AQA

AS

MATHEMATICS

7356/1

Paper 1

Mark scheme

June 2019

Version: 1.0 Final

I96A7356I/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

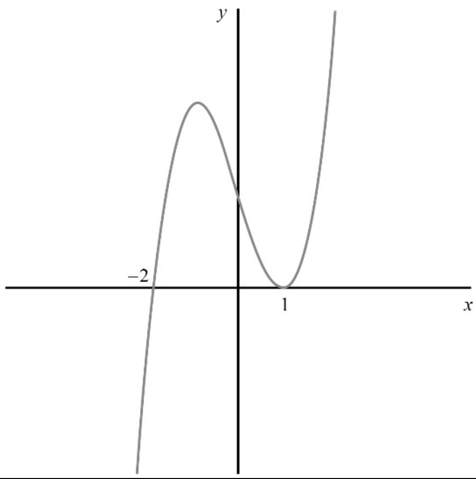
When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	1.1b	B1	4
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	2.3	B1	$n = 6$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Substitutes $x = -1$ or $x = 3$ into f(x) to obtain one equation or uses identity eg $f(x) \equiv (x+1)(x-3)(ax+b)$	3.1a	M1	$x = -1$ $\quad -p - 3 + 8 + q = 0$ $x = 3$ $\quad 27p - 27 - 24 + q = 0$ $p = 2$ and $q = -3$
	Obtains two correct equations by substitution method ACF or obtains $a = 2$, $b = 1$	1.1b	A1	
	Solves to find p and q CAO	1.1b	A1	
3(b)	Uses inspection, division by quadratic factor or repeated division or finds third root $x = -\frac{1}{2}$ PI by $(x + \frac{1}{2})$	1.1a	M1	$(x+1)(x-3) = x^2 - 2x - 3$ $(x^2 - 2x - 3)(2x + 1)$ $(x+1)(x-3)(2x+1)$
	Completes factorisation	1.1b	A1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Multiplies by $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$	AO1.1a	M1	$\frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ $= \frac{\sqrt{18}+\sqrt{12}}{3-2}$ $\frac{\sqrt{18}+\sqrt{12}}{1}$ $= \sqrt{9 \times 2} + \sqrt{4 \times 3}$ $= 3\sqrt{2} + 2\sqrt{3}$
	Correctly evaluates denominator to get 3 – 2 or 1	AO1.1b	A1	
	Evaluates numerator, one term correct $\sqrt{18}$ *or* $\sqrt{12}$ or $3\sqrt{2}$ *or* $2\sqrt{3}$	AO1.1b	A1	
	Completes solution CAO	AO2.1	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Draws a correctly orientated cubic graph with a max and a min	1.1a	M1	$g(x) = 0$ at -2 and 1 (twice) 
	Shows that the curve meets x-axis at –2 and 1 Ignore an additional cutting of the axis	1.1b	A1	
	Deduces the graph touches the x-axis at 1	2.2a	B1	
5(b)	States correct lower region	2.5	B1	$x \le -2$
	Deduces that point value x = 1 solves the inequality	2.2a	B1	$x = 1$
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
6 (a)(i)	Uses identity to replace $\sin^2\theta$ with $(1 - \cos^2\theta)$ or uses $\sin\theta = \frac{\sqrt{3}}{2}$	1.2	M1	$6(1 - \cos^2\theta) + 5\cos\theta = 7$ $6\cos^2\theta - 5\cos\theta + 1 = 0$ $(2\cos\theta - 1)(3\cos\theta - 1) = 0$ $\cos\theta = \frac{1}{2}$
	Solves quadratic equation to get one solution $\cos\theta = \frac{1}{2}$. Or verifying using $\cos\theta = \frac{1}{2}$	1.1a	A1	
6 (a)(ii)	States any two correct solutions	1.1b	B1	$\theta = 60°, 300°$, Or $\cos\theta = \frac{1}{3}$ $\theta = 71°, 289°$
	States two additional correct solutions. Condone answers of 70.5 and 289.5 or greater accuracy. Ignore any additional answers outside the range but any additional answers inside range lose second B1	1.1b	B1	
6(b)	Writes down a set that is half the values given as their solutions in part (a) Accept 36° or 144°	2.2a	M1	$\theta = 30°, 150°, 35°, 145°$
	Writes down an additional set that is 180° more than the first set. Condone AWRT integer values	1.1b	A1F	210°, 330°, 215°, 325°
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Expands at least one bracket – must reach a quartic	3.1a	M1	$(2 + 3y)^4 = 2^4 + 4\times2^3\times(3y) + 6\times2^2\times(3y)^2 + 4\times2\times(3y)^3 + (3y)^4$ $= 16 + 96y + 216y^2 + 216y^3 + 81y^4$ $(2 - 3y)^4 = 2^4 + 4\times2^3\times(-3y) + 6\times2^2\times(-3y)^2 + 4\times2\times(-3y)^3 + (-3y)^4$ $= 16 - 96y + 216y^2 - 216y^3 + 81y^4$ $(2 + 3y)^4 + (2 - 3y)^4$ $= 32 + 432y^2 + 162y^4$ $y^2 \geq 0$ and $y^4 \geq 0$ for all y $(2 + 3y)^4 + (2 - 3y)^4 \geq 32$ for all y
	Obtains at least one correct expansion – not necessarily simplified	1.1b	A1	
	Obtains two correct expansions – not necessarily simplified	1.1b	A1	
	Combines their expansions to obtain a sum containing only even power terms.	1.1a	M1	
	Explains that y^2 and y^4 are always positive **or zero** (for $y \in \mathbb{R}$) Or finds minimum value using calculus and justifies this as not just a local minimum	2.4	E1	
	Reaches correct conclusion. Sets out a well-constructed mathematical argument. R1 can be awarded if E1 not given.	2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
8	Selects differentiation as the first step. At least one term correct	1.1a	M1	$y = 2x^5 + 5x^4 + 10x^3 - 8$ $\frac{dy}{dx} = 10x^4 + 20x^3 + 30x^2$
	Differentiates fully correctly	1.1b	A1	
