Paper 1: Core Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	$\frac{1}{(r+1)(r+3)} \equiv \frac{A}{(r+1)}+\frac{B}{(r+3)} \Rightarrow A=...,\ B=...$	M1	3.1a
	$\sum_{r=1}^{n}\frac{1}{(r+1)(r+3)}=$ $\frac{1}{2\times2}-\frac{1}{2\times4}+\frac{1}{2\times3}-\frac{1}{2\times5}+...+\frac{1}{2n}-\frac{1}{2(n+2)}+\frac{1}{2(n+1)}-\frac{1}{2(n+3)}$	M1	2.1
	$=\frac{1}{4}+\frac{1}{6}-\frac{1}{2(n+2)}-\frac{1}{2(n+3)}$	A1	2.2a
	$=\frac{5(n+2)(n+3)-6(n+3)-6(n+2)}{12(n+2)(n+3)}$	M1	1.1b
	$=\frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
		(5)	
	Alternative by induction: $n=1\Rightarrow\frac{1}{8}=\frac{a+b}{12\times3\times4},\ n=2\Rightarrow\frac{1}{8}+\frac{1}{15}=\frac{2(2a+b)}{12\times4\times5}$ $a+b=18,\ 2a+b=23\Rightarrow a=...,\ b=...$	M1	3.1a
	Assume true for $n=k$ so $\sum_{r=1}^{k}\frac{1}{(r+1)(r+3)}=\frac{k(5k+13)}{12(k+2)(k+3)}$		
	$\sum_{r=1}^{k+1}\frac{1}{(r+1)(r+3)}=\frac{k(5k+13)}{12(k+2)(k+3)}+\frac{1}{(k+2)(k+4)}$	M1	2.1
	$\frac{k(5k+13)}{12(k+2)(k+3)}+\frac{1}{(k+2)(k+4)}=\frac{k(5k+13)(k+4)+12(k+3)}{12(k+2)(k+3)(k+4)}$	A1	2.2a
	$=\frac{5k^3+33k^2+52k+12k+36}{12(k+2)(k+3)(k+4)}=\frac{(k+1)(k+2)(5k+18)}{12(k+2)(k+3)(k+4)}$	M1	1.1b
	$=\frac{(\underline{k+1})(5(\underline{k+1})+13)}{12(\underline{k+1}+2)(\underline{k+1}+3)}$ So true for $n=k+1$ So $\sum_{r=1}^{n}\frac{1}{(r+1)(r+3)}=\frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
		(5)	
			(5 marks)

A1: Correct area of the glass following fully correct working. **Do not award for the correct answer following incorrect working.**
M1: Subtracts their area of the glass from their area of the rectangle, as long as it does not give a negative area
A1: awrt 0.155 or awrt $0.155\,\text{m}^2$ (If the units are stated they must be correct)

Note: Using a calculator to find the area scores a maximum of B1M0M0A0M0A0M1A1

Question	Scheme	Marks	AOs
4	$\frac{1}{(r+1)(r+2)(r+3)} \equiv \frac{A}{r+1}+\frac{B}{r+2}+\frac{C}{r+3} \Rightarrow A=\ldots,\ B=\ldots,\ C=\ldots$ $\left(\text{NB } A=\frac{1}{2}\quad B=-1\quad C=\frac{1}{2}\right)$	M1	3.1a
	$r=0$ $\quad \frac{1}{2}\left[\frac{1}{1}-\frac{2}{2}+\frac{1}{3}\right]$ or $\frac{1}{2\ 1}-\frac{1}{2}+\frac{1}{2\ 3}$ or $\frac{1}{2}-\frac{1}{2}+\frac{1}{6}$	M1	2.1
	$r=1$ $\quad \frac{1}{2}\left[\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right]$ or $\frac{1}{2\ 2}-\frac{1}{3}+\frac{1}{2\ 4}$ or $\frac{1}{4}-\frac{1}{3}+\frac{1}{8}$		
	$r=n-1$ $\quad \frac{1}{2}\left[\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right]$ or $\frac{1}{2\ n}-\frac{1}{n+1}+\frac{1}{2\ n+2}$ or $\frac{1}{2n}-\frac{1}{n+1}+\frac{1}{2n+4}$		
	$r=n$ $\quad \frac{1}{2}\left[\frac{1}{n+1}-\frac{2}{n+2}+\frac{1}{n+3}\right]$ or $\frac{1}{2\ n+1}-\frac{1}{n+2}+\frac{1}{2\ n+3}$ or $\frac{1}{2n+2}-\frac{1}{n+2}+\frac{1}{2n+6}$		
	$\frac{1}{2}-\frac{1}{2}+\frac{1}{4}+\frac{1}{2(n+2)}-\frac{1}{n+2}+\frac{1}{2(n+3)}$ or $\frac{1}{4}-\frac{1}{2(n+2)}+\frac{1}{2(n+3)}$	A1	1.1b
	$=\frac{n^2+5n+6+2n+6-4n-12+2n+4}{4(n+2)(n+3)}$	M1	1.1b
	$=\frac{(n+1)(n+4)}{4(n+2)(n+3)}$	A1	2.2a
		(5)	
		(5 marks)	

