Question	Scheme	Marks	AOs
6	(a) $\mathrm{f}(x) = -3x^3 + 8x^2 - 9x + 10, \quad x \in \mathbb{R}$		
(a)	(i) $\{\mathrm{f}(2) = -24 + 32 - 18 + 10 \Rightarrow\}\ \mathrm{f}(2) = 0$	B1	1.1b
	(ii) $\{\mathrm{f}(x) =\}\ (x-2)(-3x^2 + 2x - 5)$ or $(2-x)(3x^2 - 2x + 5)$	M1	2.2a
		A1	1.1b
		(3)	
(b)	$-3y^6 + 8y^4 - 9y^2 + 10 = 0 \Rightarrow (y^2 - 2)(-3y^4 + 2y^2 - 5) = 0$		
	Gives a partial explanation by • explaining that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions with a reason, e.g. $b^2 - 4ac = (2)^2 - 4(-3)(-5) = -56 < 0$ • or stating that $y^2 = 2$ has 2 {real} solutions or $y = \pm\sqrt{2}$ {only}	M1	2.4
	Complete proof that the given equation has exactly two {real} solutions	A1	2.1
		(2)	
(c)	$3\tan^3\theta - 8\tan^2\theta + 9\tan\theta - 10 = 0;\ 7\pi \le \theta < 10\pi$		
	{Deduces that} there are 3 solutions	B1	2.2a
		(1)	
			(6 marks)

Notes for Question 6

(a)(i)	
B1:	$\mathrm{f}(2) = 0$ or 0 stated by itself in part (a)(i)
(a)(ii)	
M1:	Deduces that $(x-2)$ or $(2-x)$ is a factor and attempts to find the other quadratic factor by • using long division to obtain either $\pm 3x^2 \pm kx + \dots,\ k = \text{value} \ne 0$ or $\pm 3x^2 \pm \alpha x + \beta,\ \beta = \text{value} \ne 0,\ \alpha$ can be 0 • factorising to obtain their quadratic factor in the form $(\pm 3x^2 \pm kx \pm c),\ k = \text{value} \ne 0$, c can be 0, or in the form $(\pm 3x^2 \pm \alpha x \pm \beta),\ \beta = \text{value} \ne 0,\ \alpha$ can be 0
A1:	$(x-2)(-3x^2 + 2x - 5)$, $(2-x)(3x^2 - 2x + 5)$ or $-(x-2)(3x^2 - 2x + 5)$ stated together as a product
(b)	
M1:	See scheme
A1:	See scheme. Proof must be correct ***with no errors***, e.g. giving an incorrect discriminant value
Note:	Correct calculation e.g. $(2)^2 - 4(-3)(-5)$, $4 - 60$ or -56 must be given for the first explanation
Note:	Note that M1 can be allowed for • a correct follow through calculation for the discriminant of their "$-3y^4 + 2y^2 - 5$" which would lead to a value < 0 together with an explanation that $-3y^4 + 2y^2 - 5 = 0$ has no {real} solutions • **or** for the omission of < 0
Note:	< 0 must also been stated in a discriminant method for A1
Note:	Do not allow A1 for incorrect working, e.g. $(2)^2 - 4(-3)(-5) = -54 < 0$
Note:	$y^2 = 2 \Rightarrow y = \pm 2$, so 2 solutions is not allowed for A1, but can be condoned for M1
Note:	Using the formula on $-3y^4 + 2y^2 - 5 = 0$ or $-3x^2 + 2x - 5 = 0$ gives y^2 or $x = \dfrac{-2 \pm \sqrt{-56}}{-6}$ or $\dfrac{-1 \pm \sqrt{-14}}{-3}$

Question	Scheme	Marks	AOs
7	(i) $4\sin x = \sec x,\ 0 \le x < \frac{\pi}{2}$; (ii) $5\sin\theta - 5\cos\theta = 2,\ 0 \le \theta < 360°$		
(i) Way 1	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 2\sin 2x = 1 \Rightarrow \sin 2x = \frac{1}{2}$	M1	3.1a
	$x = \frac{1}{2}\arcsin\left(\frac{1}{2}\right)$ or $\frac{1}{2}\left(\pi - \arcsin\left(\frac{1}{2}\right)\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(i) Way 2	For $\sec x = \frac{1}{\cos x}$	B1	1.2
	$\{4\sin x = \sec x \Rightarrow\}\ 4\sin x\cos x = 1 \Rightarrow 16\sin^2 x\cos^2 x = 1$ $16\sin^2 x(1-\sin^2 x) = 1$ | $16(1-\cos^2 x)\cos^2 x = 1$ $16\sin^4 x - 16\sin^2 x + 1 = 0$ | $16\cos^4 x - 16\cos^2 x + 1 = 0$ $\sin^2 x$ or $\cos^2 x = \frac{16 \pm \sqrt{192}}{32}\ \left\{= \frac{2\pm\sqrt{3}}{4} \text{ or } 0.933...,\ 0.066...\right\}$	M1	3.1a
	$x = \arcsin\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right)$ or $x = \arccos\left(\sqrt{\frac{2\pm\sqrt{3}}{4}}\right) \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$	dM1	1.1b
		A1	1.1b
		(4)	
(ii)	Complete strategy, i.e. • Expresses $5\sin\theta - 5\cos\theta = 2$ in the form $R\sin(\theta - \alpha) = 2$, finds both R and α, and proceeds to $\sin(\theta-\alpha) = k,\ \lvert k\rvert < 1,\ k \ne 0$ • Applies $(5\sin\theta - 5\cos\theta)^2 = 2^2$, followed by applying both $\cos^2\theta + \sin^2\theta = 1$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to proceed to $\sin 2\theta = k,\ \lvert k\rvert < 1,\ k \ne 0$	M1	3.1a
	$R = \sqrt{50}$ $\tan\alpha = 1 \Rightarrow \alpha = 45°$ | $(5\sin\theta - 5\cos\theta)^2 = 2^2 \Rightarrow$ $25\sin^2\theta + 25\cos^2\theta - 50\sin\theta\cos\theta = 4$ $\Rightarrow 25 - 25\sin 2\theta = 4$	M1	1.1b
	$\sin(\theta - 45°) = \frac{2}{\sqrt{50}}$ | $\sin 2\theta = \frac{21}{25}$	A1	1.1b
	dependent on the first M mark e.g. $\theta = \arcsin\left(\frac{2}{\sqrt{50}}\right) + 45°$ | e.g. $\theta = \frac{1}{2}\left(\arcsin\left(\frac{21}{25}\right)\right)$	dM1	1.1b
	$\theta = \text{awrt } 61.4°,\ \text{awrt } 208.6°$	A1	2.1
	Note: Working in radians does not affect any of the first 4 marks		
		(5)	
			(9 marks)

Question	Scheme	Marks	AOs
12	$1-\cos 2\theta \equiv \tan\theta\sin 2\theta,\ \theta \neq \frac{(2n+1)\pi}{2},\ n\in\mathbb{Z}$		
(a) Way 1	$\tan\theta\sin 2\theta = \left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta)$	M1	1.1b
	$=\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)(2\sin\theta\cancel{\cos\theta}) = 2\sin^2\theta = 1-\cos 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
(a) Way 2	$1-\cos 2\theta = 1-(1-2\sin^2\theta) = 2\sin^2\theta$	M1	1.1b
	$=\left(\frac{\sin\theta}{\cos\theta}\right)(2\sin\theta\cos\theta) = \tan\theta\sin 2\theta$ *	M1	1.1b
		A1*	2.1
		(3)	
	$(\sec^2 x-5)(1-\cos 2x) = 3\tan^2 x\sin 2x,\ -\frac{\pi}{2}<x<\frac{\pi}{2}$		
(b) Way 1	$(\sec^2 x-5)\tan x\sin 2x = 3\tan^2 x\sin 2x$ **or** $(\sec^2 x-5)(1-\cos 2x) = 3\tan x(1-\cos 2x)$		
	Deduces $x=0$	B1	2.2a
	Uses $\sec^2 x = 1+\tan^2 x$ and cancels/factorises out $\tan x$ or $(1-\cos 2x)$ e.g. $(1+\tan^2 x-3\tan x-5)\tan x=0$ **or** $(1+\tan^2 x-3\tan x-5)(1-\cos 2x)=0$ **or** $1+\tan^2 x-5=3\tan x$	M1	2.1
	$\tan^2 x-3\tan x-4=0$	A1	1.1b
	$(\tan x-4)(\tan x+1)=0 \Rightarrow \tan x=...$	M1	1.1b
	$x=-\frac{\pi}{4},\ 1.326$	A1	1.1b
		A1	1.1b
		(6)	
			(9 marks)

Notes for Question 12

(a)	**Way 1**
M1:	Applies $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to $\tan\theta\sin 2\theta$
M1:	Cancels as scheme (may be implied) and attempts to use $\cos 2\theta = 1-2\sin^2\theta$
A1*:	For a correct proof showing all steps of the argument
(a) Way 2	
M1:	For using $\cos 2\theta = 1-2\sin^2\theta$
Note:	If the form $\cos 2\theta = \cos^2\theta-\sin^2\theta$ or $\cos 2\theta = 2\cos^2\theta-1$ is used, the mark cannot be awarded until $\cos^2\theta$ has been replaced by $1-\sin^2\theta$
M1:	Attempts to write their $2\sin^2\theta$ in terms of $\tan\theta$ and $\sin 2\theta$ using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and
	$\sin 2\theta = 2\sin\theta\cos\theta$ within the given expression
A1*:	For a correct proof showing all steps of the argument
Note:	If a proof meets in the middle; e.g. they show LHS = $2\sin^2\theta$ and RHS = $2\sin^2\theta$; then some indication must be given that the proof is complete. E.g. $1-\cos 2\theta \equiv \tan\theta\sin 2\theta$, QED, box

Question	Scheme	Marks	AOs
6 (a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9\times\dfrac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\dfrac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
			(8 marks)

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = ...\sin\theta\cos\theta$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for θ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \dfrac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

