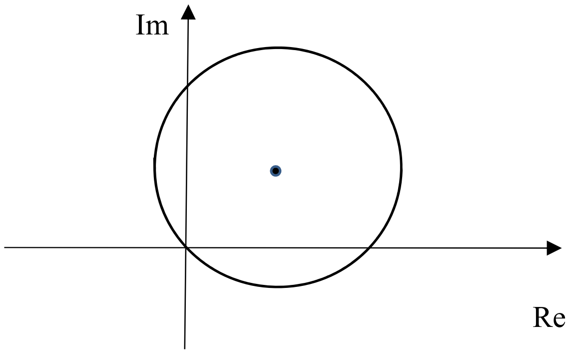
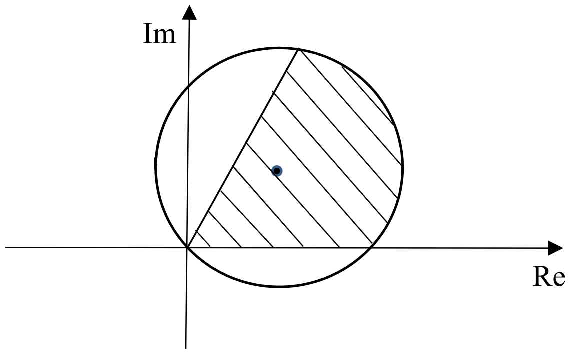
Question	Scheme	Marks	AOs
4(a)	$z^n + z^{-n} = \cos n\theta + \mathrm{i}\sin n\theta + \cos n\theta - \mathrm{i}\sin n\theta$	M1	2.1
	$= 2\cos n\theta$ *	A1*	1.1b
		(2)	
(b)	$\left(z + z^{-1}\right)^4 = 16\cos^4\theta$	B1	2.1
	$\left(z + z^{-1}\right)^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$	M1	2.1
	$= z^4 + z^{-4} + 4\left(z^2 + z^{-2}\right) + 6$	A1	1.1b
	$= 2\cos 4\theta + 4\left(2\cos 2\theta\right) + 6$	M1	2.1
	$\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$*	A1*	1.1b
		(5)	
			(7 marks)

Notes:

(a)
M1: Identifies the correct form for z^n and z^{-n} and adds to progress to the printed answer
A1*: Achieves printed answer with no errors

(b)
B1: Begins the argument by using the correct index with the result from part (a)
M1: Realises the need to find the expansion of $\left(z + z^{-1}\right)^4$
A1: Terms correctly combined
M1: Links the expansion with the result in part (a)
A1*: Achieves printed answer with no errors

Question	Scheme	Marks	AOs
6(a)(i)		M1 A1	1.1b 1.1b
(a)(ii)	$\lvert z-4-3\text{i}\rvert=5\Rightarrow\lvert x+\text{i}y-4-3\text{i}\rvert=5\Rightarrow(x-4)^2+(y-3)^2=...$	M1	2.1
	$(x-4)^2+(y-3)^2=25$ or any correct form	A1	1.1b
	$(r\cos\theta-4)^2+(r\sin\theta-3)^2=25$ $\Rightarrow r^2\cos^2\theta-8r\cos\theta+16+r^2\sin^2\theta-6r\sin\theta+9=25$ $\Rightarrow r^2-8r\cos\theta-6r\sin\theta=0$	M1	2.1
	$\therefore r=8\cos\theta+6\sin\theta$*	A1*	2.2a
		(6)	
(b)(i)		B1 B1ft	1.1b 1.1b
(b)(ii)	$A=\frac{1}{2}\int r^2\,\text{d}\theta=\frac{1}{2}\int(8\cos\theta+6\sin\theta)^2\,\text{d}\theta$ $=\frac{1}{2}\int\left(64\cos^2\theta+96\sin\theta\cos\theta+36\sin^2\theta\right)\text{d}\theta$	M1	3.1a
	$=\frac{1}{2}\int\left(32(\cos2\theta+1)+96\sin\theta\cos\theta+18(1-\cos2\theta)\right)\text{d}\theta$	M1	1.1b
	$=\frac{1}{2}\int(14\cos2\theta+50+48\sin2\theta)\,\text{d}\theta$	A1	1.1b
	$=\frac{1}{2}\left[7\sin2\theta+50\theta-24\cos2\theta\right]_0^{\frac{\pi}{3}}=\frac{1}{2}\left\{\left(\frac{7\sqrt{3}}{2}+\frac{50\pi}{3}+12\right)-(-24)\right\}$	M1	2.1
	$=\frac{7\sqrt{3}}{4}+\frac{25\pi}{3}+18$	A1	1.1b
		(7)	

Question	Scheme	Marks	AOs
4(a) Way 1	$\mathrm{C}+\mathrm{iS}=\cos\theta+\mathrm{i}\sin\theta+\frac{1}{2}(\cos 5\theta+\mathrm{i}\sin 5\theta)\left(+\frac{1}{4}(\cos 9\theta+\mathrm{i}\sin 9\theta)+\ldots\right)$	M1	1.1b
	$=\mathrm{e}^{\mathrm{i}\theta}+\frac{1}{2}\mathrm{e}^{5\mathrm{i}\theta}\left(+\frac{1}{4}\mathrm{e}^{9\mathrm{i}\theta}+\ldots\right)$	A1	2.1
	$\mathrm{C}+\mathrm{iS}=\frac{\mathrm{e}^{\mathrm{i}\theta}}{1-\frac{1}{2}\mathrm{e}^{4\mathrm{i}\theta}}$	M1	3.1a
	$=\frac{2\mathrm{e}^{\mathrm{i}\theta}}{2-\mathrm{e}^{4\mathrm{i}\theta}}$ *	A1*	1.1b
		(4)	
(a) Way 2	$\mathrm{C}+\mathrm{iS}=\cos\theta+\mathrm{i}\sin\theta+\frac{1}{2}(\cos 5\theta+\mathrm{i}\sin 5\theta)\left(+\frac{1}{4}(\cos 9\theta+\mathrm{i}\sin 9\theta)+\ldots\right)$	M1	1.1b
	$\mathrm{C}+\mathrm{iS}=\cos\theta+\mathrm{i}\sin\theta+\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)^5\left(+\frac{1}{4}(\cos\theta+\mathrm{i}\sin\theta)^9+\ldots\right)$	A1	2.1
	$\mathrm{C}+\mathrm{iS}=\frac{\cos\theta+\mathrm{i}\sin\theta}{1-\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)^4}=\frac{\mathrm{e}^{\mathrm{i}\theta}}{1-\frac{1}{2}\mathrm{e}^{4\mathrm{i}\theta}}$	M1	3.1a
	$=\frac{2\mathrm{e}^{\mathrm{i}\theta}}{2-\mathrm{e}^{4\mathrm{i}\theta}}$ *	A1*	1.1b
		(4)	
(b) Way 1	$\frac{2\mathrm{e}^{\mathrm{i}\theta}}{2-\mathrm{e}^{4\mathrm{i}\theta}}\times\frac{2-\mathrm{e}^{-4\mathrm{i}\theta}}{2-\mathrm{e}^{-4\mathrm{i}\theta}}$	M1	3.1a
	$\frac{4\mathrm{e}^{\mathrm{i}\theta}-2\mathrm{e}^{-3\mathrm{i}\theta}}{4-2\mathrm{e}^{-4\mathrm{i}\theta}-2\mathrm{e}^{4\mathrm{i}\theta}+1}$	A1	1.1b
	$\frac{4\cos\theta+4\mathrm{i}\sin\theta-2\cos 3\theta+2\mathrm{i}\sin 3\theta}{5-2\cos 4\theta+2\mathrm{i}\sin 4\theta-2\cos 4\theta-2\mathrm{i}\sin 4\theta}$ **Dependent on the first M**	**d**M1	2.1
	$S=\frac{4\sin\theta+2\sin 3\theta}{5-4\cos 4\theta}$ *	A1*	1.1b
		(4)	
(b) Way 2	$\frac{2\mathrm{e}^{\mathrm{i}\theta}}{2-\mathrm{e}^{4\mathrm{i}\theta}}=\frac{2(\cos\theta+\mathrm{i}\sin\theta)}{2-(\cos 4\theta+\mathrm{i}\sin 4\theta)}\times\frac{2-(\cos 4\theta-\mathrm{i}\sin 4\theta)}{2-(\cos 4\theta-\mathrm{i}\sin 4\theta)}$	M1	3.1a
	$\frac{4\cos\theta+4\mathrm{i}\sin\theta-2\cos\theta\cos 4\theta-2\sin\theta\sin 4\theta+2\mathrm{i}\sin 4\theta\cos\theta-2\mathrm{i}\sin\theta\cos 4\theta}{4+\cos^2 4\theta+\sin^2 4\theta-4\cos 4\theta}$	A1	1.1b
	$\frac{4\cos\theta+4\mathrm{i}\sin\theta-2\cos 3\theta+2\mathrm{i}\sin 3\theta}{5-2\cos 4\theta+2\mathrm{i}\sin 4\theta-2\cos 4\theta-2\mathrm{i}\sin 4\theta}$ **Dependent on the first M**	**d**M1	2.1
	$S=\frac{4\sin\theta+2\sin 3\theta}{5-4\cos 4\theta}$ *	A1*	1.1b
	(8 marks)		