Question	Scheme	Marks	AOs
4(a)	A complete attempt to find the sum of the cubes of the first *n* odd numbers using three of the standard summation formulae. Attempts to find $\sum(2r+1)^3$ or $\sum(2r-1)^3$ by expanding and using summation formulae	M1	3.1a
	$\sum_{r=1}^{n}(2r-1)^3=\sum_{r=1}^{n}(8r^3-12r^2+6r-1)=8\sum_{r=1}^{n}r^3-12\sum_{r=1}^{n}r^2+6\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$ or $\sum_{r=0}^{n-1}(2r+1)^3=\sum_{r=0}^{n-1}(8r^3+12r^2+6r+1)=8\sum_{r=0}^{n-1}r^3+12\sum_{r=0}^{n-1}r^2+6\sum_{r=0}^{n-1}r+\sum_{r=0}^{n-1}1$	M1	1.1b
	$=8\frac{n^2}{4}(n+1)^2-12\frac{n}{6}(n+1)(2n+1)+6\frac{n}{2}(n+1)-n$ or $=8\frac{(n-1)^2}{4}(n)^2+12\frac{(n-1)}{6}(n)(2n-1)+6\frac{(n-1)}{2}(n)+n$	M1 A1	1.1b 1.1b
	Multiplies out to achieve a correct intermediate line for example $n\ \ n+1\ \ 2n^2-2n+1\ \ -n=2n^4-2n^3+n^2+2n^3-2n^2+n-n$ $2n^4+4n^3+2n^2-4n^3-6n^2-2n+3n^2+3n-n$ leading to $=n^2\left(2n^2-1\right)$ cso *	A1 *	2.1
		(5)	
(b)	$\sum_{r=n}^{n+9}(2r-1)^3=\sum_{r=1}^{n+9}(2r-1)^3-\sum_{r=1}^{n-1}(2r-1)^3$ $=(n+9)^2\left(2(n+9)^2-1\right)-(n-1)^2\left(2(n-1)^2-1\right)=99800$ or $\sum_{r=n+1}^{n+10}(2r-1)^3=\sum_{r=1}^{n+10}(2r-1)^3-\sum_{r=1}^{n}(2r-1)^3$ $=(n+10)^2\left(2(n+10)^2-1\right)-(n)^2\left(2n^2-1\right)=99800$ or $\sum_{r=n-9}^{n}(2r-1)^3=\sum_{r=1}^{n}(2r-1)^3-\sum_{r=1}^{n-10}(2r-1)^3$ $=(n)^2\left(2(n)^2-1\right)-(n-10)^2\left(2(n-10)^2-1\right)=99800$	M1	3.1a
	$80n^3+960n^2+5820n-86760=0$ or $80n^3+1200n^2+7980n-79900=0$ or $80n^3-1200n^2+7980n-119700=0$	A1	1.1b
	Solves cubic equation	dM1	1.1b

	Achieves $n=6$ and the smallest number as 11 or Achieves $n=5$ and the smallest number as 11 or Achieves $n=15$ and the smallest number as 11	A1	2.3
		(4)	
			(9 marks)

Notes:

(a)

M1: A complete attempt to find the sum of the cubes of *n* odd numbers using three of the standard summation formulae.

M1: Expands $\sum_{r=1}^{n}(2r-1)^3$ or $\sum_{r=0}^{n-1}(2r+1)^3$ and splits into fours appropriate sums.