	Equates their derivative to zero	1.1a	M1	$10x^2(x^2 + 2x + 3) = 0$
	States $x = 0$ is one solution or verifies $x = 0$ is a solution	1.1b	A1	$x = 0$ or $x^2 + 2x + 3 = 0$
	Deduces the quadratic factor has no real roots using discriminant, completing the square, using formula Or uses a sketch from their calculator Or finds roots of quartic but discounts non-real roots (only real root is x = 0)	2.2a	M1	discriminant $= b^2 - 4ac =4 - 12$ $= -8$ negative so no real solutions Only stationary point at (0, –8)
	Deduces that there are no further stationary points and concludes that (0, –8) is the only one.	2.1	R1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)	Decides to integrate $\frac{24}{x^3}$	3.1a	M1	$y = \int \frac{24}{x^3}\,\mathrm{d}x$
	Obtains correct integral with or without c	1.1b	A1	$= -\frac{12}{x^2} + c$
	Includes c and substitutes (2, 0) to calculate c or uses definite integral	1.1a	M1	$0 = -\frac{12}{2^2} + c$
	Evaluates c and states correct equation ACF Follow through only on sign error in integral (c = –3)	1.1b	A1F	$c = 3$ $y = -\frac{12}{x^2} + 3$
9(b)	Finds midpoint of AB	1.1b	B1	Midpoint of AB is (–4, 2) Gradient of AB $= \frac{8-(-4)}{-2-(-6)} = 3$ Gradient of perpendicular bisector $\frac{-1}{3}$ Equation is $y = \frac{-1}{3}x + \frac{2}{3}$ When $x = 2$, $y = \frac{-1}{3} \times 2 + \frac{2}{3} = 0$ So passes through (2,0) Gradient of curve at (2, 0) is $\frac{24}{2^3} = 3$ Gradient of normal is $\frac{-1}{3}$ So perpendicular bisector is the normal
	Finds gradient of AB (unsimplified)	1.1b	B1	
	Finds gradient of curve at (2, 0)	1.1b	B1	
	Uses perpendicular gradients property	3.1a	M1	
	Finds the correct equation of the perpendicular bisector using (-4, 2), **or** the correct equation of the normal using (2, 0) ACF PI by correct "m & c" pair	3.1a	M1	
	Completes rigorous argument to show that the perpendicular bisector is the normal. Must include showing normal and bisector have the same gradient and that (2, 0) lies on the bisector, or that (-4, 2) lies on the normal, or that the equations are identical.	2.1	R1	
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
10 (a)(i)	States A = 12	3.3	B1	A = 12
10 (a)(ii)	States B = 6	3.3	B1	B = 6
10 (a)(iii)	Applies formula using their A and B 2 sf or better provided their answer >12 and <24	1.1b	B1F	12 + 6 sin 58 = 17.1
10 (a)(iv)	Substitutes 17.4 into formula with their A and B	1.1a	M1	17.4 = 12 + 6 sin t sin t = 0.9 t = 64 second value of t is 116 So answer is 53 days
	Evaluates first value of t. AWFW 64 to 65	1.1b	A1	
	Evaluates second value of t *as* 180 – their first value ±1 Or Subtracts their first value from 90 and doubles	3.4	M1	
	Obtains final answer AWRT 50	3.2a	A1	
10 (a)(v)	Explains 360 days is not the same as a year. Must mention 360	3.5b	E1	Jude's model will repeat after 360 days but a year has 365 days.
10(b)	Explains that Anisa's model adjusts the repeating pattern to match the number of days in a year. Mark may be supported by response seen in part (a)(v)	3.5c	E1	Anisa's model will repeat after 365 days because of the fraction
	Total		**9**	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.1b	B1	2 ms^{-1}
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Ticks correct box	3.3	B1	T is directly proportional to m
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
13(a)	Integrates given velocity equation, with at least one term correct PI	3.4	M1	$s = \int_0^{10} 0.48t^2 - 0.024t^3\, dt$
	Obtains fully correct integral PI	1.1b	A1	$s = [0.16t^3 - 0.006t^4]_0^{10}$
	Substitutes correct limits to obtain correct answer. CAO. Condone omission of units.	1.1b	A1	$s = 100$ m
13(b)	Differentiates v (at least one term correct) PI	3.4	M1	$\frac{dv}{dt} = 0.96t - 0.072t^2$
	Equates their $\frac{dv}{dt}$ equation to zero PI	1.1a	M1	$0.96t - 0.072t^2 = 0$
	Finds correct non-zero value for t	1.1b	A1	$t = 0 \text{ or } t = \frac{40}{3}$
	Finds correct maximum speed 28.4 Condone exact answer $28\frac{4}{9}$. Condone omission of units.	1.1b	A1	When $t = \frac{40}{3}$ then $v = 28.4$ ms^{-1}
13(c)	Deduces lower critical value of t with correct associated inequality Follow through their value of t from part (b) provided $0 < t < 15$	2.2a	R1F	$\frac{40}{3} < t \leq 15$
	States $t \leq 15$	2.5	R1	
	Total		**9**	