Question	Scheme	Marks	AOs
6(a)	Examples: $\begin{pmatrix}\cos 120 & -\sin 120\\ \sin 120 & \cos 120\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}=\ldots$ **or** $(6+2\text{i})\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\text{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{2\pi}{3}\right)+\text{i}\sin\left(\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)+\text{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{2\pi}{3}\right)\right)$ **or** $\sqrt{40}\text{e}^{\text{i}\arctan\left(\frac{2}{6}\right)}\text{e}^{\text{i}\left(\frac{2\pi}{3}\right)}$	M1	3.1a
	$\left(-3-\sqrt{3}\right)$ **or** $\left(3\sqrt{3}-1\right)\text{i}$	A1	1.1b
	$\left(-3-\sqrt{3}\right)+\left(3\sqrt{3}-1\right)\text{i}$	A1	1.1b
	Examples: $\begin{pmatrix}\cos 240 & -\sin 240\\ \sin 240 & \cos 240\end{pmatrix}\begin{pmatrix}6\\2\end{pmatrix}=\ldots$ **or** $(6+2\text{i})\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}\right)$ **or** $\sqrt{40}\left(\cos\arctan\left(\frac{2}{6}\right)+\text{i}\sin\arctan\left(\frac{2}{6}\right)\right)\left(\cos\left(\frac{4\pi}{3}\right)+\text{i}\sin\left(\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\left(\cos\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)+\text{i}\sin\left(\arctan\left(\frac{2}{6}\right)+\frac{4\pi}{3}\right)\right)$ **or** $\sqrt{40}\text{e}^{\text{i}\arctan\left(\frac{2}{6}\right)}\text{e}^{\text{i}\left(\frac{4\pi}{3}\right)}$	M1	3.1a
	$\left(-3+\sqrt{3}\right)$ **or** $\left(-3\sqrt{3}-1\right)\text{i}$	A1	1.1b
	$\left(-3+\sqrt{3}\right)+\left(-3\sqrt{3}-1\right)\text{i}$	A1	1.1b
		(6)	
(b) Way 1	Area ABC = $3\times\frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$ or Area AOB = $\frac{1}{2}\sqrt{6^2+2^2}\sqrt{6^2+2^2}\sin 120°$	M1	2.1
	Area DEF = $\frac{1}{4}ABC$ or $\frac{3}{4}AOB$	**d**M1	3.1a
	$=\frac{3}{8}\times 40\times\frac{\sqrt{3}}{2}=\frac{15\sqrt{3}}{2}$	A1	1.1b
		(3)	

Question	Scheme	Marks	AOs
2(a)	Centre of circle C is (1, –1)	B1	1.1b
	$r=\sqrt{(5-1)^2+(-4+1)^2}=5$ or $r=\sqrt{(-3-1)^2+(2+1)^2}=5$ or $r=\frac{1}{2}\sqrt{(-3-5)^2+(2+4)^2}=5$	M1	3.1a
	$\lvert z-1+\mathrm{i}\rvert=5$ or $\lvert z-(1-\mathrm{i})\rvert=5$	A1	2.5
		(3)	
(b)	$(x-1)^2+(y+1)^2=25,\quad (x-2)^2+(y-3)^2=4$ $x^2-2x+1+y^2+2y+1=25$ $x^2-4x+4+y^2-6y+9=4$ $\Rightarrow 2x+8y=32$	M1	3.1a
	$(16-4y)^2-4(16-4y)+4+y^2-6y+9=4$ or $x^2-4x+4+\left(\frac{16-x}{4}\right)^2-6\left(\frac{16-x}{4}\right)+9=4$	M1	1.1b
	$17y^2-118y+201=0$ or $17x^2-72x+16=0$	A1	1.1b
	$17y^2-118y+201=0\Rightarrow(17y-67)(y-3)=0\Rightarrow y=\frac{67}{17},3$ or $17x^2-72x+16=0\Rightarrow(17x-4)(x-4)=0\Rightarrow x=\frac{4}{17},4$	M1	1.1b
	$y=\frac{67}{17},3\Rightarrow x=\frac{4}{17},4$ or $x=\frac{4}{17},4\Rightarrow y=\frac{67}{17},3$	M1	2.1
	$4+3\mathrm{i},\ \frac{4}{17}+\frac{67}{17}\mathrm{i}$	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)
B1: Correct coordinates of centre
M1: Fully correct strategy for identifying the radius. If the diameter is calculated this must be halved to achieve this mark.
A1: Correct equation using the required notation
(b)
M1: Begins the process of finding z_1 and z_2 by using the Cartesian equations to obtain the equation of the line of intersection
M1: Substitutes back into the equation of one of the circles to obtain an equation in one variable
A1: Correct 3 term quadratic
M1: Solves their 3TQ
M1: Substitutes to find values of the other variable to complete the process of finding z_1 and z_2
A1: Correct complex numbers

Question	Scheme	Marks	AOs
4(a)	$(\cos\theta+\mathrm{i}\sin\theta)^7=\cos^7\theta+\binom{7}{1}\cos^6\theta(\mathrm{i}\sin\theta)+\binom{7}{2}\cos^5\theta(\mathrm{i}\sin\theta)^2+...$ Some simplification may be done at this stage e.g. $c^7+7c^6\mathrm{i}s-21c^5s^2-35c^4\mathrm{i}s^3+35c^3s^4+21c^2\mathrm{i}s^5-7cs^6-\mathrm{i}s^7$	M1	1.1b
	$\mathrm{i}\sin7\theta={}^7\mathrm{C}_1c^6\mathrm{i}s+{}^7\mathrm{C}_3c^4\mathrm{i}^3s^3+{}^7\mathrm{C}_5c^2\mathrm{i}^5s^5+\mathrm{i}^7s^7$ or $=7c^6\mathrm{i}s+35c^4\mathrm{i}^3s^3+21c^2\mathrm{i}^5s^5+\mathrm{i}^7s^7$	M1	2.1
	$\sin7\theta=7c^6s-35c^4s^3+21c^2s^5-s^7$	A1	1.1b
	$=7(1-s^2)^3s-35(1-s^2)^2s^3+21(1-s^2)s^5-s^7$ $=7(1-3s^2+3s^4-s^6)s-35(1-2s^2+s^4)s^3+21(1-s^2)s^5-s^7$	M1	2.1
	$\{7s-21s^3+21s^5-7s^7-35s^3+70s^5-35s^7+21s^5-21s^7-s^7\}$ leading to $\sin7\theta=7\sin\theta-56\sin^3\theta+112\sin^5\theta-64\sin^7\theta$ *	A1*	1.1b
		(5)	
(b)	$1+\sin7\theta=0\Rightarrow\sin7\theta=-1$	M1	3.1a
	$7\theta=-450,-90,270,630,...$ or $7\theta=-\frac{5\pi}{2},-\frac{\pi}{2},\frac{3\pi}{2},\frac{7\pi}{2},...$	A1	1.1b
	$\theta=-\frac{450}{7},-\frac{90}{7},\frac{270}{7},\frac{630}{7},..\Rightarrow\sin\theta=...$ or $\theta=-\frac{5\pi}{14},-\frac{\pi}{14},\frac{3\pi}{14},\frac{7\pi}{14},...\Rightarrow\sin\theta=...$	M1	2.2a
	$x=\sin\theta=-0.901,\ -0.223,\ 0.623,1$	A1 A1	1.1b 2.3
		(5)	
			(10 marks)

Notes

(a)
M1: Attempts to expand $(\cos\theta+\mathrm{i}\sin\theta)^7$ including a recognisable attempt at binomial coefficients
Some simplification may be done at this stage. (May only see imaginary terms)
M1: Identifies imaginary terms with sin 7θ
A1: Correct expression with coefficients evaluated and i's dealt with correctly
M1: Replaces $\cos^2\theta$ with $1-\sin^2\theta$ and applies the expansions of $(1-\sin^2\theta)^2$ and $(1-\sin^2\theta)^3$ to their expression
A1*: Reaches the printed answer with no errors and expansion of brackets seen.
(b)
M1: Makes the connection with part (a) and realises the need to solve $\sin7\theta=-1$
A1: At least one correct value for 7θ
M1: Divides by 7 and deduces that *x* values are found by finding at least one value for $\sin\theta$
A1: Awrt 2 correct values for *x*
A1: Awrt all 4 *x* values correct and no extras

Question	Scheme	Marks	AOs
1(a) (i)	$\lvert z_1 z_2 \rvert = 3\sqrt{2}$	B1	1.1b
(ii)	$\arg(z_1 z_2) = \frac{\pi}{3} + \left(-\frac{\pi}{12}\right) = \frac{\pi}{4}$ o.e.	B1	1.1b
		(2)	
(b) (i)	$n = 8$	B1ft	2.2a
(ii)	$\lvert w^n \rvert = (\text{'their} \lvert z_1 z_2 \rvert\text{'})^{\text{their } n}$	M1	1.1b
	$\lvert w^n \rvert = 104\,976$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a)

(i)

B1: Deduces $\lvert z_1 z_2 \rvert = 3\sqrt{2}$

(ii)

B1: Deduces $\arg(z_1 z_2) = \frac{\pi}{4}$ o.e

These marks may be awarded for $z_1 z_2 = 3\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(b)

(i)

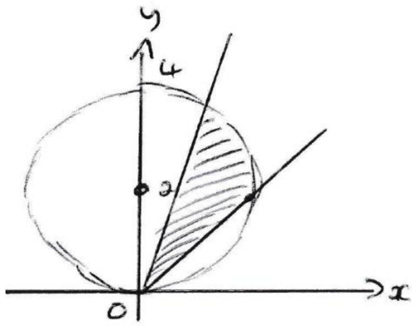
B1ft: 2π divided by their $\arg(z_1 z_2)$ found in part (a) (ii) to give an integer

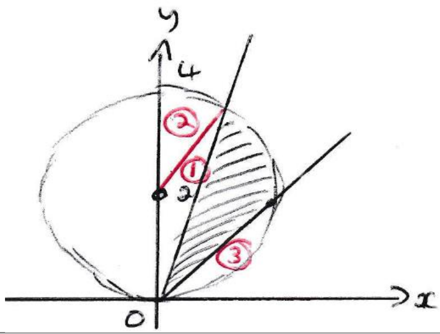
Alternatively smallest positive integer multiple required to make their argument a multiple of 2π

(ii)

M1: Their answer to (a) (i) to the power of their *n*.