M1: Applies the result for at least three summations $\sum_{r=0}^{n-1}r^3, \sum_{r=0}^{n-1}r^2, \sum_{r=0}^{n-1}r$ and $\sum_{r=0}^{n-1}1$ or $\sum_{r=1}^{n}r^3, \sum_{r=1}^{n}r^2, \sum_{r=1}^{n}r$ and $\sum_{r=1}^{n}1$ as appropriate to their expansion provided that there is an attempt at cubing some values.

A1: Correct unsimplified expression.

A1 *: Multiplies out to achieve a correct intermediate expression which clearly leads to the correct expression. cso

Special case: If uses $\sum_{r=1}^{n}(2r+1)^3$ leading to $=8\frac{n^2}{4}(n+1)^2+12\frac{n}{6}(n+1)(2n+1)+6\frac{n}{2}(n+1)+n$ max score is M1 M0 M1 A1 A0

(b)

M1: Uses the answer to part (a) to find the sum of the cubes of the first $N+10$ odd numbers minus the sum of the first *N* odd numbers and sets equal to 99800 or equivalent.

A1: Correct simplified cubic equation.

dM1: Uses their calculator to solve their cubic equation, dependent on previous method mark.

A1: cao

Question	Scheme	Marks	AOs
4(a)	Applies $ln\left(\frac{r+1}{r-1}\right) = ln(r+1) - ln(r-1)$ to the problem in order to apply differences.	M1	3.1a
	$\sum_{r=2}^{n}(ln(r+1) - ln(r-1))$ $= (ln(3) - ln(1)) + (ln(4) - ln(2)) + (ln(5) - ln(3)) + \ldots$ $+(ln(n) - ln(n-2)) + (ln(n+1) - ln(n-1))$	dM1	1.1b
	$ln(n) + ln(n+1) - ln\,2$	A1	1.1b
	$ln\left(\frac{n(n+1)}{2}\right)$ * cso	A1 *	2.1
		(4)	
(b)	$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right) = \sum_{r=2}^{100} ln\left(\frac{r+1}{r-1}\right) - \sum_{r=2}^{50} ln\left(\frac{r+1}{r-1}\right)$ $= ln\left(\frac{100 \times 101}{2}\right) - ln\left(\frac{50 \times 51}{2}\right)$	M1	1.1b
	$\sum_{r=51}^{100} ln\left(\frac{r+1}{r-1}\right)^{35} = 35\,ln\left(\frac{100\times101}{2} \div \frac{50\times51}{2}\right)$	M1	3.1a
	$= 35\,ln\left(\frac{202}{51}\right)$	A1	1.1b
		(3)	
		(7 marks)	

Notes:

(a)

M1: Uses the subtraction laws of logs to start the method of differences process.

dM1: Demonstrates the method of differences process, should have a minimum of e.g $r = 2$, $r = 3$, $r = 4$, $r = n - 1$ and $r = n$ shown -- enough to establish *at least one cancelling term* and *all non-disappearing terms* though the latter may be implied by correct extraction if only the first few cases are shown. Allow this mark if an extra term for $r = 1$ has been included.

A1: Correct terms that do not cancel - must not contradict their list of terms so e.g. if $r = 1$ was included, then A0A0 follows. The ln 1 may be included for this mark.

A1*: Achieves the printed answer, with no errors or omissions **and** must have had a complete list (as per dM1) before extraction (but condone missing brackets on ln terms). If working with r throughout, they must replace by n to gain the last A, but all other marks are available.

NB For attempts at combining log terms instead of using differences, full marks may be awarded for the equivalent steps, but attempts that do not make progress in combining terms will score no marks.

(b) Condone a bottom limit of 0 or 1 being used throughout part (b).

M1: Attempts to split into (the sum up to 100) – (the sum up to k) where k is 49, 50 or 51 **and** apply the result of (a) in some way. Condone slips with the power.

M1: Having attempted to apply (a), uses difference and power log laws correctly to reach an expression of the required form.