Question	Scheme	Marks	AOs
12	$\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) Way 1	$\{\text{LHS} =\}\ \dfrac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos(3\theta-\theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 2	$\{\text{LHS} =\}\ \dfrac{\cos 2\theta\cos\theta - \sin 2\theta\sin\theta}{\sin\theta} + \dfrac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\cos\theta}$		
	$= \dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos 2\theta(\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS} =\}\ \dfrac{2\cos 2\theta}{\sin 2\theta} = \dfrac{2\cos(3\theta-\theta)}{\sin 2\theta} = \dfrac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$= \dfrac{2(\cos 3\theta\cos\theta + \sin 3\theta\sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$= \dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k$; $k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90° < \theta < 180°,\ \tan 2\theta = \dfrac{1}{2} \Rightarrow\right\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow \dfrac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)} = 4 \Rightarrow 2(1-\tan^2\theta) = 8\tan\theta$ $\Rightarrow \tan^2\theta + 4\tan\theta - 1 = 0 \Rightarrow \tan\theta = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$ $\{\Rightarrow \tan\theta = -2 \pm \sqrt{5}\} \Rightarrow \tan\theta = k$; $k \neq 0 \Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90° < \theta < 180°,\ \tan\theta = -2 - \sqrt{5} \Rightarrow\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
			(7 marks)

Question	Scheme	Marks	AOs
12 (a)	States or uses $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	B1	1.2
	$\operatorname{cosec}\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$	M1	2.1
	$= \frac{\cos^2\theta}{\sin\theta} = \cos\theta \times \frac{\cos\theta}{\sin\theta} = \cos\theta\cot\theta$ *	A1*	2.1
		(3)	
(b)	$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50°)$ $\Rightarrow \cos x \cot x = \cos x \cot(3x - 50°)$		
	$\cot x = \cot(3x - 50°) \Rightarrow x = 3x - 50°$	M1	3.1a
	$x = 25°$	A1	1.1b
	Also $\cot x = \cot(3x - 50°) \Rightarrow x + 180° = 3x - 50°$	M1	2.1
	$x = 115°$	A1	1.1b
	Deduces $x = 90°$	B1	2.2a
		(5)	
			(8 marks)
Notes:			

(a) Condone a full proof in *x* (or other variable) instead of $\theta's$ here

B1: States or uses $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ Do not accept $\operatorname{cosec}\theta = \frac{1}{\sin}$ with the θ missing

M1: For the key step in forming a single fraction/common denominator

E.g. $\operatorname{cosec}\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$. Allow if written separately $\frac{1}{\sin\theta} - \sin\theta = \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$

Condone missing variables for this M mark

A1*: Shows careful work with all necessary steps shown leading to given answer. See scheme for necessary steps. There should not be any notational or bracketing errors.

(b) Condone $\theta's$ instead of *x*'s here

M1: Uses part (a), cancels or factorises out the $\cos x$ term, to establish that one solution is found when $x = 3x - 50°$.

You may see solutions where $\cot A - \cot B = 0 \Rightarrow \cot(A-B) = 0$ or $\tan A - \tan B = 0 \Rightarrow \tan(A-B) = 0$.

As long as they don't state $\cot A - \cot B = \cot(A-B)$ or $\tan A - \tan B = \tan(A-B)$ this is acceptable

A1: $x = 25°$

M1: For the key step in realising that $\cot x$ has a period of $180°$ and a second solution can be found by solving $x + 180° = 3x - 50°$. The sight of $x = 115°$ can imply this mark provided the step $x = 3x - 50°$ has been seen. Using reciprocal functions it is for realising that $\tan x$ has a period of $180°$

A1: $x = 115°$ Withhold this mark if there are additional values in the range (0,180) but ignore values outside.

B1: Deduces that a solution can be found from $\cos x = 0 \Rightarrow x = 90°$. Ignore additional values here.

....

Solutions with limited working. The question demands that candidates show all stages of working.

SC: $\cos x \cot x = \cos x \cot(3x - 50°) \Rightarrow \cot x = \cot(3x - 50°) \Rightarrow x = 25°, 115°$

They have shown some working so can score B1, B1 marked on epen as 11000

Question	Scheme	Marks	AOs
10 (a)	$\cos 3A = \cos(2A + A) = \cos 2A\cos A - \sin 2A\sin A$	M1	3.1a
	$= \left(2\cos^2 A - 1\right)\cos A - \left(2\sin A\cos A\right)\sin A$	dM1	1.1b
	$= \left(2\cos^2 A - 1\right)\cos A - 2\cos A\left(1 - \cos^2 A\right)$	ddM1	2.1
	$= 4\cos^3 A - 3\cos A$ *	A1*	1.1b
		(4)	
(b)	$1 - \cos 3x = \sin^2 x \Rightarrow \cos^2 x + 3\cos x - 4\cos^3 x = 0$	M1	1.1b
	$\Rightarrow \cos x\left(4\cos^2 x - \cos x - 3\right) = 0$ $\Rightarrow \cos x\left(4\cos x + 3\right)\left(\cos x - 1\right) = 0$ $\Rightarrow \cos x = \ldots$	dM1	3.1a
	Two of $-90°$, 0, $90°$, awrt $139°$	A1	1.1b
	All four of $-90°$, 0, $90°$, awrt $139°$	A1	2.1
		(4)	
			(8 marks)

Question Number	Scheme	Marks
2	$\frac{\tan 2x+\tan 50°}{1-\tan 2x\tan 50°}=2 \quad \Rightarrow \tan(2x+50°)=2$	M1A1
	$\Rightarrow 2x+50°=63.43°, (243.43°, 423.43°)$	
	$\Rightarrow x=$ awrt $6.72°$ or $96.72°$ or $186.72°$	dM1, A1
	$\Rightarrow 2x+50°=243.43°\ (423.43°) \Rightarrow x=..$	dM1
	$x=$ awrt $6.72°, 96.72°, 186.72°$	A1
		(6 marks)

Notes

M1 Uses the compound angle identity $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ to write the equation in the form $\tan(2x\pm 50°)=2$. Accept a sign error in bracket.

A1 $\tan(2x+50°)=2$

dM1 Uses the correct order of operations to find one solution in the range.
Moves from $\tan(2x\pm 50°)=2 \Rightarrow 2x\pm 50°=\arctan 2 \Rightarrow x=...$
This is dependent upon having scored the first M1

A1 One correct answer, usually awrt $6.72°$, but accept any of $6.72°, 96.72°, 186.72°$

dM1 Uses the correct order of operations to find a second solution in the range.
This can be scored by $2x\pm 50°=180+\text{their } 63$ *or* $360+\text{their } 63 \Rightarrow x=..$
It may be implied by $90+\text{their } 6.7$, *or* $180+\text{their } 6.7$ as long as no incorrect working is seen.
This is dependent upon having scored the first M1

A1 All three answers in the range, $x=$ awrt $6.72°, 96.72°, 186.72°$
Any extra solutions in the range withhold the last A mark.
Ignore any solutions outside the range $0\leqslant x\leqslant 270°$
Radian solutions will be unlikely, but could be worth marks only if $50°\to 0.873$ radians.
$\tan(2x+50)°=2 \Rightarrow 2x+50°=1.107..$ will score M1A1dM0 and nothing else.

Question Number	Scheme	Marks
8 (a)	$2\text{cosec}2A - \cot A = \dfrac{2}{\sin 2A} - \dfrac{1}{\tan A}$ $\quad 2\cos\text{ec}2A = \dfrac{2}{\sin 2A}$	B1
	$= \dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A}$	M1
	$= \dfrac{2 - 2\cos^2 A}{2\sin A\cos A}$	M1
	$\dfrac{2(1-\cos^2 A)}{2\sin A\cos A} = \dfrac{2\sin^2 A}{2\sin A\cos A} = \dfrac{\sin A}{\cos A} = \tan A$	A1*
		(4)
(b)(i)	$2\text{cosec}4\theta - \cot 2\theta = \sqrt{3} \Rightarrow \tan 2\theta = \sqrt{3}$	M1
	$\Rightarrow \theta = \dfrac{\arctan\sqrt{3}}{2} = \dfrac{\pi}{6}$ Accept awrt 0.524	A1
(ii)	$\tan\theta + \cot\theta = 5 \Rightarrow \text{cosec}2\theta = \dfrac{5}{2}$	M1
	$\Rightarrow \theta = \dfrac{1}{2}\arcsin\left(\dfrac{2}{5}\right) =$ awrt 0.206,1.37	dM1A1A1
		(6)
		(10 marks)
Alt 8 (a)	$2\text{cosec}2A - \cot A = \tan A \Rightarrow \dfrac{2}{\sin 2A} - \dfrac{1}{\tan A} = \tan A$	B1
	$\Rightarrow \dfrac{2}{2\sin A\cos A} - \dfrac{\cos A}{\sin A} = \dfrac{\sin A}{\cos A}$	M1
	$\times 2\sin A\cos A \Rightarrow 2 - 2\cos^2 A = 2\sin^2 A$	M1
	$\Rightarrow 2(1-\cos^2 A) = 2\sin^2 A$	
	$\Rightarrow 2\sin^2 A = 2\sin^2 A$ *QED* (minimal statement must be seen)	A1* **(4)**
Alt 8b(ii)	$\tan\theta + \cot\theta = 5 \Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = 5 \Rightarrow \dfrac{1}{\frac{1}{2}\sin 2\theta} = 5 \Rightarrow \sin 2\theta = \dfrac{2}{5}$ This can now score all of the marks as it is effectively using part (a)	M1
SC 8b(ii)	$\tan\theta + \cot\theta = 5 \Rightarrow \tan\theta + \dfrac{1}{\tan\theta} = 5 \Rightarrow \tan^2\theta - 5\tan\theta + 1 = 0$ $\Rightarrow \theta =$ awrt 0.206,1.37 This is not using part (a) and is a special case with one mark per correct answer. One answer =1000 Two answers=1100	1,1,0,0

Question Number	Scheme	Marks
2	$2\cos 2\theta = 5 - 13\sin\theta \Rightarrow 4\sin^2\theta - 13\sin\theta + 3 = 0$	M1A1
	$\Rightarrow (4\sin\theta - 1)(\sin\theta - 3) = 0$	
	$\sin\theta = \frac{1}{4}$	M1
	$\theta = \text{awrt } 0.253, \quad 2.889 \text{ (3dp)}$	A1,A1 cso
		(5 marks)

M1 Uses $\cos 2\theta = 1 - 2\sin^2\theta$ to get a quadratic equation in just $\sin\theta$.

