

## Paper 1: Core Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	$\frac{1}{(r+1)(r+3)} \equiv \frac{A}{(r+1)} + \frac{B}{(r+3)} \Rightarrow A = \dots, B = \dots$	M1	3.1a
	$\sum_{r=1}^n \frac{1}{(r+1)(r+3)} =$ $\frac{1}{2 \times 2} - \frac{1}{2 \times 4} + \frac{1}{2 \times 3} - \frac{1}{2 \times 5} + \dots + \frac{1}{2n} - \frac{1}{2(n+2)} + \frac{1}{2(n+1)} - \frac{1}{2(n+3)}$	M1	2.1
	$= \frac{1}{4} + \frac{1}{6} - \frac{1}{2(n+2)} - \frac{1}{2(n+3)}$	A1	2.2a
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{12(n+2)(n+3)}$	M1	1.1b
	$= \frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
	<b>(5)</b>		
	<b>Alternative by induction:</b> $n=1 \Rightarrow \frac{1}{8} = \frac{a+b}{12 \times 3 \times 4}, n=2 \Rightarrow \frac{1}{8} + \frac{1}{15} = \frac{2(2a+b)}{12 \times 4 \times 5}$ $a+b=18, 2a+b=23 \Rightarrow a = \dots, b = \dots$	M1	3.1a
	Assume true for $n=k$ so $\sum_{r=1}^k \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)}$		
	$\sum_{r=1}^{k+1} \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)}$	M1	2.1
	$\frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)} = \frac{k(5k+13)(k+4) + 12(k+3)}{12(k+2)(k+3)(k+4)}$	A1	2.2a
	$= \frac{5k^3 + 33k^2 + 52k + 12k + 36}{12(k+2)(k+3)(k+4)} = \frac{(k+1)(k+2)(5k+18)}{12(k+2)(k+3)(k+4)}$	M1	1.1b
	$= \frac{(k+1)(5(k+1)+13)}{12(\underline{k+1}+2)(\underline{k+1}+3)}$ So true for $n=k+1$	A1	1.1b
	So $\sum_{r=1}^n \frac{1}{(r+1)(r+3)} = \frac{n(5n+13)}{12(n+2)(n+3)}$		
	<b>(5)</b>		
<b>(5 marks)</b>			

**Question 1 notes:****Main Scheme**

**M1:** Valid attempt at partial fractions

**M1:** Starts the process of differences to identify the relevant fractions at the start and end

**A1:** Correct fractions that do not cancel

**M1:** Attempt common denominator

**A1:** Correct answer

**Alternative by Induction:**

**M1:** Uses  $n = 1$  and  $n = 2$  to identify values for  $a$  and  $b$

**M1:** Starts the induction process by adding the  $(k + 1)^{\text{th}}$  term to the sum of  $k$  terms

**A1:** Correct single fraction

**M1:** Attempt to factorise the numerator

**A1:** Correct answer and conclusion

Question	Scheme	Marks	AOs
2	When $n = 1$ , $2^{3n+1} + 3(5^{2n+1}) = 16 + 375 = 391$ $391 = 17 \times 23$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $2^{3k+1} + 3(5^{2k+1})$ is divisible by 17	M1	2.4
	$f(k+1) - f(k) = 2^{3k+4} + 3(5^{2k+3}) - 2^{3k+1} - 3(5^{2k+1})$	M1	2.1
	$= 7 \times 2^{3k+1} + 7 \times 3(5^{2k+1}) + 17 \times 3(5^{2k+1})$		
	$= 7f(k) + 17 \times 3(5^{2k+1})$	A1	1.1b
	$f(k+1) = 8f(k) + 17 \times 3(5^{2k+1})$	A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$ , the statement is true for all positive integers $n$	A1	2.4
		(6)	
<b>(6 marks)</b>			
<b>Notes:</b>			
<b>B1:</b> Shows the statement is true for $n = 1$			
<b>M1:</b> Assumes the statement is true for $n = k$			
<b>M1:</b> Attempts $f(k+1) - f(k)$			
<b>A1:</b> Correct expression in terms of $f(k)$			
<b>A1:</b> Correct expression in terms of $f(k)$			
<b>A1:</b> Obtains a correct expression for $f(k + 1)$			
<b>A1:</b> Correct complete conclusion			