A1: 104 976

Question	Scheme	Marks	AOs
8(i)	$\lvert z\rvert = \sqrt{6^2+6^2} = \ldots\ 6\sqrt{2}$ or $\sqrt{72}$ and $\arg z = \tan^{-1}\left(\frac{6}{6}\right) = \ldots \left\{\frac{\pi}{4}\right\}$ Can be implied by $r = 6\sqrt{2}e^{\frac{\pi}{4}i}$	M1 A1	3.1a 1.1b
	Adding multiples of $\frac{2\pi}{5}$ to their argument $z = 6\sqrt{2}e^{\frac{\pi}{4}i} \times e^{\frac{2\pi k}{5}i}$ or $z = 6\sqrt{2}\left[\cos\left(\frac{\pi}{4}+\frac{2\pi k}{5}\right)+i\sin\left(\frac{\pi}{4}+\frac{2\pi k}{5}\right)\right]$	M1	1.1b
	$z = re^{\left(\theta+\frac{2\pi}{5}\right)i},\ re^{\left(\theta+\frac{4\pi}{5}\right)i},\ re^{\left(\theta+\frac{6\pi}{5}\right)i},\ re^{\left(\theta+\frac{8\pi}{5}\right)i}$ o.e. or $z = re^{\left(\theta+\frac{2\pi}{5}\right)i},\ re^{\left(\theta-\frac{2\pi}{5}\right)i},\ re^{\left(\theta-\frac{6\pi}{5}\right)i},\ re^{\left(\theta-\frac{8\pi}{5}\right)i}$ o.e.	A1ft	1.1b
	$z = 6\sqrt{2}e^{\frac{13\pi}{20}i},\ 6\sqrt{2}e^{\frac{21\pi}{20}i},\ 6\sqrt{2}e^{\frac{29\pi}{20}i},\ 6\sqrt{2}e^{\frac{37\pi}{20}i}$ o.e. or $z = 6\sqrt{2}e^{\frac{13\pi}{20}i},\ 6\sqrt{2}e^{-\frac{19\pi}{20}i},\ 6\sqrt{2}e^{-\frac{11\pi}{20}i},\ 6\sqrt{2}e^{-\frac{3\pi}{20}i}$ o.e.	A1	1.1b
		(5)	
(ii)(a)	Circle centre (0, 2) and radius 2 or  with the point on the origin	B1	1.1b
	Fully correct 	B1	1.1b
		(2)	
(ii)(b)	area $= \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (4\sin\theta)^2\, d\theta$ or area $= \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\alpha\sin\theta)^2\, d\theta$	M1	3.1a
	Uses $\sin^2\theta = \frac{1}{2}-\frac{1}{2}\cos 2\theta$ and integrates to the form $A\theta + B\sin 2\theta$ area $= 8\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \sin^2\theta\, d\theta = 4\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (1-\cos 2\theta)\, d\theta = 4\theta - 2\sin 2\theta$	M1	3.1a
	Uses the limits of $\frac{\pi}{4}$ and $\frac{\pi}{3}$ and subtracts the correct way around $\left[4\left(\frac{\pi}{3}\right)-2\sin\left(\frac{2\pi}{3}\right)\right]-\left[4\left(\frac{\pi}{4}\right)-2\sin\left(\frac{2\pi}{4}\right)\right]$	M1	1.1b

	Area = $\frac{\pi}{3}-\sqrt{3}+2$	A1	1.1b
		(4)	
	Alternative 		
	Finds either the areas 1 or 2 Area 1 $=\frac{1}{2}\times 2^2\times\sin\left(\frac{2\pi}{3}\right)\left\{=\sqrt{3}\right\}$ Area 2 $=\frac{1}{2}\times 2^2\times\frac{\pi}{3}\left\{=\frac{2\pi}{3}\right\}$	M1	1.1b
	A complete method to find area 3 Area 3 $=\frac{1}{4}\pi\times 2^2-\frac{1}{2}\times 2^2\left\{=\pi-2\right\}$	M1	3.1a
	A complete method to find the required area Shaded area $=$ Area of semi circle $-$ area 1 $-$ area 2 $-$ area 3 $=\left[\frac{1}{2}\pi\times 2^2\right]-\left[\frac{1}{2}\times 2^2\times\sin\left(\frac{2\pi}{3}\right)\right]-\left[\frac{1}{2}\times 2^2\times\frac{\pi}{3}\right]-\left[\frac{1}{4}\pi\times 2^2-\frac{1}{2}\times 2^2\right]$ $=2\pi-\sqrt{3}-\frac{2\pi}{3}-(\pi-2)$ Or Shaded area $=$ Area of sector $-$ area 1 $-$ area 3 $=\left[\frac{1}{2}\times 4\times\left(\frac{2\pi}{3}\right)\right]-\left[\frac{1}{2}\times 2^2\times\sin\left(\frac{2\pi}{3}\right)\right]-\left[\frac{1}{4}\pi\times 2^2-\frac{1}{2}\times 2^2\right]$ $=\frac{4\pi}{3}-\sqrt{3}-(\pi-2)$	M1	3.1a
	Area = $\frac{\pi}{3}-\sqrt{3}+2$	A1	1.1b
		(4)	
(11 marks)			
Notes:			
(i) **M1:** Finds the modulus and argument of z **A1:** Correct modulus and argument of z			

Question	Scheme	Marks	AOs
9(a)	$\frac{1}{1-z}$	B1	2.2a
		(1)	
(b)(i)	$1+z+z^2+z^3+\ldots$ $=1+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^2+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^3+\ldots$ $=1+\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)+\frac{1}{4}(\cos 2\theta+\mathrm{i}\sin 2\theta)+\frac{1}{8}(\cos 3\theta+\mathrm{i}\sin 3\theta)+\ldots$	M1	3.1a
	$\frac{1}{1-z}=\frac{1}{1-\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)}\times\frac{1-\frac{1}{2}\cos\theta+\frac{1}{2}\mathrm{i}\sin\theta}{1-\frac{1}{2}\cos\theta+\frac{1}{2}\mathrm{i}\sin\theta}$ or $\frac{1}{1-z}=\frac{2}{2-(\cos\theta+\mathrm{i}\sin\theta)}\times\frac{2-(\cos\theta-\mathrm{i}\sin\theta)}{2-(\cos\theta-\mathrm{i}\sin\theta)}$	M1	3.1a
	$\left\{\frac{1}{2}(\sin\theta)+\frac{1}{4}(\sin 2\theta)+\frac{1}{8}(\sin 3\theta)+\ldots\right\}=\frac{\frac{1}{2}\sin\theta}{\left(1-\frac{1}{2}\cos\theta\right)^2+\left(\frac{1}{2}\sin\theta\right)^2}$ or $\left\{\frac{1}{2}(\sin\theta)+\frac{1}{4}(\sin 2\theta)+\frac{1}{8}(\sin 3\theta)+\ldots\right\}=\frac{2\sin\theta}{(2-\cos\theta)^2+(\sin\theta)^2}$	M1	2.1
	$\left(1-\frac{1}{2}\cos\theta\right)^2+\left(\frac{1}{2}\sin\theta\right)^2=1-\cos\theta+\frac{1}{4}\cos^2\theta+\frac{1}{4}\sin^2\theta$ $=\frac{5}{4}-\cos\theta$ or $(2-\cos\theta)^2+(\sin\theta)^2=4-4\cos\theta+\cos^2\theta+\sin^2\theta$ $=5-4\cos\theta$	M1	1.1b
	$\frac{1}{2}\sin\theta+\frac{1}{4}\sin 2\theta+\frac{1}{8}\sin 3\theta+\ldots=\frac{\frac{1}{2}\sin\theta}{\frac{5}{4}-\cos\theta}=\frac{2\sin\theta}{5-4\cos\theta}$ *	A1*	1.1b
	Alternative $1+z+z^2+z^3+\ldots$ $=1+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^2+\left(\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)\right)^3+\ldots$ $=1+\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)+\frac{1}{4}(\cos 2\theta+\mathrm{i}\sin 2\theta)+\frac{1}{8}(\cos 3\theta+\mathrm{i}\sin 3\theta)+\ldots$	M1	3.1a