If candidate uses $\cos 2\theta = \cos^2\theta - \sin^2\theta$ *or* $2\cos^2\theta - 1$ they must use $\cos^2\theta = 1 - \sin^2\theta$ to form a quadratic equation in just $\sin\theta$ before scoring the M.

A1 $\pm(4\sin^2\theta - 13\sin\theta + 3) = 0$. The = 0 may be implied by subsequent working

M1 Solves their 3TQ in $\sin\theta$ with usual rules by factorisation, formula or completing the square. They must proceed as far as $\sin\theta = ..$ Accept an answer from a calculator. You may have to pick up a calculator to check their values.

A1 Either of $\theta = \text{awrt } 0.25, 2.89 \text{ (2dp)}$ in radians or either of $\theta = \text{awrt } 14.5, 165.5 \text{ (1dp)}$ in degrees
Accept either of awrt $0.08\pi, 0.92\pi$

A1 Correct solution with only two solutions $\theta = \text{awrt } 0.253, 2.889 \text{ (3dp)}$ within the given range.
Accept equivalents such as awrt $0.0804\pi, 0.9196\pi$
Ignore any extra answers outside the range.
Note that incorrect factorisation $(4\sin\theta - 1)(\sin\theta + 3) = 0$ would lead to correct answers. As this mark is cso, it would be withheld in such circumstances.

Question Number	Scheme	Marks
7(a)	$2\cos(x+30)^\circ = \sin(x-30)^\circ$	
	$2(\cos x^\circ \cos 30^\circ - \sin x^\circ \sin 30^\circ) = \sin x^\circ \cos 30^\circ - \cos x^\circ \sin 30^\circ$	M1A1
	$2\cos 30^\circ - 2\tan x^\circ \sin 30^\circ = \tan x^\circ \cos 30^\circ - \sin 30^\circ$	
	$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$	B1
	$\tan x^\circ = \frac{2\sqrt{3}+1}{\sqrt{3}+2} \Rightarrow \times \frac{\sqrt{3}-2}{\sqrt{3}-2} \Rightarrow \tan x^\circ = 3\sqrt{3}-4$	dM1A1*
		(5)
(b)	$\tan(2\theta+10)^\circ = 3\sqrt{3}-4$	M1
	$2\theta+10 = 50.1, (230.1) \Rightarrow \theta = ..$	dM1
	$\theta = 20.1, \quad 110.1$	A1,A1
		(4)
		(9 marks)

Question Number	Scheme	Marks
7(a)	$4\tan 2x - 3\cot x\sec^2 x = 0 \Rightarrow 4\times\dfrac{2t}{1-t^2} - 3\times\dfrac{1}{t}\times(1+t^2) = 0$	B1M1A1
	So $4\times 2t^2 - 3\times(1+t^2)(1-t^2) = 0$ and $3t^4 + 8t^2 - 3 = 0$ *	A1*
		(4)
(b)	$3t^4 + 8t^2 - 3 = 0 \Rightarrow (3t^2-1)(t^2+3) = 0$ so $t =$	M1
	$\tan x\ (t) = \pm\dfrac{1}{\sqrt{3}}$ *or* ± 0.5774	A1
	$x = \dfrac{\pi}{6}, \dfrac{7\pi}{6}, \dfrac{5\pi}{6}, \dfrac{11\pi}{6}$	M1A1
		(4)
		(8 marks)

Question Number	Scheme	Marks
2 (a)	$\text{cosec}^2 x - \text{cosec}x - 12 = 0$ or $k = -12$	B1 [1]
(b)	$\text{cosec}^2 x - \text{cosec}\,x - 12 = 0$ so $(\text{cosec}x - 4)(\text{cosec}x + 3) = 0 \Rightarrow \text{cosec}x = ...$	M1
	$\sin x = \frac{1}{4}$ or $-\frac{1}{3}$	dM1
	$\Rightarrow x = 14.5°$ or $165.5°$ or $340.5°$ or $199.5°$	dM1, A1 A1 [5]
		(6 marks)

(a)

B1: Accept $\text{cosec}^2 x - \text{cosec}x - 12 = 0$ or $k = -12$. No working is required.
If they write $\text{cosec}^2 x - \text{cosec}x - 12 = 0$ followed by $k = 12$ allow isw

(b)

M1 Solves quadratic in cosec x by any method – factorising, formula (accept answers to 1 dp), completion of square. Correct answers (for cosec x of 4 and –3) imply this M mark.
Quadratic equations that have 'imaginary' roots please put into review.

dM1 Uses $\sin x = \frac{1}{\text{cosec}x}$ by taking the reciprocal of at least one of their previous answers
This is dependent upon having scored the first M1

dM1 For using arcsin to produce one answer inside the range 0 to 360 from their values.
Implied by any of $14.5°$ or $165.5°$ or $340.5°$ or $199.5°$ following (cosec x -4) (cosec x+3) =0

A1 Two correct answers inside the range 0 to 360

A1 All four answers in the range, $x = \text{awrt}\,14.5°\ 165.5°\ 340.5°\ 199.5°$
Any extra solutions in the range withhold the last A mark.
Ignore any solutions outside the range $0 \leqslant x \leqslant 360°$

....

Radian solutions will be unlikely, but could be worth dM1 for one solution and dM1A1 A0 for all four solutions (maximum penalty is 1 mark) but accuracy marks are awarded for solutions to 3dp
FYI: Solutions awrt are 0.253, 2.889, 3.481, 5.943

....

The first two M marks may be achieved 'the other way around' if a candidate uses $\text{cosec}x = \frac{1}{\sin x}$ in line 1 and produces a quadratic in $\sin x$.

Award M1 for using $\text{cosec}x = \frac{1}{\sin x}$ (twice) and producing a quadratic in $\sin x$ and dM1 for solving as above.

Question Number	Scheme		Marks
8 (a)	$\sin 2x - \tan x \equiv 2\sin x\cos x - \frac{\sin x}{\cos x}$	Uses a **correct** identity for sin2x	M1
	$\equiv \frac{2\sin x\cos x\cos x}{\cos x} - \frac{\sin x}{\cos x}$	Obtains common denominator. This is **NOT** dependent upon the previous M so accept expressions like, $\sin 2x - \tan x \equiv \sin 2x - \frac{\sin x}{\cos x}$ $= \frac{\sin 2x\cos x - \sin x}{\cos x}$	M1
	$\equiv \frac{2\cos^2 x\sin x - \sin x}{\cos x}$	Correct fraction with just sin x and cos x	A1
	$\equiv \frac{(2\cos^2 x - 1)\sin x}{\cos x} \equiv \cos 2x\tan x$*	Uses a correct identity for cos2x and completes correctly with no errors. An error could be for example, mixed variables used or loss of an x along the way.	A1*
			(4)
	Alternative 1 for (a)		
	$\sin 2x - \tan x \equiv 2\sin x\cos x - \frac{\sin x}{\cos x}$	Uses a **correct** identity for sin2x	M1
	$\frac{\sin x}{\cos x}\left(2\cos^2 x - 1\right)$	M1: Takes out a factor of $\frac{\sin x}{\cos x}$ A1: Correct expression	M1A1
	$\equiv \tan x\cos 2x$*	Completes correctly with no errors.	A1*
	Alternative 2 for (a)		
	$2\sin x\cos x - \frac{\sin x}{\cos x} \equiv \frac{\sin x}{\cos x}\left(\cos^2 x - \sin^2 x\right)$	Uses a **correct** identity for sin2x	M1
	$2\sin x\cos^2 x - \sin x \equiv \sin x\left(\cos^2 x - \sin^2 x\right)$	Multiplies **both sides** by cos x	M1
	$2\cos^2 x - 1 \equiv \left(\cos^2 x - \sin^2 x\right)$	Correct identity	A1
	This is true*	Conclusion provided	A1*
	Alternative 3 for (a)		
	$\tan x\cos 2x \equiv \frac{\sin x}{\cos x}\left(2\cos^2 x - 1\right)$	Uses a **correct** identity for cos2x	M1
	$\equiv 2\sin x\cos x - \frac{\sin x}{\cos x}$	M1: Multiplies out A1: Correct expression	M1A1
	$\equiv \sin 2x - \tan x$*	A1: Obtains lhs with no errors	A1*

8(b)(i)	$\sin 2\theta - \tan\theta = \sqrt{3}\cos 2\theta \Rightarrow \tan\theta\cos 2\theta = \sqrt{3}\cos 2\theta$		
	$\tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} = (\text{awrt } 1.05)$	M1: $\tan\theta = \pm\sqrt{3} \Rightarrow \theta = ...$ A1: $\theta = \frac{\pi}{3}$ Accept awrt 1.05. Ignore solutions outside the range but withhold the A mark for extra solutions in range.	M1A1
	$\cos 2\theta = 0 \Rightarrow \theta = \frac{\pi}{4}\ (\text{awrt } 0.785)$	M1: $\cos 2\theta = 0 \Rightarrow \theta = ...$ A1: $\theta = \frac{\pi}{4}$ Accept awrt 0.785. Ignore solutions outside the range but withhold the A mark for extra solutions in range.	M1A1
(b)(ii)	$\tan(\theta+1)\cos(2\theta+2) - \sin(2\theta+2) = 2 \Rightarrow \tan(\theta+1) = -2$ M1: $\tan(\theta+1) = \pm 2$		M1
	$\Rightarrow \theta = \arctan(-2) - 1$	Correct order of operations i.e. $\theta = \arctan(\pm 2) - 1$. This may be implied by $\theta = -2.1...$	dM1
	$\Rightarrow \theta = 1.03$	awrt $\theta = 1.03$. Ignore solutions outside the range but withhold the A mark for extra solutions in range.	A1
			(7)
			(11 marks)