14(a)	**Condone omission of units throughout this question** Calculates the magnitude of *AB* Ignore one sign error.	1.1a	M1	Distance = $\sqrt{(13-3)^2+(-22-2)^2}$ = 26
	Obtains correct distance.	1.1b	A1	
14(b)	Explains that A remains at rest implies resultant force = 0 and shows $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$ or shows addition of $\mathbf{F}_1$ and $\mathbf{F}_2$ and states that $\mathbf{F}_3$ is the opposite	2.4	E1	$(-2\mathbf{i}+4\mathbf{j})+(6\mathbf{i}-10\mathbf{j}) = 4\mathbf{i}-6\mathbf{j}$ so $\mathbf{F}_3$ is the opposite = $-4\mathbf{i} + 6\mathbf{j}$
14 (c)(i)	Calculates magnitude of given force **Or** uses Newton's second law to obtain $\boldsymbol{a}$ = 6.25**i** – 15**j**	1.1a	M1	$\lvert\boldsymbol{F}\rvert = 13$ 13 = 0.8a $a = 16.25$
	Completes calculation correctly AG	2.1	R1	
14 (c)(ii)	Uses appropriate suvat equation with a = 16.25 and their s used	3.3	M1	$26 = 0.5 \times 16.25 \times t^2$ $t^2 = 3.2$ $t = 1.8$
	Solves to find the correct value of time to 2sf	1.1b	A1	
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Models overall system as a single particle using Newton's second law, one side of equation correct. If two separate equations used, must eliminate T to obtain a single equation	3.3	M1	By $F = ma$, $11080 - 160 - 600 = 0.8(4m + m)$
	Obtains fully correct equation	1.1b	A1	$10320 = 4m$
	Obtains correct value for m	1.1b	A1	$m = 2580$
15(b)	Models either tractor or trailer separately using resistance force, T and *their* m **value**. Tractor: $11080 - 160 - T = 0.8 \times m$	3.3	M1	Trailer: $T - 600 = 0.8 \times 4 \times 2580$ $T = 8856$ newtons
	Obtains correct value for T using their m value Condone omission of units.	1.1b	A1F	
15(c)	Models trailer using only resistance force = ±600 and *their* $4m$ value to find a from Newton's second law. or finds s using energy.	3.4	M1	$-600 = 10320a$ $\Rightarrow a = -\frac{5}{86}$ m s^{-2}
	Finds the correct value of a or s (161.25)	1.1b	A1	18 km h^{-1} = 5 m s^{-1}
	Selects suitable suvat equation to find required time, using their calculated value for a or s and consistent units	3.4	M1	Using $v = u + at$: $2.5 = 5 + \left(-\frac{5}{86}\right)t$
	Obtains correct value for t **including units** CAO	1.1b	A1	Time taken, $t = 43$ seconds
	Total		**9**	