A1: Correct answer**.** Accept equivalents in required form, such as $35\,ln\frac{5050}{1275}$

Question	Scheme	Marks	AOs
7(a)	All the even terms are positive and all the odd ones are negative. or $\sum_{r=1}^{2n}(-1)^r \mathrm{f}(r) = -\mathrm{f}(1)+\mathrm{f}(2)-\mathrm{f}(3)+\mathrm{f}(4)-\ldots$	**M1**	2.4
	$\sum_{r=1}^{2n}(-1)^r \mathrm{f}(r) = -\mathrm{f}(1)+\mathrm{f}(2)-\mathrm{f}(3)+\mathrm{f}(4)-\ldots-\mathrm{f}(2n-1)+\mathrm{f}(2n)$ $= \mathrm{f}(2)+\mathrm{f}(4)+\ldots+\mathrm{f}(2n)-\left(\mathrm{f}(1)+\mathrm{f}(3)+\ldots+\mathrm{f}(2n-1)\right)$ $= \sum_{r=1}^{n}\left(\mathrm{f}(2r)-\mathrm{f}(2r-1)\right)$ *	**A1***	3.1a
		(2)	
(b)	$\sum_{r=1}^{2n} r\left((-1)^r+2r\right)^2 = \sum_{r=1}^{2n} r\left(1+4r(-1)^r+4r^2\right)$	**M1**	2.1
	$= \frac{1}{2}(2n)(2n+1)+4\frac{(2n)^2}{4}(2n+1)^2+4\sum_{r=1}^{2n}(-1)^r r^2$	**M1**	1.1b
	$\sum_{r=1}^{2n}(-1)^r r^2 = \sum_{r=1}^{n}\left((2r)^2-(2r-1)^2\right)$	**M1**	3.1a
	$\sum_{r=1}^{2n} r\left((-1)^r+2r\right)^2 = n(2n+1)+4n^2(2n+1)^2+\underline{4\left(4\frac{n}{2}(n+1)-n\right)}$	**B1**	1.1b
	$= n(2n+1)+4\left(n(2n+1)\right)^2+4n(2n+1)$ $= n(2n+1)\left(1+4n(2n+1)+4\right)$	**dM1**	2.1
	$= n(2n+1)\left(8n^2+4n+5\right)$ *	**A1***	1.1b
		(6)	
(c)	$\sum_{r=14}^{50} r\left((-1)^r+2r\right)^2 = \sum_{r=1}^{50} r\left((-1)^r+2r\right)^2 - \sum_{r=1}^{13} r\left((-1)^r+2r\right)^2$	**M1**	1.1b
	$= \sum_{r=1}^{50} r\left((-1)^r+2r\right)^2 - \sum_{r=1}^{12} r\left((-1)^r+2r\right)^2 - 13\times 25^2$ or $= \sum_{r=1}^{50} r\left((-1)^r+2r\right)^2 - \sum_{r=1}^{14} r\left((-1)^r+2r\right)^2 + 14\times 29^2$	**M1**	3.1a
	$= 25\times 51\times 5105 - 6\times 13\times 317 - 13\times 25^2$ $\left(= 6508875 - 24726 - 8125\right)$ or $25\times 51\times 5105 - 7\times 15\times 425 + 14\times 29^2$ $\left(= 6508875 - 44625 + 11774\right)$	**M1**	2.1
	$= 6476024$	**A1**	1.1b
		(4)	
			(12 marks)

Question	Scheme	Marks	AOs
6(a)	$(3r-2)^2 = 9r^2 - 12r + 4$	B1	1.1b
	$\sum_{r=1}^{n}(9r^2 - 12r + 4) = 9\times\frac{1}{6}n(n+1)(2n+1) - 12\times\frac{1}{2}n(n+1) + ...$	M1	2.1
	$= 9\times\frac{1}{6}n(n+1)(2n+1) - 12\times\frac{1}{2}n(n+1) + 4n$	A1	1.1b
	$= \frac{1}{2}n\left[3(n+1)(2n+1) - 12(n+1) + 8\right]$	dM1	1.1b
	$= \frac{1}{2}n\left[6n^2 - 3n - 1\right]$*	A1*	1.1b
		(5)	
(b)	$\sum_{r=5}^{n}(3r-2)^2 = \frac{1}{2}n(6n^2 - 3n - 1) - \frac{1}{2}(4)\left(6(4)^2 - 3\times4 - 1\right)$	M1	3.1a
	$\sum_{r=1}^{28} r\cos\left(\frac{r\pi}{2}\right) = 0 - 2 + 0 + 4 + 0 - 6 + 0 + 8 + 0 - 10 + 0 + 12 + ...$	M1	3.1a
	$3n^3 - \frac{3}{2}n^2 - \frac{1}{2}n - 166 + 103\times14 = 3n^3$ $\Rightarrow 3n^2 + n - 2552 = 0$	A1	1.1b
	$\Rightarrow 3n^2 + n - 2552 = 0 \Rightarrow n = ...$	M1	1.1b
	$n = 29$	A1	2.3
		(5)	
			(10 marks)