Qu	Scheme	Marks
8 (a)	$\tan(2x+x)=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$	M1
	$=\dfrac{\dfrac{2\tan x}{1-\tan^2 x}+\tan x}{1-\dfrac{2\tan x}{1-\tan^2 x}\tan x}$	dM1
	$=\dfrac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan^2 x}$ OR $=\dfrac{\dfrac{2\tan x+\tan x-\tan^3 x}{1-\tan^2 x}}{\dfrac{1-\tan^2 x-2\tan^2 x}{1-\tan^2 x}}$ oe	A1
	So $\tan 3x=\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ *	A1*cso
		(4)
(b)	Put $\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}=11\tan x$ so $3\tan x-\tan^3 x=11\tan x(1-3\tan^2 x)$	M1
	$32\tan^3 x=8\tan x$	A1
	So $\tan x=\pm\dfrac{1}{2}$ or $0\Rightarrow x=..$	dM1
	$\Rightarrow x=$ awrt $26.6°$, $-26.6°$, 0	A1 A1
		(5)
		(9 marks)

(a)

M1: Expands $\tan(2x+x)=\dfrac{\tan 2x+\tan x}{1-\tan 2x\tan x}$ condoning sign errors

dM1: Uses the correct double angle formula $\dfrac{2\tan x}{1-\tan^2 x}$ both times within their expression for tan(2*x*+*x*)

A1: Multiplies both numerator and denominator by $1-\tan^2 x$ to obtain a correct intermediate line

Eg $=\dfrac{2\tan x+\tan x(1-\tan^2 x)}{1-\tan^2 x-2\tan^2 x}$ or similar.

Alternatively they write both numerator and denominator as single correct fractions.

They cannot just write down the final given answer for this mark

A1*: Correct printed answer achieved with no errors and all of the lines in the markscheme (c.s.o.)

Withhold the final A1 for candidates who use poor notation or mixed variables.

Examples of poor notation would include $\tan \leftrightarrow \tan x$ $\tan^2 x \leftrightarrow \tan x^2$ $\tan 2x=\dfrac{2\tan\theta}{1-\tan^2\theta}$

(b)

M1: Attempts to use the given identity and multiplies by $1-3\tan^2 x$. Condone slips

A1: Obtain $32\tan^3 x=8\tan x$ or equivalent. Accept $32\tan^2 x=8$ for this mark

dM1: Obtains one value of *x* from tan *x* =... using a correct method for their equation. The order of operations to find *x* must be correct but can be scored from $\tan x=0\Rightarrow x=0$

A1: Either one of $x=26.6°$ or $-26.6°$ or in radians ± 0.46

A1: CAO $x=$ awrt $26.6°$, awrt $-26.6°$, 0 (do not need degrees symbol) with no extras within the range

Note: Answers only scores 0 marks. Answers from a correct cubic/quadratic scores M1 A1 dM1 (implied) then as scheme

Question Number	Scheme	Marks
	Examples:	
7(a)	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\left(1-2\sin^2 x\right)}{1+\left(2\cos^2 x-1\right)}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M0dM0A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}$ $=\frac{\cos^2 x+\sin^2 x-\cos^2 x+\sin^2 x}{2\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
		(3)
(b)	$\frac{2-2\cos 2\theta}{1+\cos 2\theta}-2=7\sec\theta$	
	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta\Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$\Rightarrow 2\left(\sec^2\theta-1\right)-2=7\sec\theta$	M1
	$\Rightarrow 2\sec^2\theta-7\sec\theta-4=0$	A1
	$\Rightarrow (2\sec\theta+1)(\sec\theta-4)=0$	
	$\Rightarrow \sec\theta=-\frac{1}{2},4$	
	$\Rightarrow \cos\theta=-2,\frac{1}{4}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1,A1
		(6)
		(9 marks)
7(a) alt1	$\tan^2 x=\frac{\sin^2 x}{\cos^2 x}=\frac{\frac{1}{2}\left(1-\cos 2x\right)}{\frac{1}{2}(1+\cos 2x)}=\frac{\left(1-\cos 2x\right)}{(1+\cos 2x)}$	M1dM1A1
7(a) alt2	$\frac{1-\cos 2x}{1+\cos 2x}=\tan^2 x\Rightarrow 1-\cos 2x=\tan^2 x\left(1+\cos 2x\right)$ $1-\left(1-2\sin^2 x\right)=\tan^2 x\left(1+2\cos^2 x-1\right)$ $2\sin^2 x=\frac{\sin^2 x}{\cos^2 x}\left(2\cos^2 x\right)$ $2\sin^2 x=2\sin^2 x$	M1dM1A1

Question Number	Scheme	Marks
4 (a)	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x} = \frac{2\sin x\sin x}{2\sin x\cos x}$ Allow $\frac{1-\cos 2x}{\sin 2x} \equiv \frac{2\sin^2 x}{2\sin x\cos x}$	M1A1
	$= \frac{\sin x}{\cos x} = \tan x$	A1*
		(3)
	Examples	
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{1-1+2\sin^2 x}{2\sin x\cos x} = \frac{\sin x}{\cos x} = \tan x$	M1A1A1
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x} = \frac{\sin x}{\cos x} = \tan x$	M1A1A1
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{2\sin^2 x}{2\sin x\cos x} = \tan x$	M1A1A0
	$\frac{1-\cos 2x}{\sin 2x} \equiv \frac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2}\cancel{\sin} x\cos x} = \tan x$	M1A1A1
		(3)
(b)	$3\sec^2\theta - 7 = \frac{1-\cos 2\theta}{\sin 2\theta} \Rightarrow 3\sec^2\theta - 7 = \tan\theta$	M1
	$\Rightarrow 3\left(1+\tan^2\theta\right) - 7 = \tan\theta$	M1
	$\Rightarrow 3\tan^2\theta - \tan\theta - 4 = 0$	A1
	$\Rightarrow (3\tan\theta - 4)(\tan\theta + 1) = 0$	
	$\Rightarrow \tan\theta = \frac{4}{3}, \tan\theta = -1$	**d**M1
	$\theta = 0.927, 4.069, \frac{3}{4}\pi\left(2.356\right), \frac{7}{4}\pi\left(5.498\right)$	A1 A1
		(6)
		(9 marks)

(a)

M1: Score for using $\cos 2x = 1 - 2\sin^2 x$ **and** $\sin 2x = 2\sin x\cos x$

If $\cos 2x = \cos^2 x - \sin^2 x$ is used first there must be an attempt to change into just $\sin^2 x$ by using the identity $\sin^2 x + \cos^2 x = 1$. Condone missing brackets for this mark.

A1: A correct intermediate line of e.g. $\frac{a\sin x\sin x}{a\sin x\cos x}$ or $\frac{a\sin^2 x}{a\sin x\cos x}$ or $\frac{1-1+2\sin^2 x}{2\sin x\cos x}$ or $\frac{1-\left(1-2\sin^2 x\right)}{2\sin x\cos x}$

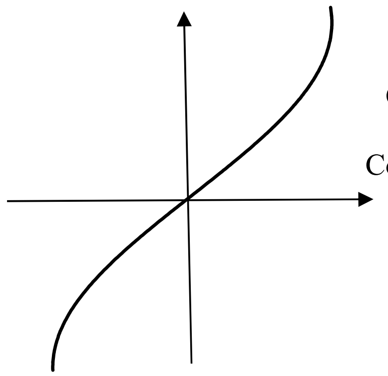
A1*: Correctly proceeds to given answer with no errors or omissions including all bracketing. There must be an intermediate line of either $\frac{\cancel{2}\cancel{\sin} x\sin x}{\cancel{2}\cancel{\sin} x\cos x}$ showing cancelling or $\frac{\sin x}{\cos x}$ or $\frac{2\sin x}{2\cos x}$ before $\tan x$ is seen and if their working necessitates the appearance of the 2's in the numerator and denominator and they are not shown, this mark can be withheld. If the candidate uses θ instead of x, the final mark should be withheld.

(b)

M1: Uses the identity from part (a) to get an equation in just $\sec^2\theta$ or $\frac{1}{\cos^2\theta}$ and $\tan\theta$

M1: Uses the identity $\sec^2\theta = \pm 1 \pm \tan^2\theta$ to get an equation in just $\tan\theta$.

A1: A correct equation in $\tan\theta$. Look for $3\tan^2\theta - \tan\theta - 4 = 0$ or equivalent.

Question	Scheme	Marks
7(a)	 Correct position **or** curvature	M1
	Correct position **and** curvature	A1
		(2)
(b)	$3\arcsin(x+1)+\pi=0 \Rightarrow \arcsin(x+1)=-\frac{\pi}{3}$	M1
	$\Rightarrow (x+1)=\sin\left(-\frac{\pi}{3}\right)$	
	$\Rightarrow x=-1-\frac{\sqrt{3}}{2}$	dM1A1
		(3)
		(5 marks)

(a) Ignore any scales that appear on the axes

M1 Accept for the method mark
Either one of the two sections with correct curvature passing through (0,0),
Or both sections condoning dubious curvature passing through (0,0) (but do not accept any negative gradients)
Or a curve with a different range or an "extended range"
See the next page for a useful guide for clarification of this mark.

A1 A curve only in quadrants one and three passing through the point (0,0) with a gradient that is always positive. The gradient should appear to be approx ∞ at each end. If you are unsure use review
If range and domain are given then ignore.