	$\dfrac{1}{1-z}=\dfrac{1}{1-\frac{1}{2}e^{i\theta}}\times\dfrac{1-\frac{1}{2}e^{-i\theta}}{1-\frac{1}{2}e^{-i\theta}}$	M1	3.1a
	$\dfrac{1-\frac{1}{2}e^{-i\theta}}{1-\frac{1}{4}e^{i\theta}-\frac{1}{4}e^{-i\theta}+\frac{1}{4}}=\dfrac{4-2e^{-i\theta}}{5-2\left(e^{i\theta}+e^{-i\theta}\right)}=\dfrac{4-2(\cos\theta-i\sin\theta)}{5-2(2\cos\theta)}$	M1	2.1
	Select the imaginary part $\dfrac{2\sin\theta}{5-4\cos\theta}$	M1	1.1b
	$\frac{1}{2}\sin\theta+\frac{1}{4}\sin 2\theta+\frac{1}{8}\sin 3\theta+...=\dfrac{2\sin\theta}{5-4\cos\theta}$ *	A1*	1.1b
		(5)	
(b)(ii)	$\dfrac{1-\frac{1}{2}\cos\theta}{\frac{5}{4}-\cos\theta}=0\Rightarrow\cos\theta=2$	M1	3.1a
	As $(-1\le)\cos\theta\le1$ therefore there is no solution to $\cos\theta=2$ so there will also be a real part, hence the sum cannot be purely imaginary.	A1	2.4
	Alternative 1 Imaginary part is $\dfrac{4-2\cos\theta}{5-4\cos\theta}=\dfrac{1}{2}+\dfrac{3}{2(5-4\cos\theta)}$	M1	3.1a
	$-1\le\cos\theta\le1$ therefore $\dfrac{1}{6}<\dfrac{3}{2(5-4\cos\theta)}<\dfrac{3}{2}$ so sum must contain real part	A1	2.4
	Alternative 2 $\dfrac{1}{1-z}=ki\Rightarrow z=1+\dfrac{i}{k}$	M1	3.1a
	mod $z>1$ contradiction hence cannot be purely imaginary	A1	2.4
		(2)	
			(8 marks)

Notes:

(a)
B1: See scheme

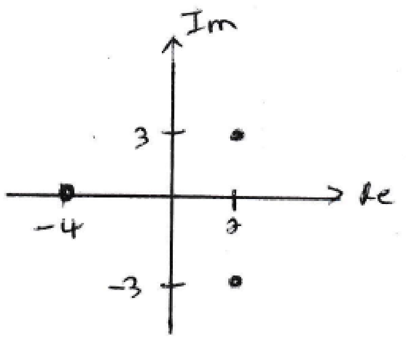
(b)(i)

M1: Substitutes $z=\frac{1}{2}(\cos\theta+i\sin\theta)$ into at least 3 terms of the series and applies de Moivre's theorem.

M1: Substitutes $z=\frac{1}{2}(\cos\theta+i\sin\theta)$ into their answer to part (a) and rationalises the denominator.

M1: Equates the imaginary terms.

M1: Multiplies out the denominator and simplifies by using the identity $\cos^2\theta+\sin^2\theta=1$

Question	Scheme	Marks	AOs
1(a)	$2+3i$	B1	1.1b
		(1)	
(b) (i)	$z*=2+3i$ so $z+z*=4$, $zz*=13$ $z+z*+\alpha=0 \Rightarrow \alpha=\ldots$ or $\alpha zz*=-52 \Rightarrow \alpha=-\frac{52}{"13"}=\ldots$ or $z^2-(\text{sum roots})z+(\text{product roots})=0$ or $\left(z-(2+3i)\right)\left(z-(2-3i)\right)=\ldots$ $\Rightarrow (z^2-4z+13)\underline{(z+4)} \Rightarrow z=\ldots$	M1	3.1a
	$z=2\pm 3\text{i},\ -4$	A1	1.1b
(ii)	$(z^2-4z+13)(z+4)$ expands the brackets to find value for a Or a = pair sum $=-4(2+3i+2-3i)+13=\ldots$ Or $f(-4)/f(2\pm 3i)=0 \Rightarrow\ldots\Rightarrow a=\ldots$	M1	1.1b
	$a=-3$	A1	2.2a
		(4)	
(c)		B1ft	1.1b
		(1)	
			(6 marks)

Question	Scheme	Marks	AOs
7(a)	$z* = a - bi$ then $zz* = (a+bi)(a-bi) = \ldots$	M1	1.1b
	$zz* = a^2 + b^2$ therefore, a real number	A1	2.4
		(2)	
(b)	$\frac{z}{z*} = \frac{a+bi}{a-bi} = \frac{(a+bi)(a+bi)}{(a-bi)(a+bi)} = \frac{(a^2-b^2)+2abi}{a^2+b^2} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$ or $\frac{z}{z^*} = \frac{z^2}{zz^*} = \frac{z^2}{18} \Rightarrow$ $z^2 = 14 + 8\sqrt{2}i$ or $a + bi = \left(\frac{7}{9} + \frac{4\sqrt{2}i}{9}\right)(a - bi) = ..+..i$	M1	1.1b
	Forms two equations from $a^2 + b^2 = 18$ or $\frac{a^2-b^2}{18} = \frac{7}{9}$ or $\frac{a^2-b^2}{a^2+b^2} = \frac{7}{9}$ or $\frac{2ab}{18} = \frac{4\sqrt{2}}{9}$ or $\frac{2ab}{a^2+b^2} = \frac{4\sqrt{2}}{9}$ or $a = \frac{7}{9}a + \frac{4\sqrt{2}}{9}b$ oe	M1 A1	3.1a 1.1b
	Solves the equations simultaneously e.g. $a^2 + b^2 = 18$ and $a^2 - b^2 = 14$ leading to a value for *a* or *b*	dM1	1.1b
	$z = \pm(4 + \sqrt{2}i)$	A1	2.2a
		(5)	
			(7 marks)

Notes:

(a)(i)
M1: States or implies $z* = a - bi$ and finds an expression for $zz*$
A1: Achieves $zz* = a^2 + b^2$ and draws the conclusion that $zz*$ is a real number. Accept $\in \mathbb{R}$ as conclusion, but not just "no imaginary part".

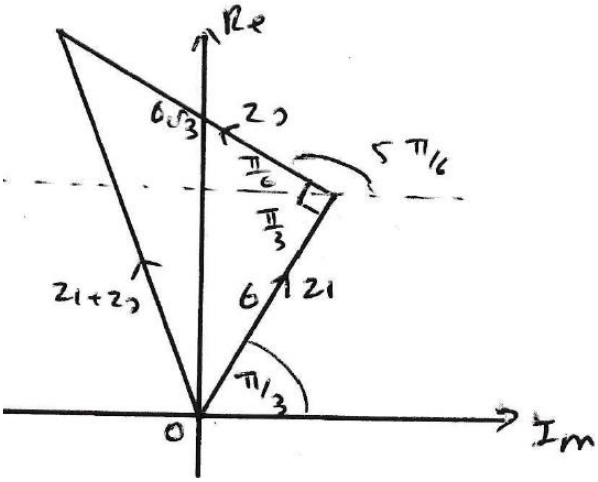
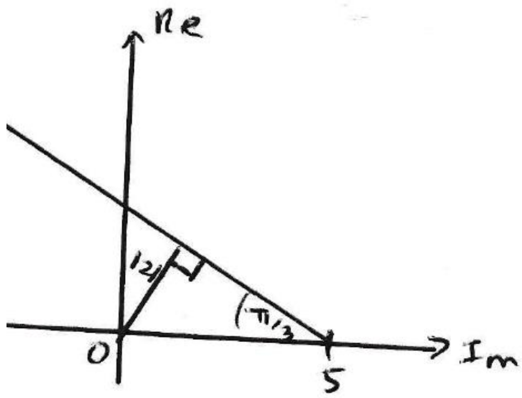
(b)
M1: Starts the process of solving by using the conjugate to form an equation with real denominators, and without z^* or i^2 in the equation. Accept as shown in scheme, or may multiply through by $a - bi$ and expand and gather terms. May be implied by correct extraction of equation(s).
M1: Uses the given information to form two equations involving *a* and *b* at least one of which includes both. It must involve equating real or imaginary parts of $\frac{z}{z*} = \frac{7}{9} + \frac{4\sqrt{2}i}{9}$
A1: Any two correct equations arising from use of both given facts. (Note: if multiplying through by $a - bi$ then equating real and imaginary terms gives the same equation.)
dM1: Dependent on previous method mark, solves the equations to find a value for either *a* or *b*.
A1: Deduces the correct complex numbers and no extras. Do not accept $\pm 4 \pm \sqrt{2}i$
Note: it is possible to solve via polar coordinates, but unlikely to succeed. If you see responses you think are worthy of credit but are unsure how to mark, use review. Example solutions shown below.