Notes

(a) Do not allow proof by induction (but the B1 could score for $(3r-2)^2 = 9r^2 - 12r + 4$ if seen)

B1: Correct expansion
M1: **Substitutes** at least one of the standard formulae into their expanded expression
A1: Fully correct expression
dM1: Attempts to factorise $\frac{1}{2}n$ having used at least one standard formula correctly. Dependent on the first M mark and dependent on there being an *n* in all terms.
A1*: Obtains the printed result with no errors seen
(b)
M1: Uses the result from part (a) by substituting *n* = 4 and subtracts from the result in (a) in order to find the first sum in terms of *n*.
M1: Identifies the periodic nature of the second sum by calculating terms. This may be implied by a sum of 14.
A1: Uses their sum and the given result to form the correct 3 term quadratic
M1: Solves their three term quadratic to obtain at least one value for *n*
A1: Obtains *n* = 29 only or obtains *n* = 29 and $n = -\frac{88}{3}$ and rejects the $-\frac{88}{3}$

Question	Scheme	Marks	AOs
6. (a)	(mean $=\bar{x}=$) $\frac{1}{n}\sum_{r=1}^{n}(7+3r)$	M1	1.1a
	$\sum_{r=1}^{n}(7+3r)=\left(7\sum_{r=1}^{n}1+3\sum_{r=1}^{n}r=\right)7n+3\frac{n}{2}(n+1)$	M1	1.1b
	$\bar{x}=7+\frac{3}{2}(n+1)=\frac{14+3n+3}{2}=\frac{1}{2}(3n+17)\,*$	A1*	2.1
		(3)	
(b)	Correct overall strategy to find the variance or standard deviation. This must include: • An attempt to find the mean • An attempt at $\sum(7+3r)^2$ as part of their formula (however poor, or if stated and followed by a value or if used with incorrect limits). • An attempt at either variance formula with their mean (allow slips in the formula)	M1	3.1a
(Mean)	mean $(=\bar{x})=136$	B1	1.1b
(Sum)	Way1: $\sum_{r=1}^{n}(7+3r)^2=\sum_{r=1}^{n}(49+42r+9r^2)$ $=\underline{\underline{49n}}+\underline{42\times\frac{1}{2}n(n+1)}+\underline{9\times\frac{1}{6}n(n+1)(2n+1)}$ Way 2: $\sum_{r=1}^{n}\left(x_i-\bar{x}\right)^2=\sum_{r=1}^{n}\left(7+3r-\text{"136"}\right)^2=a\sum_{r=1}^{n}r^2+b\sum_{r=1}^{n}r+c\sum_{r=1}^{n}1$ $=\underline{9\times\frac{1}{6}n(n+1)(2n+1)}-\text{"774"}\times\underline{\frac{1}{2}n(n+1)}+\underline{\underline{\text{"16641"}n}}$	$\underline{\text{M1}}$ $\underline{\underline{\text{B1}}}$	1.1b 1.1b
(Variance/standard deviation)	Way 1: $=\frac{\text{"2032690"}}{85}-136^2=\ldots$ or $\frac{\text{"2032690"}}{84}-\frac{85}{84}\times136^2=\ldots$ Way 2: $=\frac{\text{"460530"}}{85}=\ldots$ or $\frac{\text{"460530"}}{84}=\ldots$ (using sample standard deviation).	M1	1.1b
	So s.d = $\sqrt{5418}=73.6$ (g) Accept 74.0 (g) if sample s.d. used	A1	1.1b
		(6)	
	(9 marks)		