(b)

M1 Substitutes $g(x+1)=\arcsin(x+1)$ in $3g(x+1)+\pi=0$ and attempts to make $\arcsin(x+1)$ the subject
Accept $\arcsin(x+1)=\pm\frac{\pi}{3}$ or even $g(x+1)=\pm\frac{\pi}{3}$. Condone $\frac{\pi}{3}$ in decimal form awrt1.047

dM1 Proceeds by evaluating $\sin\left(\pm\frac{\pi}{3}\right)$ and making x the subject.
Accept for this mark $\Rightarrow x=\pm\frac{\sqrt{3}}{2}\pm1$. Accept decimal such as -1.866
Do not allow this mark if the candidate works in mixed modes (radians and degrees)
You may condone invisible brackets for both M's as long as the candidate is working correctly with the function

A1 $-1-\frac{\sqrt{3}}{2}$ oe with no other solutions. Remember to isw after a correct answer
Be careful with single fractions. $-\frac{2-\sqrt{3}}{2}$ and $\frac{-2+\sqrt{3}}{2}$ are incorrect but $-\frac{2+\sqrt{3}}{2}$ is correct

Note: It is possible for a candidate to change $\frac{\pi}{3}$ to $60°$ and work in degrees for all marks

M1 Combines the two fractions to form a single fraction with a common denominator. Cubic denominators are fine for this mark. Allow slips on the numerator but one must have been adapted. Allow 'invisible' brackets. Accept two separate fractions with the same denominator. Amongst many possible options are

Correct $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$,Invisible bracket $\frac{3x+1-2x-1}{(2x-1)(x+4)}$,

Cubic and separate $\frac{3(x+1)(x+4)}{(2x^2+7x-4)(x+4)} - \frac{2x^2+7x-4}{(2x^2+7x-4)(x+4)}$

M1 Simplifies the (now) single fraction to one with a linear numerator divided by a quadratic factorised denominator. Any cubic denominator must have been fully factorised (check first and last terms) and cancelled with terms on a fully factorised numerator (check first and last terms).

A1* Cso. This is a given solution and it must be fully correct. All bracketing/algebra must have been correct. You can however accept $\frac{x+4}{(2x-1)(x+4)}$ going to $\frac{1}{2x-1}$ without the need for 'seeing' the cancelling

For example $\frac{3(x+1)-2x-1}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A0. Incorrect line leading to solution.

Whereas $\frac{3(x+1)-(2x-1)}{(2x-1)(x+4)} = \frac{x+4}{(2x-1)(x+4)} = \frac{1}{2x-1}$ scores B1,M1,M1,A1

(b)

M1 This is awarded for an attempt to make x or a swapped y the subject of the formula. The minimum criteria is that they start by multiplying by (2*x*-1) and finish with x= or swapped y=. Allow 'invisible' brackets.

M1 For applying the order of operations correctly. Allow maximum of one 'slip'. Examples of this are

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2x-1 = \frac{1}{y} \to x = \frac{\frac{1}{y}\pm 1}{2}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to y(2x-1) = 1 \to 2xy - y = 1 \to 2xy = 1 \pm y \to x = \frac{1\pm y}{2y}$ (allow slip on sign)

$y = \frac{1}{2x-1} \to 2x-1 = \frac{1}{y} \to 2x = \frac{1}{y} + 1 \to x = \frac{1}{2y} + 1$ (allow slip on ÷2)

A1 Must be written in terms of x but can be $y = \frac{1+x}{2x}$ or equivalent inc $y = \frac{\frac{1}{x}+1}{2}, y = \frac{x^{-1}+1}{2}, y = \frac{1}{2x} + \frac{1}{2}$

(c)

B1 Accept x>0, (0,∞), domain is all values more than 0. **Do not accept** x≥ 0 , y>0, [0, ∞], $f^{-1}(x) > 0$

(d)

M1 Attempt to write down fg(x) and set it equal to 1/7.
The order must be correct but accept incorrect or lack of bracketing. Eg $\frac{1}{2\ln x+1-1} = \frac{1}{7}$

A1 Achieving correctly the line $\ln(x+1) = 4$. Accept also $\ln(x+1)^2 = 8$

M1 Moving from $\ln(x \pm A) = c \quad A \neq 0 \text{ to } x =$ The ln work must be correct
Alternatively moving from $\ln(x+1)^2 = c$ to $x = \cdots$
Full solutions to calculate *x* leading from $gf(x) = \frac{1}{7}$, that is $\ln\left(\frac{1}{2x-1} + 1\right) = \frac{1}{7}$ can score this mark.

A1 Correct answer only $= e^4 - 1$. Accept $e^4 - e^0$

Question No	Scheme	Marks
8	(a) $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$	M1A1
	$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A\sin B}{\cos A\cos B}}$ $(\div \cos A\cos B)$	M1

	$= \dfrac{tanA + tanB}{1 - tanAtanB}$	A1 *	
			(4)
	(b) $\tan\left(\theta + \frac{\pi}{6}\right) = \frac{tan\theta + tan\frac{\pi}{6}}{1 - tan\theta tan\frac{\pi}{6}}$	M1	
	$= \frac{tan\theta + \frac{1}{\sqrt{3}}}{1 - tan\theta\frac{1}{\sqrt{3}}}$	M1	
	$= \frac{\sqrt{3}tan\theta + 1}{\sqrt{3} - tan\theta}$	A1 *	
			(3)
	(c) $\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta)$.	M1	
	$\left(\theta + \frac{\pi}{6}\right) = (\pi - \theta)$	dM1	
	$\theta = \frac{5}{12}\pi$	ddM1 A1	
	$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(2\pi - \theta)$	dddM1	
	$\theta = \frac{11}{12}\pi$	A1	
			(6)
		(13 MARKS)	

(a)

M1 Uses the identity $\{\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}\} = \frac{sinAcosB \pm cosAsinB}{cosAcosB \mp sinAsinB}$. Accept incorrect signs for this. Just the right hand side is acceptable.

A1 Fully correct statement in terms of cos and sin $\{\tan(A+B)\} = \frac{sinAcosB + cosAsinB}{cosAcosB - sinAsinB}$

Question	Scheme	Marks	AOs
1	Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in $\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$	M1	1.1b
	Attempts both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2} \rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta}$ and attempts to simplify	M1	2.1
	$=\frac{4}{3}$ oe	A1	1.1b
		(3)	
			(3 marks)

M1: Attempts either $\sin 3\theta \approx 3\theta$ or $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$ in the given expression.

See below for description of marking of $\cos 4\theta$

M1: Attempts to substitute both $\sin 3\theta \approx 3\theta$ and $\cos 4\theta \approx 1-\frac{(4\theta)^2}{2}$

$$\rightarrow \frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} \text{ and attempts to simplify.}$$

Condone missing bracket on the 4θ so $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$ would score the method

Expect to see it simplified to a single term which could be in terms of θ

Look for an answer of k but condone $k\theta$ following a slip

A1: Uses both identities and simplifies to $\frac{4}{3}$ or exact equivalent with no incorrect lines BUT allow recovery on missing bracket for $\cos 4\theta \approx 1-\frac{4\theta^2}{2}$.

Eg. $\frac{1-\left(1-\frac{(4\theta)^2}{2}\right)}{2\theta \times 3\theta} = \frac{8\theta^2}{6\theta} = \frac{4}{3}$ is M1 M1 A0

Condone awrt 1.33.

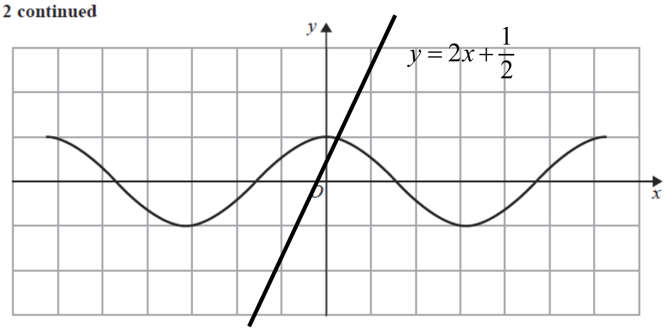
...

Alt: $\frac{1-\cos 4\theta}{2\theta \sin 3\theta} = \frac{1-\left(1-2\sin^2 2\theta\right)}{2\theta \sin 3\theta} = \frac{2\sin^2 2\theta}{2\theta \sin 3\theta} = \frac{2\times(2\theta)^2}{2\theta \times 3\theta} = \frac{4}{3}$

M1 For an attempt at $\sin 3\theta \approx 3\theta$ or the identity $\cos 4\theta = 1-2\sin^2 2\theta$ with $\sin 2\theta \approx 2\theta$

M1 For both of the above and attempts to simplify to a single term.

A1 $\frac{4}{3}$ oe

Question	Scheme	Marks	AOs
2(a)	2 continued  Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2+\sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

(a)

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$ Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator. Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2+\sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

Question	Scheme	Marks	AOs
8 (a)	$D = 5 + 2\sin(30\times 6.5)° = \text{awrt } 4.48\text{ m}$ with units	B1	3.4
		(1)	
(b)	$3.8 = 5 + 2\sin(30t)° \Rightarrow \sin(30t)° = -0.6$	M1 A1	1.1b 1.1b
	$t = 10.77$	dM1	3.1a
	10:46 a.m. or 10:47 a.m.	A1	3.2a
		(4)	
			(5 marks)

Notes:

(a)

B1: Scored for using the model ie. substituting $t = 6.5$ into $D = 5 + 2\sin(30t)°$ and stating $D = \textit{awrt}\ 4.48\text{m}$. The units must be seen somewhere in (a) . So allow when $D = 4.482.. = 4.5\text{ m}$

Allow the mark for a correct answer without any working.

(b)

M1: For using $D = 3.8$ and proceeding to $\sin(30t)° = k, \quad |k| \le 1$

A1: $\sin(30t)° = -0.6$ This may be implied by any correct answer for *t* such as $t = 7.2$

If the A1 implied, the calculation must be performed in degrees.

dM1: For finding **the first value** of *t* for their $\sin(30t)° = k$ after $t = 8.5$.