Question	Scheme	Marks	AOs
3	$z = 3 - 2i$ is also a root	B1	1.2
	$(z - (3 + 2i))(z - (3 - 2i)) = \dots$ or Sum of roots = 6, Product of roots = 13 $\Rightarrow \dots$	M1	3.1a
	$= z^2 - 6z + 13$	A1	1.1b
	$(z^4 + az^3 + 6z^2 + bz + 65) = (z^2 - 6z + 13)(z^2 + cz + 5) \Rightarrow c = \dots$	M1	3.1a
	$z^2 + 2z + 5 = 0$	A1	1.1b
	$z^2 + 2z + 5 = 0 \Rightarrow z = \dots$	M1	1.1a
	$z = -1 \pm 2i$	A1	1.1b
		B1 $3 \pm 2i$ Plotted correctly	1.1b
	B1ft $-1 \pm 2i$ Plotted correctly	1.1b	
<b>(9 marks)</b>			
<b>Notes:</b>			
<p><b>B1:</b> Identifies the complex conjugate as another root</p> <p><b>M1:</b> Uses the conjugate pair and a correct method to find a quadratic factor</p> <p><b>A1:</b> Correct quadratic</p> <p><b>M1:</b> Uses the given quartic and their quadratic to identify the value of <math>c</math></p> <p><b>A1:</b> Correct 3TQ</p> <p><b>M1:</b> Solves their second quadratic</p> <p><b>A1:</b> Correct second conjugate pair</p> <p><b>B1:</b> First conjugate pair plotted correctly and labelled</p> <p><b>B1ft:</b> Second conjugate pair plotted correctly and labelled (Follow through their second conjugate pair)</p>			

Question	Scheme	Marks	AOs
4	$4 + \cos 2\theta = \frac{9}{2} \Rightarrow \theta = \dots$	M1	3.1a
	$\theta = \frac{\pi}{6}$	A1	1.1b
	$\frac{1}{2} \int (4 + \cos 2\theta)^2 d\theta = \frac{1}{2} \int (16 + 8\cos 2\theta + \cos^2 2\theta) d\theta$	M1	3.1a
	$\cos^2 2\theta = \frac{1}{2} + \frac{1}{2} \cos 4\theta \Rightarrow A = \frac{1}{2} \int \left( 16 + 8\cos 2\theta + \frac{1}{2} + \frac{1}{2} \cos 4\theta \right) d\theta$	M1	3.1a
	$= \frac{1}{2} \left[ 16\theta + 4\sin 2\theta + \frac{\sin 4\theta}{8} + \frac{\theta}{2} \right]$	A1	1.1b
	Using limits 0 and their $\frac{\pi}{6}$ : $\frac{1}{2} \left[ \frac{33\pi}{12} + 2\sqrt{3} + \frac{\sqrt{3}}{16} - (0) \right]$	M1	1.1b
	Area of triangle = $\frac{1}{2} (r \cos \theta)(r \sin \theta) = \frac{1}{2} \times \frac{81}{4} \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$	M1	3.1a
	Area of R = $\frac{33\pi}{24} + \frac{33\sqrt{3}}{32} - \frac{81\sqrt{3}}{32}$	M1	1.1b
	$= \frac{11}{8}\pi - \frac{3\sqrt{3}}{2} \left( p = \frac{11}{8}, q = -\frac{3}{2} \right)$	A1	1.1b
<b>(9 marks)</b>			
<b>Notes:</b>			
<b>M1:</b> Realises the angle for A is required and attempts to find it			
<b>A1:</b> Correct angle			
<b>M1:</b> Uses a correct area formula and squares r to achieve a 3TQ integrand in cos 2θ			
<b>M1:</b> Use of the correct double angle identity on the integrand to achieve a suitable form for integration			
<b>A1:</b> Correct integration			
<b>M1:</b> Correct use of limits			
<b>M1:</b> Identifies the need to subtract the area of a triangle and so finds the area of the triangle			
<b>M1:</b> Complete method for the area of R			
<b>A1:</b> Correct final answer			