Question	Scheme	Marks	AOs
1(a) (i) (a) (ii)	$\{arg(z_1) =\}\, tan^{-1}\left(\frac{-3}{3}\right)$ or $\{arg(z_1) =\}\, tan^{-1}(-1)$ or $\{arg(z_1) =\} - tan^{-1}\left(\frac{3}{3}\right)$ or $\{arg(z_1) =\} - \frac{\pi}{4}$ or $\{arg(z_1) =\} 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$ or states should be – 3 not 3 on top	B1	2.3
	States that $\left\{arg\left(\frac{z_1}{z_2}\right) =\right\} arg(z_1) - arg(z_2)$ Or states that the arguments should be subtracted	B1	2.3
		(2)	
(b)	$\left\{arg\left(\frac{z_1}{z_2}\right) = \left(\text{their} - \frac{\pi}{4}\right) - \frac{\pi}{6} =\right\} - \frac{5\pi}{12}$ Or $\left\{arg\left(\frac{z_1}{z_2}\right) = \left(\text{their}\,\frac{7\pi}{4}\right) - \frac{\pi}{6}\right\} = \frac{19\pi}{12}$	B1ft	2.2a
		(1)	
		(3 marks)	

Notes:

(a) (i)
B1: See scheme, Condone – 45
Any incorrect arguments seen is B0.
$arg(z_1) = tan^{-1}\left(\frac{3}{-3}\right)$ is B0
Note: They used 3 instead of – 3 is B0, there are two 3's in line 1 do they mean both should – 3
It should be negative is B0
(a) (ii)
B1: See scheme
(b)
B1ft: States a correct value for $arg\left(\frac{z_1}{z_2}\right)$ Follow through on their answer to part (a) (i), do not ISW

Question	Scheme	Marks	AOs
4(i)	$z_1 = 6\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right] = \ldots\{3 + 3\sqrt{3}i\}$ $z_2 = 6\sqrt{3}\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] = \ldots\{-9 + 3\sqrt{3}i\}$ $\{z_1 + z_2 =\}(3 + 3\sqrt{3}i) + (-9 + 3\sqrt{3}i) = \ldots\{-6 + 6\sqrt{3}i\}$ Or $\{z_1 + z_2 =\}6\left[\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right] + 6\sqrt{3}\left[\cos\left(\frac{5\pi}{6}\right) + i\sin\left(\frac{5\pi}{6}\right)\right] = a + bi$ where a and b are constants, the trig function must be evaluated	M1	3.1a
	Clearly show the method to find modulus **and** argument for $z_1 + z_2$ $arg(z_1 + z_2) = \pi - \tan^{-1}\left(\frac{6\sqrt{3}}{6}\right)$ or $\tan^{-1}\left(\frac{6\sqrt{3}}{-6}\right) = \ldots\left\{\frac{2\pi}{3}\right\}$ **and** $\lvert z_1 + z_2\rvert = \sqrt{6^2 + (6\sqrt{3})^2} = \ldots\{12\}$ **Alternative 1** $-6 + 6\sqrt{3}i = 12\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$ $= 12\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$ **Alternative 2** $12e^{\frac{2\pi}{3}i} = 12\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ $= \ldots\{-6 + 6\sqrt{3}i\}$	dM1	2.1
	$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$ * $12e^{\frac{2\pi}{3}i} = -6 + 6\sqrt{3}i$ Therefore $z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$*	A1*	1.1b
		(3)	
	Alternative 3 $z_1 + z_2 = 6\boldsymbol{e}^{\frac{\pi}{3}\boldsymbol{i}} + 6\sqrt{3}\boldsymbol{e}^{\frac{5\pi}{6}\boldsymbol{i}}$ $= 12\left[\frac{1}{2}\boldsymbol{\cos}\left(\frac{\pi}{3}\right) + \frac{1}{2}\boldsymbol{i\sin}\left(\frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}\boldsymbol{\cos}\left(\frac{5\pi}{6}\right) + \frac{\sqrt{3}}{2}\boldsymbol{i\sin}\left(\frac{5\pi}{6}\right)\right]$	M1	3.1a
	$12\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 12\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$	dM1	2.1
	$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}$*	A1*	1.1b
		(3)	
	Alternative 4 $z_1 + z_2 = 6e^{\frac{\pi}{3}i} + 6\sqrt{3}e^{\frac{5\pi}{6}i} = 6e^{\frac{\pi}{3}i}\left(1 + \sqrt{3}e^{\frac{\pi}{2}i}\right) = 6e^{\frac{\pi}{3}i}(1 + \sqrt{3}i)$	M1	
	Either $r = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$ **and** $arg = \arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$	dM1	

	Or $6e^{\frac{\pi}{3}i}(1+\sqrt{3}i) = 12e^{\frac{\pi}{3}i}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i()e^{\frac{\pi}{3}i}\left(cos\left(\frac{\pi}{3}\right)+i\,sin\left(\frac{\pi}{3}\right)\right)\right)$		
	$z_1 + z_2 = 12e^{\frac{\pi}{3}i}e^{\frac{\pi}{3}i} = 12e^{\frac{2\pi}{3}i}*$	A1*	
		(3)	
	Alternative 5 Uses geometry to show that $z_1,\ z_2$ and $z_1 + z_2$ form a right-angled triangle 	M1	3.1a
	$arg(z_1+z_2) = \frac{\pi}{3} + tan^{-1}\left(\frac{6\sqrt{3}}{6}\right) = \ldots\left\{\frac{2\pi}{3}\right\}$ $\lvert z_1+z_2\rvert = \sqrt{(6)^2+(6\sqrt{3})^2} = \ldots\{12\}$	dM1	1.1b
	$z_1 + z_2 = 12e^{\frac{2\pi}{3}i}*$	A1*	1.1b
		(3)	
(ii)		M1	3.1a
	$sin\left(\frac{\pi}{3}\right) = \frac{\lvert z\rvert}{5} \Rightarrow \lvert z\rvert = \ldots$	M1	1.1b
	$\lvert z\rvert = \frac{5\sqrt{3}}{2}$	A1	1.1b
		(3)	

Question	Scheme	Marks	AOs
3(a)	e.g. $\lvert z_1\rvert = \sqrt{(-4)^2+4^2}$ or $\arg z_1 = \pi - \frac{\pi}{4}$ oe	M1	1.1b
	$(z_1 =)4\sqrt{2}\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right)$ or e.g. $(z_1 =)\sqrt{32}\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right)$	A1	1.1b
		(2)	
(b)(i)	$\frac{z_1}{z_2} = \frac{"4\sqrt{2}"}{3}\left(\cos\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)+\mathrm{i}\sin\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)\right)=...$ **or** $\frac{z_1}{z_2} = \frac{"4\sqrt{2}"\mathrm{e}^{"\frac{3\pi}{4}\mathrm{i}"}}{3\mathrm{e}^{\frac{17\pi}{12}\mathrm{i}}} = \frac{"4\sqrt{2}"}{3}\mathrm{e}^{\left("\frac{3\pi}{4}"-\frac{17\pi}{12}\right)\mathrm{i}}$ **or** $\frac{z_1}{z_2} = \frac{-4+4\mathrm{i}}{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)} \times \frac{\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)+\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)}{\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)+\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)}=...$	M1	3.1a
	$=-\frac{2\sqrt{2}}{3}-\frac{2\sqrt{6}}{3}\mathrm{i}$ or $-\frac{2\sqrt{2}}{3}-\mathrm{i}\frac{2\sqrt{6}}{3}$ or $-\frac{2\sqrt{2}}{3}+\mathrm{i}\left(-\frac{2\sqrt{6}}{3}\right)$	A1	1.1b
		(2)	

Notes

(a) <u>Correct answer with no working scores both marks in (a)</u>

M1: Any correct expression for $\lvert z_1\rvert$ or $\arg z_1$ e.g. $\lvert z_1\rvert = \sqrt{(-4)^2+4^2}$ or $\arg z_1 = \pi - \frac{\pi}{4}$

A1: Correct <u>expression</u>. The $"z_1 ="$ is not required.

This mark is not for correct modulus and correct argument it is for the complex number written in the required form. Condone the missing closing bracket e.g. $(z_1 =)\sqrt{32}\left(\cos\frac{3\pi}{4}+\mathrm{i}\sin\frac{3\pi}{4}\right.$

(b)(i) <u>Correct answer with no working scores no marks in (b)(i)</u>

M1: Employs a correct method to find the quotient. E.g.

- uses modulus argument form and divides moduli and subtracts arguments the right way round
- uses exponential form and divides moduli and subtracts arguments the right way round
- converts z_2 to Cartesian form and multiplies numerator and denominator by the complex conjugate of the denominator. Allow if the “3” is missing for this method. Allow with decimals for this method e.g. $\frac{z_1}{z_2} = \frac{-4+4\mathrm{i}}{-0.258...-0.965...\mathrm{i}} \times \frac{-0.258...+0.965...\mathrm{i}}{-0.258...+0.965...\mathrm{i}} = ...$

<u>If they convert z_2 to Cartesian form it must be correct as shown or correct decimals.</u>

A1: Correct exact answer in the required form.

Do not allow e.g. $-\frac{2}{3}\left(\sqrt{2}+\sqrt{6}\mathrm{i}\right)$ or $\frac{-2\sqrt{2}-2\sqrt{6}\mathrm{i}}{3}$ unless a correct form is seen previously then apply isw.