You may well see other values as well which is not an issue for this dM mark

(Note that $\sin(30t)° = -0.6 \Rightarrow 30t = 216.9°$ as well but this gives $t = 7.2$)

For the correct $\sin(30t)° = -0.6 \Rightarrow 30t = 323.1 \Rightarrow t = \text{awrt } 10.8$

For the incorrect $\sin(30t)° = +0.6 \Rightarrow 30t = 396.9 \Rightarrow t = \text{awrt } 13.2$

So award this mark if you see $30t = \text{invsin } \textit{their} - 0.6$ to give the first value of *t* where $30t > 255$

A1: Allow 10:46 a.m. (12 hour clock notation) or 10:46 (24 hour clock notation) oe

Allow 10:47 a.m. (12 hour clock notation) or 10:47 (24 hour clock notation) oe

DO NOT allow 646 minutes or 10 hours 46 minutes.

Question	Scheme	Marks	AOs
6 (a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha = 2 \Rightarrow \alpha = ...$	M1	1.1b
	$\alpha = 1.107$	A1	1.1b
		(3)	
	$\theta = 5+\sqrt{5}\sin\left(\frac{\pi t}{12}+1.107-3\right)$		
(b)	$\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C	B1ft	2.2a
		(1)	
(c)	$\frac{\pi t}{12}+1.107-3=\frac{\pi}{2}\Rightarrow t=$	M1	3.1b
	$t =$ awrt 13.2	A1	1.1b
	Either 13:14 or 1:14 pm or 13 hours 14 minutes after midnight.	A1	3.2a
		(3)	
			(7 marks)
Notes:			

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value of α from $\tan\alpha = \pm 2, \tan\alpha = \pm\frac{1}{2}, \sin\alpha = \pm\frac{2}{"R"}$ OR $\cos\alpha = \pm\frac{1}{"R"}$

It is implied by either awrt 1.11 (radians) or 63.4 (degrees)

A1: $\alpha =$ awrt 1.107

(b)

B1ft: Deduces that the maximum temperature is $\left(5+\sqrt{5}\right)$ °C or awrt 7.24 °C Remember to isw

Condone a lack of units. Follow through on their value of R so allow $(5+"R")$ °C

(c)

M1: An complete strategy to find t from $\frac{\pi t}{12}\pm 1.107-3=\frac{\pi}{2}$.

Follow through on their 1.107 but the angle must be in radians.

It is possible via degrees but only using $15t \pm 63.4-171.9=90$

A1: awrt $t=13.2$

A1: The question asks for the time of day so accept either 13:14, 1:14 pm, 13 hours 14 minutes after midnight, 13h 14, or 1 hour 14 minutes after midday. If in doubt use review

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It is possible to attempt parts (b) and (c) via differentiation but it is unlikely to yield correct results.

$\frac{d\theta}{dt}=\frac{\pi}{12}\cos\left(\frac{\pi t}{12}-3\right)-\frac{2\pi}{12}\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ scores M1 A1 A1

$\frac{d\theta}{dt}=\cos\left(\frac{\pi t}{12}-3\right)-2\sin\left(\frac{\pi t}{12}-3\right)=0\Rightarrow \tan\left(\frac{\pi t}{12}-3\right)=\frac{1}{2}\Rightarrow t=13.23=13:14$ they can score M1 A0 A1 (SC)

A value of $t=1.23$ implies the minimum value has been found and therefore incorrect method M0.

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Question Number	Scheme	Marks
13. (a)	$R = \sqrt{5} =$ 2.23606... (must be given in part (a))	B1
	$\tan\alpha = \frac{1}{2}$ or $\sin\alpha = \frac{1}{\sqrt{5}}$ or $\cos\alpha = \frac{2}{\sqrt{5}}$ (see notes for other values which gain M1)	M1
	$\Rightarrow \alpha = 26.56505...^\circ$ (must be given in part (a))	A1
		[3]
(b)	Way 1: Uses distance between two lines is 4 (or half distance is 2) with correct trigonometry **may** state $4\sin\theta + 2\cos\theta = 4$ **or** show sketch	M1
	Need sketch **and** $4\sin\theta + 2\cos\theta = 4$ **and** deduction that $2\sin\theta + \cos\theta = 2$ or $\cos\theta + 2\sin\theta = 2$ *	A1 *
		[2]
	Way 2: **Alternative method:** Uses diagonal of rectangle as hypotenuse of right angle triangle and obtains $\sqrt{20}\,\sin(\theta + \alpha) = 4$	M1
	So from (a) $2\sin\theta + \cos\theta = 2$ or $\cos\theta + 2\sin\theta = 2$	A1
		[2]
	Way 3: They may state and verify the result provided the work is correct and accurate See notes below. **Substitution of 36.9** (obtained in (c) is a circular argument and is **M0A0**)	
(c)	Way1: Uses $\sqrt{5}\sin(\theta + 26.57) = 2$ to obtain $\sin(\theta + "26.57") = \frac{2}{"\sqrt{5}"}$ $(= 0.8944...)$ Way 2 $\cos^2\theta + 4\cos\theta\sin\theta + 4\sin^2\theta = 4$ See notes for variations $4\cos\theta\sin\theta - 3\cos^2\theta = 0$ $\cos\theta(4\sin\theta - 3\cos\theta) = 0$ so $\tan\theta = \frac{3}{4}$	M1
	Way 1: $\theta = \arcsin\left(\frac{2}{\text{their } "\sqrt{5}"}\right) - "26.57"$ Way 2: $\theta = \arctan\frac{3}{4}$ or equivalent	M1
	Hence, $\theta = 36.8699...^\circ$	A1
		[3]
(d)	Way 1: $"x" = \frac{2}{\tan"36.9"}$ Way 2: "y" $= \frac{4}{\sin\theta}$	B1
	Way 1: $\{h + x = 4 \Rightarrow\}\ h + \frac{2}{\tan"36.9"} = 4$ Way 2: $\{h + y = 8 \Rightarrow\}\ h + \frac{4}{\sin"36.9"} = 8$	M1
	Way 1: $h = 4 - \frac{2}{\tan 36.9} = 1.336...$ or $\frac{4}{3}$ or <u>1.3</u> (2sf) Way 2: $h = 8 - \frac{4}{\sin 36.9} = \frac{4}{3}$ or <u>1.3</u> (2sf)	<u>A1</u> **cao**
		[3]
		11

Question Number	Scheme	Marks
13(a)	$R = \sqrt{109}$	B1
	$\tan\alpha = \dfrac{3}{10} \Rightarrow \alpha = \text{awrt}16.70°$	M1A1
		(3)
(b)(i)	Max height = 12+ '$\sqrt{109}$ '= 22.44 m	M1A1
(ii)	Occurs when $30t + '16.70' = 180 \Rightarrow t = 5.44$	M1A1
		(4)
(c)	$18 = 12 - \sqrt{109}\cos(30t + 16.70) \Rightarrow \cos(30t + 16.70) = -\dfrac{6}{\sqrt{109}} \quad (-0.57..)$	M1A1
	$\Rightarrow 30t + 16.70 = \arccos\left(-\dfrac{6}{\sqrt{109}}\right) \Rightarrow t = ..$	dM1
	$t = \text{awrt } 3.61$ (2dp)	A1
		(4)
(d)	Attempting $30t = 360 \Rightarrow t = ..$ or $30t = 720 \Rightarrow t = ..$	M1
	2 revolutions in 24 **minutes**	A1
		(2)
		(13 marks)

Question Number	Scheme	Marks
11(a)	$(R=\sqrt{1.5^2+1.2^2})=$ awrt 1.921 - accept e.g. $\sqrt{3.69}$ or $\dfrac{3\sqrt{41}}{10}$	B1
	$\tan\alpha=\dfrac{1.2}{1.5}\Rightarrow\alpha=0.675$ or 0.215π	M1A1
		(3)
(b)	$H=3+1.921\sin\left(\dfrac{\pi t}{6}-0.675\right)$	
	$H_{\min}=3-\text{'}1.921\text{'}=\text{awrt}\,1.08$	M1A1
	$\left(\dfrac{\pi t}{6}-\text{"}0.675\text{"}\right)=\dfrac{3\pi}{2}\Rightarrow t=10.29$	M1A1
		(4)
(c)	$4=3+1.921\sin\left(\dfrac{\pi t}{6}-0.675\right)\Rightarrow\sin\left(\dfrac{\pi t}{6}-0.675\right)=\dfrac{1}{1.921}$	M1
	$\dfrac{\pi t}{6}-0.675=0.548\Rightarrow t=$ awrt 2.33 or 2.34	dM1A1
	$\dfrac{\pi t}{6}-0.675=\pi-0.548=2.594\Rightarrow t=$ awrt 6.24 or 6.25	ddM1A1
	Times are 2:20pm and 6:15pm or 6.14pm (14:20 and 18:15 or 18:14) – allow 2 hours 20minutes and 6 hours 15 or 14minutes or 140 minutes and 375 or 374 minutes	A1
	Extra values in the range – lose final A mark.	**(6)**
		(13 marks)

Question Number	Scheme	Marks
10. (a)	$R = \sqrt{34}$	B1
	$\tan\alpha = \frac{5}{3}$	M1
	$\Rightarrow \alpha = 1.03$ awrt 1.03	A1
		[3]
(b)	$3\sin 2x + 5\cos 2x = 4 \Rightarrow \sqrt{34}\sin(2x+1.03) = 4$	
	$\sin(2x + "1.03") = \frac{4}{"\sqrt{34}"} \quad (= 0.68599...)$	M1
	One solution in range Eg. $2x + "1.03" = 2\pi + \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x = ...$	M1
	Either $x =$ awrt 3.0 **or** awrt 0.68	A1
	Second solution in range Eg $2x + "1.03" = \pi - \arcsin\left(\frac{4}{"\sqrt{34}"}\right) \Rightarrow x = ...$	M1
	Both $x =$ awrt 2sf 3.0 **and** 0.68	A1
		[5]
(c)	Greatest value is $4\left(\sqrt{34}\right)^2 + 3 = 139$	M1 A1
	Least value is $4(0) + 3 = 3$.	M1 A1
		[4]
		(12 marks)