Question	Scheme	Marks	AOs
<b>5(a)</b>	Pond contains $1000 + 5t$ litres after $t$ days	M1	3.3
	If $x$ is the amount of pollutant in the pond after $t$ days		
	Rate of pollutant out = $20 \times \frac{x}{1000+5t}$ g per day	M1	3.3
	Rate of pollutant in = $25 \times 2$ g = 50g per day	B1	2.2a
	$\frac{dx}{dt} = 50 - \frac{4x}{200+t}$ *	A1*	1.1b
	<b>(4)</b>		
<b>(b)</b>	$I = e^{\int \frac{4}{200+t} dt} = (200+t)^4 \Rightarrow x(200+t)^4 = \int 50(200+t)^4 dt$	M1	3.1b
	$x(200+t)^4 = 10(200+t)^5 + c$	A1	1.1b
	$x = 0, t = 0 \Rightarrow c = -3.2 \times 10^{12}$	M1	3.4
	$t = 8 \Rightarrow x = 10(200+8) - \frac{3.2 \times 10^{12}}{(200+8)^4}$	M1	1.1b
	$= 370\text{g}$	A1	2.2b
	<b>(5)</b>		
<b>(c)</b>	e.g. <ul style="list-style-type: none"> <li>The model should take into account the fact that the pollutant does not dissolve throughout the pond upon entry</li> <li>The rate of leaking could be made to vary with the volume of water in the pond</li> </ul>	B1	3.5c
		<b>(1)</b>	
<b>(10 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>	<b>M1:</b> Forms an expression of the form $1000 + kt$ for the volume of water in the pond at time $t$ <b>M1:</b> Expresses the amount of pollutant out in terms of $x$ and $t$ <b>B1:</b> Correct interpretation for pollutant entering the pond <b>A1*:</b> Puts all the components together to form the correct differential equation		
<b>(b)</b>	<b>M1:</b> Uses the model to find the integrating factor and attempts solution of their differential equation <b>A1:</b> Correct solution <b>M1:</b> Interprets the initial conditions to find the constant of integration <b>M1:</b> Uses their solution to the problem to find the amount of pollutant after 8 days <b>A1:</b> Correct number of grams		
<b>(c)</b>	<b>B1:</b> Suggests a suitable refinement to the model		

Question	Scheme	Marks	AOs
<b>6(a)</b>	$f(x) = \frac{x+2}{x^2+9} = \frac{x}{x^2+9} + \frac{2}{x^2+9}$	B1	3.1a
	$\int \frac{x}{x^2+9} dx = k \ln(x^2+9) (+c)$	M1	1.1b
	$\int \frac{2}{x^2+9} dx = k \arctan\left(\frac{x}{3}\right) (+c)$	M1	1.1b
	$\int \frac{x+2}{x^2+9} dx = \frac{1}{2} \ln(x^2+9) + \frac{2}{3} \arctan\left(\frac{x}{3}\right) + c$	A1	1.1b
		<b>(4)</b>	
<b>(b)</b>	$\int_0^3 f(x) dx = \left[ \frac{1}{2} \ln(x^2+9) + \frac{2}{3} \arctan\left(\frac{x}{3}\right) \right]_0^3$ $= \frac{1}{2} \ln 18 + \frac{2}{3} \arctan\left(\frac{3}{3}\right) - \left( \frac{1}{2} \ln 9 + \frac{2}{3} \arctan(0) \right)$ $= \frac{1}{2} \ln \frac{18}{9} + \frac{2}{3} \arctan\left(\frac{3}{3}\right)$	M1	1.1b
	Mean value = $\frac{1}{3-0} \left( \frac{1}{2} \ln 2 + \frac{\pi}{6} \right)$	M1	2.1
	$\frac{1}{6} \ln 2 + \frac{1}{18} \pi^*$	A1*	2.2a
		<b>(3)</b>	
<b>(c)</b>	$\frac{1}{6} \ln 2 + \frac{1}{18} \pi + \ln k$	M1	2.2a
	$\frac{1}{6} \ln 2k^6 + \frac{1}{18} \pi$	A1	1.1b
		<b>(2)</b>	
<b>(9 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>B1:</b> Splits the fraction into two correct separate expressions			
<b>M1:</b> Recognises the required form for the first integration			
<b>M1:</b> Recognises the required form for the second integration			
<b>A1:</b> Both expressions integrated correctly and added together with constant of integration included			
<b>(b)</b>			
<b>M1:</b> Uses limits correctly and combines logarithmic terms			
<b>M1:</b> Correctly applies the method for the mean value for their integration			
<b>A1*:</b> Correct work leading to the given answer			
<b>(c)</b>			
<b>M1:</b> Realises that the effect of the transformation is to increase the mean value by $\ln k$			
<b>A1:</b> Combines $\ln$ 's correctly to obtain the correct expression			

