a 27} = -1 \Rightarrow \frac{3x}{27} = a^{-1}$ etc. scores **M1A0M0A0** Note that $\log_a x + \log_a 3 = \log_a 27 - 1$ so $\frac{\log_a x \log_a 3}{\log_a 27} = -1 \Rightarrow \frac{3x}{27} = a^{-1}$ etc. scores **no marks**	
(ii)	**M1:** Recognise and attempt to solve quadratic **A1:** Obtain both 3 and 4 (Both correct implies M1A1) **dM1:** Uses powers **correctly** to find a value for y **(Dependent on first method mark)** **A1:** Both values correct	