

Sequences & Series

Key Formulae

FB formula booklet

Arithmetic Sequences/Series:

$$U_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n(2a + (n-1)d) \text{ FB}$$

Geometric Sequences/Series:

$$U_n = ar^{n-1}$$

$$S_n = a \left(\frac{1-r^n}{1-r} \right) \text{ FB}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1 \text{ FB}$$

Key Concepts

Arithmetic

- General notation:

$$S_n = a + (a+d) + (a+2d) + \dots + a + (n-1)d$$

↑ first term
↑ common difference

Geometric

- General notation:

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

↑ first term
↑ common ratio

- $|r| < 1 \Rightarrow S_n$ is convergent as $n \rightarrow \infty$
- $|r| > 1 \Rightarrow S_n$ is divergent as $n \rightarrow \infty$
- $r < 0 \Rightarrow$ the sequence is alternating

Recurrence Relations

- A recurrence relation provides a rule to generate the next term given the current term.

eg $U_{n+1} = f(U_n)$

- If the terms repeat (ie $U_{n+k} = U_n$ for all $n \in \mathbb{N}$) the sequence is periodic.

eg 2, 8, 7, 2, 8, 7, 2, ... is periodic.

- The order or period is the number of terms required for the sequence to repeat.

Sigma Notation

- The sigma symbol, Σ , is an instruction to sum.
- The specific instruction is

$$\sum_{r=1}^n f(r) = f(1) + f(2) + f(3) + \dots + f(n)$$