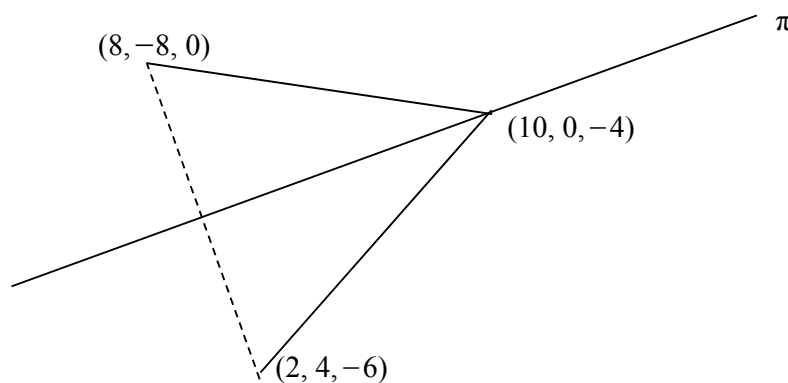
Question	Scheme	Marks	AOs
<b>7(a)</b>	$x = \cos \theta + \sin \theta \cos \theta = -y \cos \theta$	M1	2.1
	$\sin \theta = -y - 1$	M1	2.1
	$\left(\frac{x}{-y}\right)^2 = 1 - (-y - 1)^2$	M1	2.1
	$x^2 = -(y^4 + 2y^3)^*$	A1*	1.1b
		<b>(4)</b>	
<b>(b)</b>	$V = \pi \int x^2 dy = \pi \int -(y^4 + 2y^3) dy$	M1	3.4
	$= \pi \left[ -\left(\frac{y^5}{5} + \frac{y^4}{2}\right) \right]$	A1	1.1b
	$= -\pi \left[ \left(\frac{(0)^5}{5} + \frac{(0)^4}{2}\right) - \left(\frac{(-2)^5}{5} + \frac{(-2)^4}{2}\right) \right]$	M1	3.4
	$= 1.6\pi \text{ cm}^3 \text{ or awrt } 5.03 \text{ cm}^3$	A1	1.1b
		<b>(4)</b>	
<b>(8 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>M1:</b> Obtains $x$ in terms of $y$ and $\cos \theta$			
<b>M1:</b> Obtains an equation connecting $y$ and $\sin \theta$			
<b>M1:</b> Uses Pythagoras to obtain an equation in $x$ and $y$ only			
<b>A1*:</b> Obtains printed answer			
<b>(b)</b>			
<b>M1:</b> Uses the correct volume of revolution formula with the given expression			
<b>A1:</b> Correct integration			
<b>M1:</b> Correct use of correct limits			
<b>A1:</b> Correct volume			



Question	Scheme	Marks	AOs
8	$2 + 4\lambda - 2(4 - 2\lambda) - 6 + \lambda = 6 \Rightarrow \lambda = \dots$	M1	1.1b
	$\lambda = 2 \Rightarrow$ Required point is $(2 + 2(4), 4 + 2(-2), -6 + 2(1))$ $(10, 0, -4)$	A1	1.1b
	$2 + t - 2(4 - 2t) - 6 + t = 6 \Rightarrow t = \dots$	M1	3.1a
	$t = 3$ so reflection of $(2, 4, -6)$ is $(2 + 6(1), 4 + 6(-2), -6 + 6(1))$ $(8, -8, 0)$	M1	3.1a
		A1	1.1b
	$\begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ -8 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ -4 \end{pmatrix}$	M1	3.1a
	$\mathbf{r} = \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} + k \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ or equivalent e.g. $\left( \mathbf{r} - \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} \right) \times \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \mathbf{0}$	A1	2.5
	(7)		
<b>(7 marks)</b>			

