

5.

In this question you should show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

- (a) show that the profit for Year 3 will be £23 328 (1)

- (b) find the first year when the yearly profit will exceed £65 000 (3)

- (c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000 (2)

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12. The value, £ V , of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \quad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

- (a) (i) find p to 4 decimal places,
- (ii) show that A is approximately 24 800 (4)

- (b) With reference to the model, interpret
 - (i) the value of the constant A ,
 - (ii) the value of the constant p . (2)

Using the model,

- (c) find the year during which the value of the car first exceeds £100 000 (4)



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7. In a simple model, the value, £ V , of a car depends on its age, t , in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A , a possible equation linking V with t . (4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information. (2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B . (1)

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11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre. After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds, (2)

(b) show that her estimated time, in minutes, to run the r th kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1.05^{r-4} \tag{1}$$

(c) estimate the total time, in minutes and seconds, that she will take to complete the race. (4)

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5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

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9. In the first month after opening, a mobile phone shop sold 300 phones. A model for future sales assumes that the number of phones sold will increase by 5% per month, so that 300×1.05 will be sold in the second month, 300×1.05^2 in the third month, and so on.

Using this model, calculate

- (a) the number of phones sold in the 24th month, (2)

- (b) the total number of phones sold over the whole 24 months. (2)

This model predicts that, in the N th month, the number of phones sold in that month exceeds 3000 for the first time.

- (c) Find the value of N . (3)



12. A business is expected to have a yearly profit of £275 000 for the year 2016. The profit is expected to increase by 10% per year, so that the expected yearly profits form a geometric sequence with common ratio 1.1

(a) Show that the difference between the expected profit for the year 2020 and the expected profit for the year 2021 is £40 300 to the nearest hundred pounds.

(3)

(b) Find the first year for which the expected yearly profit is more than one million pounds.

(4)

(c) Find the total expected profits for the years 2016 to 2026 inclusive, giving your answer to the nearest hundred pounds.

(3)



9. The resident population of a city is 130 000 at the end of Year 1

A model predicts that the resident population of the city will increase by 2% each year, with the populations at the end of each year forming a geometric sequence.

(a) Show that the predicted resident population at the end of Year 2 is 132 600 (1)

(b) Write down the value of the common ratio of the geometric sequence. (1)

The model predicts that Year N will be the first year which will end with the resident population of the city exceeding 260 000

(c) Show that

$$N > \frac{\log_{10} 2}{\log_{10} 1.02} + 1 \quad (4)$$

(d) Find the value of N . (1)

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11. Wheat is to be grown on a farm.

A model predicts that the mass of wheat harvested on the farm will increase by 1.5% per year, so that the mass of wheat harvested each year forms a geometric sequence.

Given that the mass of wheat harvested during year one is 6000 tonnes,

- (a) show that, according to the model, the mass of wheat harvested on the farm during year 4 will be approximately 6274 tonnes. (2)

During year N , according to the model, there is predicted to be more than 8000 tonnes of wheat harvested on the farm.

- (b) Find the smallest possible value of N . (5)

It costs £5 per tonne to harvest the wheat.

- (c) Assuming the model, find the total amount that it would cost to harvest the wheat from year one to year 10 inclusive. Give your answer to the nearest £1000. (3)

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6. Each year Lin pays into a savings scheme. In year 1 she pays in £600. Her payments then increase by £80 a year, so that she pays £680 into the savings scheme in year 2, £760 in year 3 and so on. In year N , Lin pays £1000 into the savings scheme.

(a) Find the value of N . (2)

(b) Find the total amount that Lin pays into the savings scheme from year 1 to year 15 inclusive. (2)

Saima starts paying into a different savings scheme at the same time as Lin starts paying into her savings scheme.

In year 1 she pays in £ A . Her payments increase by £ A each year so that she pays £ $2A$ in year 2, £ $3A$ in year 3 and so on.

Given that Saima and Lin have each paid, in total, the same amount of money into their savings schemes after 15 years,

(c) find the value of A . (3)

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14. A new mineral has been discovered and is going to be mined over a number of years.

A model predicts that the mass of the mineral mined each year will decrease by 15% per year, so that the mass of the mineral mined each year forms a geometric sequence.

Given that the mass of the mineral mined during year 1 is 8000 tonnes,

- (a) show that, according to the model, the mass of the mineral mined during year 6 will be approximately 3550 tonnes. (2)

According to the model, there is a limit to the total mass of the mineral that can be mined.

- (b) With reference to the geometric series, state why this limit exists. (1)

- (c) Calculate the value of this limit. (2)

It is decided that a total mass of 40 000 tonnes of the mineral is required. This is going to be mined from year 1 to year N inclusive.

- (d) Assuming the model, find the value of N . (5)

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