Question	Scheme	Marks	AOs
5(a)	Volume = $r\times(r+1)\times(r+2)$	B1	1.1b
	A complete method for finding the total volume of *n* blocks and expressing it in sigma notation. This can be implied by later work. $\sum_{r=1}^{n}(r^3+3r^2+2r)$	M1	3.1b
	$V=\frac{1}{4}n^2(n+1)^2+3\times\frac{1}{6}n(n+1)(2n+1)+2\times\frac{n}{2}(n+1)$	M1	2.1
	$V=\frac{1}{4}n(n+1)[n(n+1)+2(2n+1)+4]$	dM1	1.1b
	$V=\frac{1}{4}n(n+1)[n^2+5n+6]$ $\Rightarrow V=\frac{1}{4}n(n+1)(n+2)(n+3)*$	A1*	1.1b
		(5)	
(b)	Sets $\frac{1}{4}n(n+1)(n+2)(n+3)=n^4+6n^3-11710$ $\frac{1}{4}n^4+\frac{3}{2}n^3+\frac{11}{4}n^2+\frac{3}{2}n=n^4+6n^3-11710$ simplifies $(3n^4+18n^3-11n^2-6n-46840=0)$ and solves $n=\ \ldots$	M1	1.1b
	There are 10 blocks or n = 10	A1	3.2a
		(2)	
		(7 marks)	

Notes:

(a)

B1: Correct volume of a block

M1: Expressing the total volume of all *n* blocks as a series in terms of *r*, r^2 and r^3

M1: Substitutes at least one of the standard formulae into their volume.

dM1: Attempts to factorise $\frac{1}{4}n(n+1)$ having used at least one standard formula correctly. Each term must contain a factor of $n(n+1)$

A1*: Obtains the printed result with no errors seen, no bracketing errors and following from $V=\frac{1}{4}n(n+1)[n^2+5n+6]$ o.e.

Note: Going from $\frac{1}{4}n(n^3+6n^2+11n+6)$ to $\frac{1}{4}n(n+1)(n+2)(n+3)$ with no reasoning shown scores **dM0 A0**

(b)

Question	Scheme	Marks	AOs
3(a)	$(5r-2)^2 = 25r^2 - 20r + 4$	B1	1.1b
	$\sum_{r=1}^{n} 25r^2 - 20r + 4 = \frac{25}{6}n(n+1)(2n+1) - \frac{20}{2}n(n+1) + ...$	M1	2.1
	$= \frac{25}{6}n(n+1)(2n+1) - \frac{20}{2}n(n+1) + 4n$	A1	1.1b
	$= \frac{1}{6}n\left[25(2n^2+3n+1) - 60(n+1) + 24\right]$	dM1	1.1b
	$= \frac{1}{6}n\left[50n^2 + 15n - 11\right]$	A1	1.1b
		(5)	
(b)	$\frac{1}{6}k\left[50k^2 + 15k - 11\right] = 94k^2$	M1	1.1b
	$50k^3 - 549k^2 - 11k = 0$ or $50k^2 - 549k - 11 = 0$	A1	1.1b
	$(k-11)(50k+1) = 0 \Rightarrow k = ...$	M1	1.1b
	k = 11(only)	A1	2.3
		(4)	
			(9 marks)

Notes

(a)
B1: Correct expansion
M1: Substitutes at least one of the standard formulae into their expanded expression
A1: Fully correct expression
dM1: Attempts to factorise $\frac{1}{6}n$ having used at least one standard formula correctly. Dependent on the first M mark.
A1: Obtains the correct expression or the correct values of *a*, *b* and *c*
(b)
M1: Uses their result from part (a) and sets equal to $94k^2$ and attempt to expand and collect terms.
A1: Correct cubic or quadratic
M1: Attempts to solve their 3TQ or cubic equation
A1: Identifies the correct value of *k* with no other values offered