**Notes:**

- M1:** Substitutes the parametric equation of the line into the equation of the plane and solves for  $\lambda$
- A1:** Obtains the correct coordinates of the intersection of the line and the plane
- M1:** Substitutes the parametric form of the line perpendicular to the plane passing through  $(2, 4, -6)$  into the equation of the plane to find  $t$
- M1:** Find the reflection of  $(2, 4, -6)$  in the plane
- A1:** Correct coordinates
- M1:** Determines the direction of  $l$  by subtracting the appropriate vectors
- A1:** Correct vector equation using the correct notation



Question	Scheme	Marks	AOs
<b>9(a)(i)</b>	Weight = mass $\times$ g $\Rightarrow m = \frac{30000}{g} = 3000$ But mass is in thousands of kg, so $m = 3$	M1	3.3
<b>(ii)</b>	$\frac{dx}{dt} = 40 \cos t + 20 \sin t, \frac{d^2x}{dt^2} = -40 \sin t + 20 \cos t$	M1	1.1b
	$3(-40 \sin t + 20 \cos t) + 4(40 \cos t + 20 \sin t)$ $+ 40 \sin t - 20 \cos t = \dots$	M1	1.1b
	$= 200 \cos t$ so PI is $x = 40 \sin t - 20 \cos t$	A1*	2.1
	<b>or</b>		
	Let $x = a \cos t + b \sin t$ $\frac{dx}{dt} = -a \sin t + b \cos t, \frac{d^2x}{dt^2} = -a \cos t - b \sin t$	M1	1.1b
	$4b - 2a = 200, -2b - 4a = 0 \Rightarrow a = \dots, b = \dots$	M1	2.1
	$x = 40 \sin t - 20 \cos t$	A1*	1.1b
<b>(iii)</b>	$3\lambda^2 + 4\lambda + 1 = 0 \Rightarrow \lambda = -1, -\frac{1}{3}$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t}$	A1	1.1b
	$x = PI + CF$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40 \sin t - 20 \cos t$	A1	1.1b
	<b>(8)</b>		
<b>(b)</b>	$t = 0, x = 0 \Rightarrow A + B = 20$	M1	3.4
	$x = 0, \frac{dx}{dt} = -Ae^{-t} - \frac{1}{3}Be^{-\frac{1}{3}t} + 40 \cos t + 20 \sin t = 0$ $\Rightarrow A + \frac{1}{3}B = 40$	M1	3.4
	$x = 50e^{-t} - 30e^{-\frac{1}{3}t} + 40 \sin t - 20 \cos t$	A1	1.1b
	$t = 9 \Rightarrow x = 33\text{m}$	A1	3.4
	<b>(4)</b>		
<b>(12 marks)</b>			

<b>Question 9 notes:</b>
<p><b>(a)(i)</b>  <b>M1:</b> Correct explanation that in the model, <math>m = 3</math></p>
<p><b>(ii)</b>  <b>M1:</b> Differentiates the given PI twice  <b>M1:</b> Substitutes into the given differential equation  <b>A1*:</b> Reaches <math>200\cos t</math> and makes a conclusion  <b>or</b>  <b>M1:</b> Uses the correct form for the PI and differentiates twice  <b>M1:</b> Substitutes into the given differential equation and attempts to solve  <b>A1*:</b> Correct PI</p>
<p><b>(iii)</b>  <b>M1:</b> Uses the model to form and solve the auxiliary equation  <b>A1:</b> Correct complementary function  <b>M1:</b> Uses the correct notation for the general solution by combining PI and CF  <b>A1:</b> Correct General Solution for the model</p>
<p><b>(b)</b>  <b>M1:</b> Uses the initial conditions of the model, <math>t = 0</math> at <math>x = 0</math>, to form an equation in <math>A</math> and <math>B</math>  <b>M1:</b> Uses <math>\frac{dx}{dt} = 0</math> at <math>x = 0</math> in the model to form an equation in <math>A</math> and <math>B</math>  <b>A1:</b> Correct PS  <b>A1:</b> Obtains 33m using the assumptions made in the model</p>