Question Number	Scheme	Notes	Marks
1.(a)	$R=\sqrt{34}$	Cao (Must be exact but score when first seen and ignore decimal value (5.83…))	B1
	$\tan\alpha=\pm\frac{5}{3},\ \tan\alpha=\pm\frac{3}{5}\Rightarrow\alpha=...$ (Allow $\cos\alpha=\pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}$, $\sin\alpha=\pm\frac{5}{\sqrt{34}}$ or $\pm\frac{3}{\sqrt{34}}\Rightarrow\alpha=...$) Where $\sqrt{34}$ is their R		M1
	$\alpha=59.04°$	awrt 59.04°	A1
			(3)
(b)	$\sqrt{34}\cos(\theta-59.04)=2\Rightarrow\cos(\theta-59.04)=\frac{2}{\sqrt{34}}(0.343)$ Attempts to use part (a)"$\sqrt{34}$"$\cos(\theta-$"59.04"$)=2$ and proceeds to $\cos(\theta\pm$"59.04"$)=K,\ \lvert K\rvert\leqslant 1$ May be implied by $\theta-$"59.04"$=69.94...°$ or $\theta-$"59.04"$\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)$ The $\theta-$"59.04" must be seen here or implied later		M1
	$\theta_1-59.04=69.94\Rightarrow\theta_1=$ awrt $129.0°$		A1
	$\theta_2\pm59.04=360-$'69.94'$\Rightarrow\theta_2=...$ Correct attempt at a second solution in the range. It is **dependent** upon having scored the previous M. Usually for $\theta-$ their $59.04=360-$ their '69.94'$\Rightarrow\theta=...$		**d**M1
	$\theta_2=349.1°$	awrt 349.1°	A1
	For solutions in (b) that are otherwise fully correct, if there are extra answers in range, deduct the final A mark.		
			(4)
(c)	$\theta+$ their $59.04=\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)\Rightarrow\theta=...$ Allow $\theta-$ their $59.04=\cos^{-1}\left(\frac{2}{\text{their}\sqrt{34}}\right)\Rightarrow\theta=...$ if they have $\theta+$ .. in (b) Evidence that use is being made of parts (a) and (b) to obtain a value for θ. This can be implied by the use of their answers to part (b).		M1
	$\theta=10.9°$	awrt 10.9	A1
			(2)
			(9 marks)

Qu	Scheme	Marks
11. (a)	$R = 37$	B1
	$\tan\alpha = \frac{12}{35} \Rightarrow \alpha = \text{awrt } 0.3303$	M1 A1
		(3)
(b)	$\sin(x-\alpha) = \frac{37}{2R} \quad (= 0.5...)$	M1
	$x = \arcsin\left(\frac{37}{2\times\text{their "37"}}\right) + \text{their"0.3303"}$	M1
	$x = $ awrt 0.854 or awrt 2.95	A1
	$x = $ awrt 0.854 and awrt 2.95	A1
		(4)
(c)(i)	Find $y = \frac{7000}{31+(\pm\text{"}R\text{"})^2} = 5$	M1 A1
(c)(ii)	$x - \alpha = \frac{\pi}{2} \Rightarrow x = 1.90$	M1 A1
		(4)
		(11 marks)

(a)

B1: $R = 37$ no working needed. Condone $R = \pm 37$

M1: $\tan\alpha = \pm\frac{12}{35}$ or $\tan\alpha = \pm\frac{35}{12}$ with an attempt to find alpha. Accept decimal attempts from $\tan\alpha = \text{awrt} \pm 0.343$ or $\tan\alpha = \text{awrt} \pm 2.92$ If R is used allow $\sin\alpha = \pm\frac{12}{R}$ OR $\cos\alpha = \pm\frac{35}{R}$ with an attempt to find alpha

A1: $\alpha = \text{awrt } 0.3303$. Answers in degrees are A0

(b)

M1: (Uses part (a) to solve equation) $\sin(x\pm\alpha) = \frac{37}{2\times theirR}$

M1: operations undone in the correct order to give $x =$... Accept $\sin(x\pm\alpha) = k \Rightarrow x = \arcsin k \pm \alpha$

A1: one correct answer to within required accuracy. Allow 0.272π or 0.938π.

Condone for this mark only **both** $\frac{\pi}{6} + 0.3303$ **and** $\frac{5\pi}{6} + 0.3303$

A1: both values (and no extra values in the range) correct to within required accuracy. Allow $0.272\pi, 0.938\pi$

(c) (i)

M1: For an attempt at $\frac{7000}{31+(\pm\text{"}R\text{"})^2}$

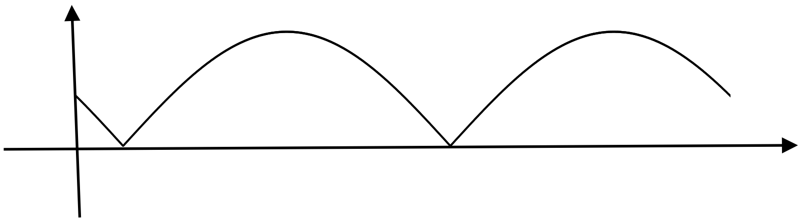
A1: 5

(c)(ii)

M1: Uses $x - \text{their } \alpha = (2n+1)\frac{\pi}{2}$ to find x This may be implied by $1.57 \pm \text{their } 0.33$ stated or calculated (2dp)

A1: Awrt 1.90 but condone 1.9 for this answer

Answers in degrees, withhold the first time seen, usually part (a). FYI (a) $18.92°$ (b) $48.9°, 168.9°$ (c)(ii) $108.9°$

Question Number	Scheme	Marks
10(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow(\alpha=)26.6°$	M1,A1
		(3)
(b)	 Shape	B1
	$(0,1)$	B1
	$("26.6",0)$ and $("206.6",0)$ (Allow in radians i.e. their α and π+α	B1ft
		(3)
(c)(i)	$5+'R'=5+\sqrt{5}$	B1ft
(c)(ii)	$15t-'26.6'=270\Rightarrow t=19.8$	M1,A1
		(3)
		(9 marks)

(a)

B1: $R=\sqrt{5}$

M1: For $\tan\alpha=\pm\frac{1}{2}$ or $\tan\alpha=\pm\frac{2}{1}$ or $\sin\alpha=\pm\frac{1}{"\sqrt{5}"}$ or $\cos\alpha=\pm\frac{2}{"\sqrt{5}"}$

A1: Awrt $\alpha=26.6°$

(b)

B1: Correct shape including cusps. A curve that starts downwards from the positive y-axis with two maxima. This mark is essentially for realising that the parts of the curve under the x-axis are reflected in the x-axis and for cusps that look “pointed” and not rounded.

B1: $(0,1)$ may be seen on the diagram or in the body of the script as coordinates or seen as x = 0, y = 1. If there is any ambiguity, the sketch takes precedence. Allow (1, 0) as long as it is marked in the correct place on the sketch.

B1**ft**: $(26.6,0)$ and $(206.6,0)$ or their 26.6 and 180 + their 26.6. May be seen on the sketch or in the body of the script as coordinates or seen as y = 0, $\theta(\text{or } x)=26.6$, $\theta(\text{or } x)=206.6$. If there is any ambiguity, the sketch takes precedence. Allow awrt 26.6 and awrt 207 or their ft values.

(c)(i)

B1ft: Follow through on $5+'R'$ including decimal answers (NB 5 + √5 = 7.24…)

(c)(ii)

M1: Attempts $15t-"26.6"=90$ or $270\Rightarrow t=...$ (Allow π/2, 3π/2 for 90, 270 if working in radians)

A1: $t=19.8$ **only**

(c)(ii) Alternative:

$f(t)=5+2\sin(15t)-\cos(15t)\Rightarrow f'(t)=30\cos(15t)+15\sin(15t)$

M1: Attempts $f'(t)=0\Rightarrow 15t=180-63.43...$ or $360-63.43$

A1: $t=19.8$ **only**

Question Number	Scheme	Marks
4.(a)	$R=\sqrt{29}$	B1
	$\tan\alpha=\frac{2}{5}\Rightarrow\alpha=\text{awrt }0.381$	M1A1
		(3)
(b)	$5\cot 2x-3\text{cosec}\,2x=2\Rightarrow 5\frac{\cos 2x}{\sin 2x}-\frac{3}{\sin 2x}=2$	M1
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1
		(2)
(c)	$5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\frac{3}{\sqrt{29}}$	M1
	$2x+0.381=\arccos\left(\frac{3}{\sqrt{29}}\right)\Rightarrow x=...$	dM1
	$x=\text{awrt }0.30,\ 2.46$	A1A1
		(4)
		(9 marks)
Alt I (c)	$5\cos 2x-2\sin 2x=3\Rightarrow 10\cos^2 x-5-4\sin x\cos x=3$	
	$\Rightarrow 4\tan^2 x+2\tan x-1=0$	M1
	$\Rightarrow\tan x=\frac{-1\pm\sqrt{5}}{4}\Rightarrow x=..$	dM1
	$x=\text{awrt }0.30,\ 2.46$	A1A1
		(4)
Alt II (c)	$5\cos 2x-2\sin 2x=3\Rightarrow(5\cos 2x)^2=(3+2\sin 2x)^2$ **&** $\cos^2 2x=1-\sin^2 2x$	
	$\Rightarrow 29\sin^2 2x+12\sin 2x-16=0$	M1
	$\Rightarrow\sin 2x=\frac{-12\pm\sqrt{2000}}{58}\Rightarrow 2x=..\Rightarrow x=..$	dM1
	$x=\text{awrt }0.30,\ 2.46$	A1A1
		(4)

(a)

B1 $R=\sqrt{29}$

Condone $R=\pm\sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$)

M1 $\tan\alpha=\pm\frac{2}{5},\ \tan\alpha=\pm\frac{5}{2}\Rightarrow\alpha=...$

If R is used to find α accept $\sin\alpha=\pm\frac{2}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=...$