Provided a correct method is shown as above, allow to go from the forms in the main scheme to the correct exact answer with no intermediate step.

(ii)	$z_2^4 = 3^4\left(\cos\left(4\times\frac{17\pi}{12}\right)+\mathrm{i}\sin\left(4\times\frac{17\pi}{12}\right)\right)$ **or** $(z_2)^4 = \left(3\mathrm{e}^{\frac{17\pi}{12}\mathrm{i}}\right)^4 = 3^4\mathrm{e}^{\frac{17\pi}{12}\times 4\mathrm{i}}$ **or** $z_2^4 = \left\{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)\right\}^4 = \ldots$	**M1**	1.1b
	$=\frac{81}{2}-\frac{81\sqrt{3}}{2}\mathrm{i}$ or $\frac{81}{2}-\mathrm{i}\frac{81\sqrt{3}}{2}$ or $\frac{81}{2}+\mathrm{i}\left(-\frac{81\sqrt{3}}{2}\right)$	**A1**	1.1b
		(2)	

(b)(ii) Correct answer with no working scores no marks in (b)(ii)

M1: Applies De Moivre's theorem correctly to z_2. E.g. uses polar form or exponential form and calculates the modulus as 3^4 and the argument as $4\times\frac{17\pi}{12}$

For attempts at $z_2^4 = \left\{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)\right\}^4$ you would need to see:

- the correct exact form used
- a clear and convincing attempt to expand the brackets e.g. by using a full binomial expansion or a complete attempt to multiply all 4 brackets together but you are not expected to check every detail
- a final answer in the required form with no obvious errors seen

So $z_2^4 = \left\{3\left(\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)-\mathrm{i}\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)\right)\right\}^4 = \frac{81}{2}-\frac{81\sqrt{3}}{2}\mathrm{i}$ scores no marks.

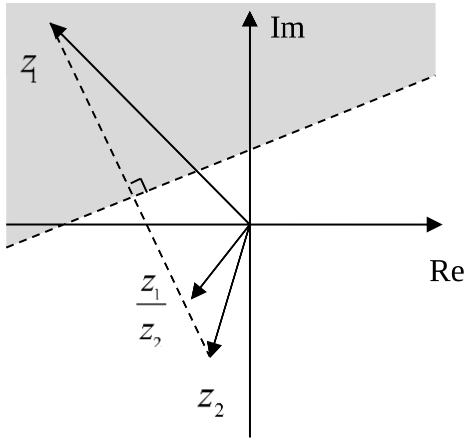
Similar guidance applies if they attempt to expand $\left\{3\left(\cos\frac{17\pi}{12}+\mathrm{i}\sin\frac{17\pi}{12}\right)\right\}^4$

A1: Correct exact answer in the required form.

Do not allow e.g. $\frac{81}{2}\left(1-\frac{81\sqrt{3}}{2}i\right)$ or $\frac{81-81\sqrt{3}\mathrm{i}}{2}$ unless a correct form is seen previously then apply isw.

Provided a correct method is shown as above, allow to go from the forms in the main scheme to the correct exact answer with no intermediate step.

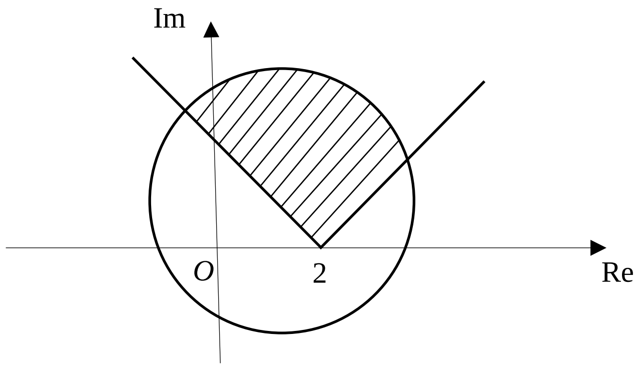
(c)(i) and (ii)



Notes:		
(c)(i) **B1:** z_1 and z_2 correctly positioned. Look for <u>correct quadrants</u> with z_1 approximately on $y = -x$ and z_2 below $y = x$ closer to the origin than z_1. Note that the points are usually labelled but mark positively if it is clear which points are which if there is no labelling.	**B1**	1.1b
B1ft: $\frac{z_1}{z_2}$ in the correct quadrant. Follow through their answer to (b)(i). Note that the point is usually labelled but mark positively if it is clear which point it is. It is sometimes labelled as z_3 which is fine.	**B1ft**	1.1b
(ii) **M1:** Draws a line (solid or dashed) that is the perpendicular bisector of z_1z_2 **or** draws a line that crosses z_1z_2 and shades one of the sides of this line.	**M1**	3.1a
A1: A line drawn (solid or dashed) that is the perpendicular bisector of z_1z_2 <u>**with either side shaded**</u> as long as it is clear they are not discounting the upper region. The B1 in part (i) may not have been scored but z_1 must be in quadrant 2 and z_2 in quadrant 3. **Note that some candidates are drawing the region on a separate diagram and this is acceptable. You do not need to see a line joining z_1 to z_2.**	**A1**	1.1b
	(4)	
		(10 marks)

Question	Scheme	Marks	AOs
5(a)	$\alpha = \frac{z_1+z_2}{2} = \frac{35-25i-29+39i}{2} = \ldots$ $\alpha = z_1 + \frac{1}{2}\overrightarrow{z_1z_2} = 35-25i+\frac{1}{2}(-64+64i) = \ldots$ $\alpha = z_2 + \frac{1}{2}\overrightarrow{z_2z_1} = -29+39+\frac{1}{2}(64-64i) = \ldots$	M1	1.1b
	$= 3+7i *$	A1*	1.1b
		(2)	
(b)	$\beta(z_1-\alpha) = \left(\frac{1+i}{64}\right)(35-25i-(3+7i)) = \left(\frac{1+i}{64}\right)(32-32i) =$ $= \frac{1}{64}(32-32i+32i-32i^2) = \frac{1}{64}(32-32i+32i+32)$	M1	1.1b
	$= \frac{1}{64}(64) = 1*$	A1*	1.1b
		(2)	
(c)(i)	Roots are $\left\{e^0\left(\text{or } 1 \text{ or } e^{i2\pi}\right)\right\}, e^{i\frac{\pi}{3}}, e^{i\frac{2\pi}{3}}, e^{i\pi}, e^{i\frac{4\pi}{3}}, e^{i\frac{5\pi}{3}}$ or $e^{i\frac{k\pi}{3}}$, $k = 0,1,2,3,4,5$ $\left\{e^0\left(\text{or } 1 \text{ or } e^{i2\pi}\right)\right\}, e^{i\frac{\pi}{3}}, e^{i\frac{2\pi}{3}}, e^{i\pi}, e^{-i\frac{\pi}{3}}, e^{-i\frac{2\pi}{3}}$ or $e^{i\frac{k\pi}{3}}$, $k = -2, -1, 0, 1, 2, 3,$	B1	1.1b
		(1)	
(c)(ii)	$w = \beta(z-\alpha) = e^{i\frac{k\pi}{3}} \Rightarrow z = \frac{e^{i\frac{k\pi}{3}}}{\beta} + \alpha$	M1	3.1a
	$\Rightarrow z = \frac{64\left(\cos\frac{k\pi}{3} + i\sin\frac{k\pi}{3}\right)(1-i)}{(1+i)(1-i)} + 3 + 7i = \ldots$	M1	1.1b
	Two of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)i$ $\quad$ $\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)i$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)i$ $\quad$ $\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)i$ Or four correct decimal answers 46.7 + 18.7i $\quad$ – 40.7 – 4.7i $\quad$ 14.7 + 50.7i $\quad$ – 8.7 – 36.7i	A1	2.5

	All four of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)\mathrm{i}$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)\mathrm{i}$ $\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)\mathrm{i}$ $\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)\mathrm{i}$	**A1**	2.2a
		(4)	
	Alternative 1 $\left(\beta(z-\alpha)\right)^6=1^6 \Rightarrow (z-\alpha)^6=\frac{1}{\beta^6}=8589934459\mathrm{i}$ $r=\sqrt[6]{858993459}=32\sqrt{2}$ or 45.25... and $\theta=\frac{\pi}{12}+\frac{k\pi}{3}$ or $\theta=-\frac{\pi}{4}+\frac{k\pi}{3}$	**M1**	3.1a
	$z=r\left(\cos\theta-\mathrm{i}\sin\theta\right)+3+7\mathrm{i}=...$	**M1**	1.1b
	Two of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)\mathrm{i}$ $\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)\mathrm{i}$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)\mathrm{i}$ $\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)\mathrm{i}$ Or four correct decimal answers 46.7 + 18.7i – 40.7 – 4.7i 14.7 + 50.7i – 8.7 – 36.7i	**A1**	2.5
	All four of $\left(19+16\sqrt{3}\right)+\left(-9+16\sqrt{3}\right)\mathrm{i}$ $\left(-13+16\sqrt{3}\right)+\left(23+16\sqrt{3}\right)\mathrm{i}$ $\left(-13-16\sqrt{3}\right)+\left(23-16\sqrt{3}\right)\mathrm{i}$ $\left(19-16\sqrt{3}\right)-\left(9+16\sqrt{3}\right)\mathrm{i}$	**A1**	2.2a
		(4)	
	Alternative 2 Rotation matrix $\begin{pmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix}$ and $\begin{pmatrix}35\\-25\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$ or $\begin{pmatrix}-29\\39\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$ Or find the exponential form for $\begin{pmatrix}35\\-25\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$ or $\begin{pmatrix}-29\\39\end{pmatrix}-\begin{pmatrix}3\\7\end{pmatrix}$	**M1**	3.1a