Question	Scheme	Marks	AOs
5(a)	$\sum_{r=1}^{n}\left(3r^2-17r-25\right)=3\times\frac{n}{6}(n+1)(2n+1)-17\times\frac{1}{2}n(n+1)-\ldots$	M1	1.1b
	$=3\times\frac{n}{6}(n+1)(2n+1)-17\times\frac{1}{2}n(n+1)-25n$	A1	1.1b
	$=n\left(\frac{1}{2}\left(2n^2+3n+1\right)-\frac{17}{2}(n+1)-25\right)$ or $=\frac{n}{2}\left(\left(2n^2+3n+1\right)-17(n+1)-50\right)$	M1	1.1b
	$=n\left(n^2-7n-33\right)$ cso (so $A = 7$ and $B = 33$)	A1 cso	2.1
		(4)	
(b)	$\sum_{r=1}^{3k} r\tan\left(60r\right)°$ $=\tan\left(60\right)°+2\tan\left(120\right)°+3\tan\left(180\right)°+4\tan\left(240\right)°+5\tan\left(300\right)°$ $+6\tan\left(360\right)°+$ $=\left(\sqrt{3}-2\sqrt{3}+0\right)+\left(4\sqrt{3}-5\sqrt{3}+0\right)+\ldots$	M1	3.1a
	Since tan has period 180° we see $\tan\left(60r\right)°$ repeats every three terms and **each group of three terms results in $-\sqrt{3}$ as a sum**, so with ***k* groups** of terms the sum is $-k\sqrt{3}$	A1	2.4
		(2)	
(c)	$\sum_{r=5}^{n}\left(3r^2-17r-25\right)=\sum_{r=1}^{n}\left(3r^2-17r-25\right)-\sum_{r=1}^{4}\left(3r^2-17r-25\right)$	M1	1.1b
	$=n\left(n^2-7n-33\right)-4\left(4^2-7\times4-33\right)$ $\left(=n\left(n^2-7n-33\right)+180\right)$	A1	1.1b
	$\sum_{r=6}^{3n} r\tan\left(60r\right)°=-n\sqrt{3}+2\sqrt{3}$ (allow for $-n\sqrt{3}--2\sqrt{3}$)	B1	2.2a
	$\Rightarrow n\left(n^2-7n-33\right)+180=15\left[-n\sqrt{3}+2\sqrt{3}\right]^2$ $\Rightarrow n^3-7n^2-33n+180=15\left(3n^2-12n+12\right)$ $\Rightarrow n^3-52n^2+147n=0$	M1	3.1a
	$\Rightarrow n^3-52n^2+147n=0\Rightarrow n=\ldots$	M1	1.1b
	But need $n > 5$ for sums to be valid, so $n = 49$ (allow if $n = 0$ also given but $n = 3$ must be rejected).	A1	2.3
		(6)	
(12 marks)			

Question	Scheme	Marks	AOs
8(a)	$(2r-1)^2 = 4r^2 - 4r + 1$	B1	1.1b
	$\sum_{r=1}^{n}(2r-1)^2 = 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$ $= 4\frac{n}{6}(n+1)(2n+1) - 4\frac{n}{2}(n+1) + n$	M1 A1	1.1b 1.1b
	$= \frac{n}{3}\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$ Or $= n\left[\frac{2}{3}(n+1)(2n+1) - 2(n+1) + 1\right]$	dM1	1.1b
	$\left\{\frac{n}{3}\left(4n^2 + 6n + 2 - 6n - 6 + 3\right)\right\}$ $= \frac{n}{3}\left(4n^2 - 1\right)$ cso	A1	2.1
		(5)	
(b)	$\sum_{r=51}^{500}(2r-1)^2$	B1	3.1a
	$\sum_{r=51}^{500}(2r-1)^2 = \sum_{r=1}^{500}(2r-1)^2 - \sum_{r=1}^{50}(2r-1)^2$ $= \frac{500}{3}\left(4(500)^2 - 1\right) - \frac{50}{3}\left(4(50)^2 - 1\right)$ $\{ = 166666500 - 166650\}$	M1	1.1b
	166 499 850	A1	1.1b
		(3)	
			(8 marks)

Notes:

(a)

B1: Correct expanded expression.

M1: Substitutes at least one of the standard formulae into their expanded expression.

A1: Fully correct unsimplified expression.

dM1: Dependent on previous method. Attempts to factorises out *n*. Must have a *n* in every term. Condone a slip with one term as long as the intention is clear.

A1: Achieves the correct answer, with a correct intermediate line of working. cso

Note If uses $\sum 1 = 1$ scores B1 M1 A0 M0 A0

An attempt at proof by induction may score B1 only

(b)

B1: Correct summation formula for the sum of the squares of all positive odd three-digit integers including limits. This can be implied by later work.