Question	Scheme	Marks
3.(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(awrt\ 0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$	A1
	$\theta+26.6°=360°-\text{their}\ '59.5°'$	dM1
	$\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$	A1
		(5)
(c)	$\theta-\text{their}\ 26.57°=\text{their}\ 59.54°\Rightarrow\theta=...$	M1
	$\theta=\text{awrt}\ 86.1°$	A1
		(2)
		(10 marks)

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\frac{1}{2}$, $\tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$

If their value of R is used to find the value of α only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=\text{awrt}\ 26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their}\ 26.6°)=K$, $|K|\leqslant 1$

A1 $\cos(\theta\pm\text{their}\ 26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt}\ 0.507)$. Can be implied by $(\theta\pm\text{their}\ 26.6°)=\text{awrt}\ 59.5°/59.6°$

A1 One solution correct, $\theta=awrt\ 33.0°$ or $\theta=awrt\ 273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm\text{their}\ 26.6°=360°-\text{their}\ 59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=awrt\ 33.0°$ and $awrt\ 273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\leqslant\theta<360°$

(c)

M1 $\theta-\text{their}\ 26.57°=\text{their}\ 59.54°\Rightarrow\theta=...$
Alternatively $-\theta+\text{their}\ 26.6°=-\text{their}\ 59.5°\Rightarrow\theta=...$
If the candidate has an incorrect sign for α, for example they used $\cos(\theta-26.57°)$ in part (b) it would be scored for $\theta+\text{their}\ 26.57°=\text{their}\ 59.54°\Rightarrow\theta=...$

A1 awrt $86.1°$ ONLY. Allow both marks following a correct (a) and (b)
They can restart the question to achieve this result. Do not award if 86.1 was their smallest answer in (b). This occurs when they have $\cos(\theta-26.57°)$ instead of $\cos(\theta+26.57°)$ in (b)

Answers in radians: Withhold only one A mark, the first time a solution in radians appears
FYI (a) $\alpha=0.46$ (b) $\theta_1=\text{awrt}\ 0.58$ and $\theta_2=\text{awrt}\ 4.78$ (c) $\theta_3=\text{awrt}\ 1.50$. Require 2 dp accuracy

Question Number	Scheme	Marks
3(a)	$4\cos 2\theta + 2\sin 2\theta = R\cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = \left(2\sqrt{5}\right)$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565°... = \text{awrt } 26.57°$	M1A1
		(3)
(b)	$\sqrt{20}\cos(2\theta - 26.6) = 1 \Rightarrow \cos(2\theta - 26.57) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57) = +77.1.. \Rightarrow \theta = ...$	dM1
	$\theta = \text{awrt } 51.8°$	A1
	$2\theta - 26.57 = \text{'}-77.1...\text{'} \Rightarrow \theta = -\text{awrt } 25.3°$	ddM1A1
		(5)
(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)

You can marks parts (a) and (b) together as one.

(a)

B1 For $R = \sqrt{20} = 2\sqrt{5}$. Condone $R = \pm\sqrt{20}$

M1 For $\alpha = \arctan\left(\pm\frac{1}{2}\right)$ or $\alpha = \arctan(\pm 2)$ leading to a solution of α

Condone any solutions coming from $\cos\alpha = 4, \sin\alpha = 2$

Condone for this mark $2\alpha = \arctan\left(\pm\frac{1}{2}\right) \Rightarrow \alpha = ..$

If R has been used to find α award for only $\alpha = \arccos\left(\pm\frac{4}{\text{'}R\text{'}}\right)$ $\alpha = \arcsin\left(\pm\frac{2}{\text{'}R\text{'}}\right)$

A1 $\alpha = \text{awrt } 26.57°$

Question Number	Scheme	Marks
7.(a)	$R=\sqrt{\left(6^2+2.5^2\right)}=6.5$	B1
	$\tan\alpha=\frac{2.5}{6}, \quad \Rightarrow \quad \alpha=\text{awrt } 0.395$	M1A1
		(3)
(b)	$(0,6)$,	B1
	awrt $(1.97,0) \quad (5.11,0)$	M1A1
		(3)
(c)	$H_{\max}=18.5, H_{\min}=5.5$	M1A1A1
		(3)
(d)	Sub $H=16$ and proceed to $'6.5'\cos\left(\frac{2\pi t}{52}\pm'0.395'\right)=4$	M1
	$\left(\frac{2\pi t}{52}-'0.395'\right)=\text{awrt } 0.91$	A1
	$t=\left(awrt\, 0.908\pm'0.395'\right)\times\frac{52}{2\pi}=11\ (10.78)$	dM1A1
	$\left(\frac{2\pi t}{52}\pm'0.395'\right)=awrt\, 2\pi-0.908\Rightarrow t=48\ (47.75)$	ddM1A1
		(6)
		(15 marks)

(a)

B1 $R=6.50, \frac{13}{2}$. Accept $R=\text{awrt } 6.50$. Do not accept $R=\pm 6.50$

M1 For reaching $\tan\alpha=\pm\frac{2.5}{6}$ *or* $\tan\alpha=\pm\frac{6}{2.5}$.

If R has been attempted first then only accept $\sin\alpha=\pm\frac{2.5}{'R'}$ *or* $\cos\alpha=\pm\frac{6}{'R'}$

A1 Correct value $\alpha=awrt\ 0.395$. The answer in degrees $22.6°$ is A0

(b)

B1 The correct y intercept. Accept $y=6$, $(0,6)$, awrt y = 6.00, $f(0)=6$ or it marked on the curve. Do not accept $(6,0)$

M1 Attempt to find either x intercept from $\frac{\pi}{2}+\text{their } 0.395$, or $\frac{3\pi}{2}+\text{their } 0.395$

If the candidate is working in degrees accept $90+\text{their } 22.6$ or $270+\text{their } 22.6$

One answer correct will imply this.

A1 Both answers correct. Accept awrt $(1.97,0)$ and $(5.11,0)$, Accept $x=1.97$ and $x=5.11$ or both being marked on the curve. Do not accept $(0,1.97)$ and $(0,5.11)$ for both marks

In degrees accept $(112.6,0)$ and $(292.6,0)$

Question Number	Scheme	Marks
9.(a)	$R=\sqrt{20}$	B1
	$\tan\alpha=\frac{4}{2}\Rightarrow\alpha=\text{awrt }1.107$	M1A1
		(3)
(b)(i)	$'4+5R^2'=104$	B1ft
(ii)	$3\theta-'1.107'=\frac{\pi}{2}\Rightarrow\theta=\text{awrt }0.89$	M1A1
		(3)
(c)(i)	4	B1
(ii)	$3\theta-'1.107'=2\pi\Rightarrow\theta=\text{awrt }2.46$	M1A1
		(3)
		(9 marks)

Question Number	Scheme	Marks
8(a)	$R=\sqrt{\left(7^2+24^2\right)}=25$	B1
	$\tan\alpha=\frac{24}{7}, \quad \Rightarrow \quad \alpha=\text{awrt}\,73.74°$	M1A1
		(3)
(b)	maximum value of 24sin$x$ + 7cos$x$ = 25 so $V_{\min}=\frac{21}{25}=(0.84)$	M1A1
		(2)
(c)	Distance $AB=\frac{7}{\sin\theta}$, with $\theta=\alpha$	M1, B1
	So distance = 7.29m $=\frac{175}{24}$m	A1
		(3)
(d)	$R\cos(\theta-\alpha)=\frac{21}{1.68}\Rightarrow\cos(\theta-\alpha)=0.5$	M1, A1
	$\theta-\alpha=60\Rightarrow\theta=..,\theta-\alpha=-60\Rightarrow\theta=..$	dM1, dM1
	$\theta=\text{awrt}\,133.7, 13.7$	A1, A1
		(6) **(14 marks)**

Notes for Question 8

(a)

B1 25. Accept 25.0 but not $\sqrt{625}$ or answers that are not exactly 25. Eg 25.0001

M1 For $\tan\alpha=\pm\frac{24}{7}, \tan\alpha=\pm\frac{7}{24}$.

If the value of R is used only accept $\sin\alpha=\pm\frac{24}{R}, \cos\alpha=\pm\frac{7}{R}$

A1 Accept answers which round to 73.74 – must be in degrees for this mark

(b)

M1 Calculates $V=\frac{21}{their\,'R'}$ NOT - R

A1 Obtains correct answer. $V=\frac{21}{25}$ Accept 0.84

Do not accept if you see incorrect working- ie from $\cos(\theta-\alpha)=-1$ or the minus just disappearing from a previous line.

Questions involving differentiation are acceptable. To score M1 the candidate would have to differentiate V by the quotient rule (or similar) , set V'=0 to find θ and then sub this back into V to find its value.

Question Number	Scheme	Marks
4.	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $p(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
	$p(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - '\mathit{their}\,\alpha' = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)

(a) M1 Using Pythagoras' Theorem with 6 and 8 to find *R*. Accept $R^2 = 6^2 + 8^2$

If α has been found first accept $R = \pm\frac{8}{\sin'\alpha'}$ or $R = \pm\frac{6}{\cos'\alpha'}$

A1 $R = 10$. Many candidates will just write this down which is fine for the 2 marks. Accept ± 10 but not -10

M1 For $\tan\alpha = \pm\frac{8}{6}$ or $\tan\alpha = \pm\frac{6}{8}$

If *R* is used then only accept $\sin\alpha = \pm\frac{8}{R}$ or $\cos\alpha = \pm\frac{6}{R}$

A1 $\alpha = \text{awrt } 0.927$. Note that 53.1^0 is A0

(b) Note that (b)(i) and (b)(ii) can be marked together

(i) M1 Award for $p(x) = \frac{4}{12 - 'R'}$.

A1 Cao $p(x)_{\max} = 2$.

The answer is acceptable for both marks as long as no incorrect working is seen

(ii) M1 For setting $\theta - '\mathit{their}\,\alpha' = \pi$ and proceeding to θ=..

If working exclusively in degrees accept $\theta - '\mathit{their}\,\alpha' = 180$

Do not accept mixed units

A1 $\theta = \text{awrt } 4.07$. If the final A mark in part (a) is lost for 53.1, then accept awrt 233.1