Question	Scheme	Marks	AOs
3(a)		M1	1.1b
		M1	1.1b
		A1	2.2a
		M1	3.1a
		A1	1.1b
		(5)	
(b)	$(x-1)^2+(y-1)^2=9,\ \ y=x-2 \Rightarrow x=...,$ or $y=...$	M1	3.1a
	$x=2+\frac{\sqrt{14}}{2}, y=\frac{\sqrt{14}}{2}$	A1	1.1b
	$\lvert w\rvert^2=\left(2+\frac{\sqrt{14}}{2}\right)^2+\left(\frac{\sqrt{14}}{2}\right)^2$	M1	1.1b
	$=11+2\sqrt{14}$	A1	1.1b
		(4)	
			(9 marks)

Notes

(a)

M1: Circle or arc of a circle with centre in first quadrant and with the circle in all 4 quadrants or arc of circle in quadrants 1 and 2

M1: A "V" shape i.e. with both branches above the *x*-axis and with the vertex on the positive real axis. Ignore any branches below the *x*-axis.

A1: Two half lines that meet on the positive real axis where the right branch intersects the circle or arc of a circle in the first quadrant and the left branch intersects the circle or arc of a circle in the second quadrant but not on the *y*-axis.

M1: Shades the region between the half-lines and within the circle

A1: Cso. A fully correct diagram including 2 marked (or implied by ticks) at the vertex on the real axis with the correct region shaded and all the previous marks scored.

(b)

M1: Identifies a suitable strategy for finding the *x* or *y* coordinate of the point of intersection.

Look for an attempt to solve equations of the form $(x\pm1)^2+(y\pm1)^2=9 \text{ or } 3$ and $y=\pm x\pm 2$

A1: Correct coordinates for the intersection (there may be other points but allow this mark if the correct coordinates are seen). (The correct coordinates may be implied by subsequent work.)

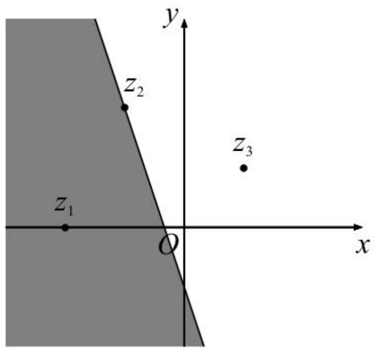
Allow equivalent exact forms and allow as a complex number e.g. $2+\frac{\sqrt{14}}{2}+\frac{\sqrt{14}}{2}\text{i}$

M1: Correct use of Pythagoras on their coordinates (There must be no i's)

A1: Correct **exact** value by cso

Note that solving $(x-1)^2+(y-1)^2=9,\ \ y=x+2$ gives $x=\frac{\sqrt{14}}{2}, y=2+\frac{\sqrt{14}}{2}$ and hence the correct answer fortuitously so scores M1A0M1A0

Question	Scheme	Marks	AOs
7	Complex roots are e.g. $\alpha \pm \beta$i or $(z^3+z^2+pz+q)\div(z-3)=z^2+4z+p+12$ or $f(3)=0 \Rightarrow 3^3+3^2+3p+q=0$ or One of: $3+z_2+z_3=-1$, $3z_2z_3=-q$, $3z_2+3z_3+z_2z_3=p$	B1	3.1a
	Sum of roots $\alpha+\beta i+\alpha-\beta i+3=-1 \Rightarrow \alpha=\ldots$ or $\alpha+\beta i+\alpha-\beta i=-4 \Rightarrow \alpha=\ldots$	M1	1.1b
	$\alpha=-2$	A1	1.1b
	So $\frac{1}{2}\times 2\beta\times 5=35 \Rightarrow \beta=7$	M1	1.1b
	$q=-3(\text{"}-2+7\text{i"})(\text{"}-2-7\text{i"})=\ldots$ or $p=3(\text{"}-2+7\text{i"})+3(\text{"}-2-7\text{i"})+(\text{"}-2+7\text{i"})(\text{"}-2-7\text{i"})$ or $(z-3)(z-(\text{"}-2+7\text{i"}))(z-(\text{"}-2-7\text{i"}))=\ldots$	M1	3.1a
	$q=-159$ or $p=41$	A1	1.1b
	$3p+q=-36 \Rightarrow p=\frac{-36-q}{3}=41$ and $q=-159$	A1	1.1b
		(7)	
	Alternative		
	$(z^3+z^2+pz+q)\div(z-3)=z^2+4z+p+12$	B1	3.1a
	$z^2+4z+p+12=0 \Rightarrow z=\frac{-4\pm\sqrt{4^2-4(p+12)}}{2}\left(=-2\pm \text{i}\sqrt{p+8}\right)$	M1	1.1b
	$\alpha=-2$	A1	1.1b
	$\beta=\sqrt{p+8}$	M1	1.1b
	$\frac{1}{2}\times(3+2)\times 2\sqrt{p+8}=35 \Rightarrow p=\ldots$	M1	3.1a
	$p=41$	A1	1.1b
	$3p+q=-36 \Rightarrow q=-159$	A1	1.1b
		(7)	
			(7 marks)

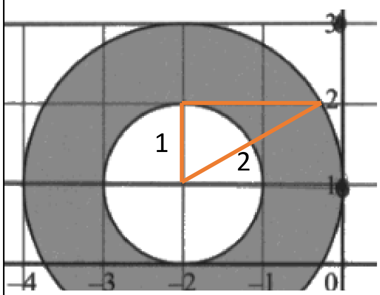
Question	Scheme	Marks	AOs
5 (a)	Complex roots of a real polynomial occur in conjugate pairs	M1	1.2
	so a polynomial with z_1, z_2 and z_3 as roots also needs z_2* and z_3* as roots, so 5 roots in total, but a quartic has at most 4 roots, so no quartic can have z_1, z_2 and z_3 as roots.	A1	2.4
		(2)	
(b)	$\frac{z_2 - z_1}{z_3 - z_1} = \frac{-1+2i-(-2)}{1+i-(-2)} = \frac{1+2i}{3+i} \times \frac{3-i}{3-i} = \ldots$	M1	1.1b
	$= \frac{3-i+6i+2}{9+1} = \frac{5+5i}{10} = \frac{1}{2}+\frac{1}{2}i$ oe	A1	1.1b
	As $\frac{1}{2}+\frac{1}{2}i$ is in the first quadrant (may be shown by diagram), hence $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arctan\left(\frac{1/2}{1/2}\right)\left(=\arctan(1)\right) = \frac{\pi}{4}$ *	A1*	2.1
		(3)	
(c)	$\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arg(z_2 - z_1) - \arg(z_3 - z_1) = \arg(1+2i) - \arg(3+i)$	M1	1.1b
	Hence $\arctan(2) - \arctan\left(\frac{1}{3}\right) = \frac{\pi}{4}$ *	A1*	2.1
		(2)	
(d)	 Line passing through z_2 and the negative imaginary axis drawn.	B1	1.1b
	Area below and left of their line shaded, where the line must have negative gradient passing through negative imaginary axis but need not pass through z_2	B1	1.1b
	Unless otherwise indicated by the student mark Diagram 1(if used) if there are multiple attempts.		
		(2)	
	(9 marks)		

Question	Scheme	Marks	AOs
2(a)	$\lvert z_1\rvert=\sqrt{13}$ and $\arg z_1=\tan^{-1}\left(\frac{3}{2}\right)$	B1	1.1b
	$z_1=\sqrt{13}\left(\cos 0.9828+\mathrm{i}\sin 0.9828\right)$	B1ft	1.1b
		(2)	
(b)	A complete method to find the modulus of z_2 e.g. $\lvert z_1\rvert=\sqrt{13}$ and uses $\lvert z_1z_2\rvert=\lvert z_1\rvert\times\lvert z_2\rvert=39\sqrt{2}\Rightarrow\lvert z_2\rvert=3\sqrt{26}$ or $\sqrt{234}$	M1 A1	3.1a 1.1b
	A complete method to find the argument of z_2 e.g. $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)=\frac{\pi}{4}\Rightarrow\arg(z_2)=...$ $\arg(z_2)=\frac{\pi}{4}-\tan^{-1}\left(\frac{3}{2}\right)$ or $\frac{\pi}{4}-0.9828$ or $-0.1974...$	M1 A1	3.1a 1.1b
	$z_2=3\sqrt{26}\left(\cos('-0.1974...')+\mathrm{i}\sin('-0.1974...')\right)$ or $z_2=a+b\mathrm{i}\Rightarrow a^2+b^2=234$ and $\tan^{-1}(-0.1974)=\frac{b}{a}\Rightarrow\frac{b}{a}=-0.2$ $\Rightarrow a=...$ and $b=...$	ddM1	1.1b
	Deduces that $z_2=15-3\mathrm{i}$ only	A1	2.2a
	Alternative $z_1z_2=(a+b\mathrm{i})(2+3\mathrm{i})=(2a-3b)+(3a+2b)\mathrm{i}$		
	$(2a-3b)^2+(3a+2b)^2=(39\sqrt{2})^2$ or 3042 $\Rightarrow a^2+b^2=234$ or $\lvert z_1z_2\rvert=\lvert z_1\rvert\times\lvert z_2\rvert=39\sqrt{2}\Rightarrow\lvert z_2\rvert=3\sqrt{26}$ or $\sqrt{234}$ $\Rightarrow a^2+b^2=234$	M1 A1	3.1a 1.1b
	$\arg\left[(2a-3b)+(3a+2b)\mathrm{i}\right]=\frac{\pi}{4}\Rightarrow\tan^{-1}\left(\frac{3a+2b}{2a-3b}\right)=\frac{\pi}{4}\Rightarrow\frac{3a+2b}{2a-3b}=1$ $\Rightarrow a=-5b$	M1 A1	3.1a 1.1b
	Solves $a=-5b$ and $a^2+b^2=234$ to find values for a and b	ddM1	1.1b
	Deduces that $z_2=15-3\mathrm{i}$ only	A1	2.2a
		(6)	
			(8 marks)

Question	Scheme	Marks	AOs
10	$(x-3)^2+(y-5)^2=(2r)^2$ and $y=-x+2$	B1	1.1b
	$(x-3)^2+(-x+2-5)^2=(2r)^2$ or $(-y+2-3)^2+(y-5)^2=(2r)^2$	M1	3.1a
	$2x^2+18-4r^2=0$ or $2y^2-8y+26-4r^2=0$	A1	1.1b
	$b^2-4ac>0\Rightarrow 0^2-4(2)(18-4r^2)>0\Rightarrow r>...$ or $x^2=9-2r^2\Rightarrow 9-2r^2>0\Rightarrow r>...$ or $b^2-4ac>0\Rightarrow(-8)^2-4(2)(26-4r^2)>0\Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2\Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
	Alternative Using a circle with centre (3, 5) and radius $2r$ and $y=-x+2$	B1	1.1b
	$y-5=1(x-3)\Rightarrow y=x+2$ $x+2=-x+2\Rightarrow x=...$	M1	3.1a
	$(0,2)$	A1	1.1b
	$2r>\sqrt{(3-0)^2+(5-2)^2}\Rightarrow r>...$	dM1	3.1a
	Finds a maximum value for r $(2r)^2=5^2+(3-2)^2\Rightarrow r=...$	M1	3.1a
	$\frac{3\sqrt{2}}{2}<r<\frac{\sqrt{26}}{2}$ o.e.	A1 A1	1.1b 1.1b
		(7)	
			(7 marks)

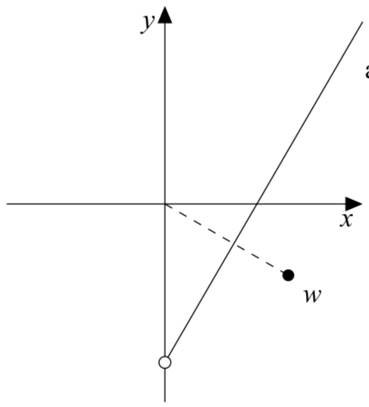
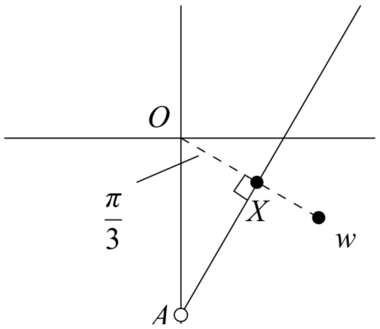
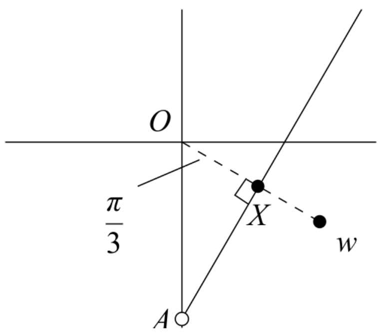
Notes:

B1: Correct equations for each loci of points
M1: A complete method to find a 3TQ involving one variable using equations of the form $(x\pm3)^2+(y\pm5)^2=(2r)^2$ or $2r^2$ or r^2 and $y=\pm x\pm 2$
A1: Correct quadratic equation

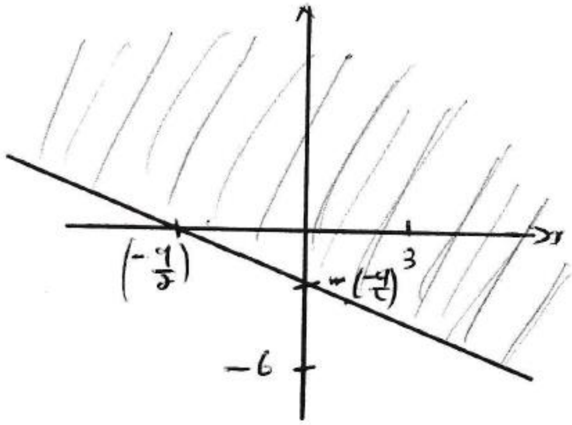
Question	Scheme	Marks	AOs
5(a)	$a = 1, d = 2$	B1	1.1b
	$b = 2$	B1	1.1b
	$c = -1$	B1	1.1b
		(3)	
(b)	$\lvert z-\mathrm{i}\rvert = \lvert z-3\mathrm{i}\rvert \Rightarrow y = 2$	B1	2.2a
	Area between the circles $= \pi\times 2^2 - \pi\times 1^2$	M1	1.1a
	 Angle subtended at centre = $2\times\cos^{-1}\left(\frac{1}{2}\right)$ Alternatively $(x+2)^2+(y-1)^2=4,\ y=2\Rightarrow x=...$ Or $x=\sqrt{2^2-1^2}$ Leading to Angle subtended at centre = $2\times\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$	M1	3.1a
	Segment area = $\frac{1}{2}\times\frac{2\pi}{3}\times 2^2 - \frac{1}{2}\times 2^2\times\sin\left("\frac{2\pi}{3}"\right)\left\{=\frac{4}{3}\pi-\sqrt{3}\right\}$	M1 A1	2.1 1.1b
	Area of Q: $\pi\times 2^2 - \pi\times 1^2 - \left(\frac{1}{2}\times\frac{2\pi}{3}\times 2^2 - \frac{1}{2}\times 2^2\times\sin\left(\frac{2\pi}{3}\right)\right)$	M1	3.1a
	$= \frac{5\pi}{3}+\sqrt{3}$	A1	1.1b
		(7)	
			(10 marks)

Notes

(a)
B1: Correct values for *a* and *d*
B1: Correct value for *b*
B1: Correct value for *c*
(b)
B1: Deduces that $\lvert z-\mathrm{i}\rvert = \lvert z-3\mathrm{i}\rvert$ is a perpendicular bisector with equation $y = 2$, this may be drawn on a diagram.
M1: Selects the correct procedure to find the area of the large circle – the area of the small circle.
M1: Correct method to find the angle at the centre (or half this angle).
Recognises that the hypotenuse is the radius of the larger circle and the adjacent is the radius if the smaller circle and using cosine
Alternatively find where the perpendicular bisector intersects the larger circle so uses their $y = 2$ and the equation of the larger circle in an attempt to establish the x values for the intersection points or uses geometry and Pythagoras to identify the required length and then uses tangent.
M1: Correct method for the area of the minor segment (allow equivalent work)

Question	Scheme	Marks	AOs
2(a)	$\lvert w\rvert=\sqrt{\left(4\sqrt{3}\right)^2+(-4)^2}=8$	B1	1.1b
	$\arg w=\arctan\left(\dfrac{\pm4}{4\sqrt{3}}\right)=\arctan\left(\pm\dfrac{1}{\sqrt{3}}\right)$	M1	1.1b
	$=-\dfrac{\pi}{6}$	A1	1.1b
	So $(w=)8\left(\cos\left(-\dfrac{\pi}{6}\right)+\mathrm{i}\sin\left(-\dfrac{\pi}{6}\right)\right)$	A1	1.1b
		(4)	
(b)	 (i) w in 4[th] quadrant with either $\left(4\sqrt{3},-4\right)$ seen or $-\dfrac{\pi}{4}<\arg w<0$	B1	1.1b
	(ii) half line with positive gradient emanating from imaginary axis.	M1	1.1b
	The half line should pass between O and w starting from a point on the imaginary axis below w	A1	1.1b
		(3)	
(c)	 ΔOAX is right angled at X so $OX=10\sin\dfrac{\pi}{6}=5$ (oe)	M1	3.1a
	So shortest distance is $WX=OW-OX=$ '8' $-5=\ldots$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 1  A complete method to find the coordinates of X. Finds the equation of the line from O to w, $y=-\dfrac{1}{\sqrt{3}}x$ and the equation of the half line $y=\sqrt{3}x-10$, solves to find the point of intersection $X\left(\dfrac{5\sqrt{3}}{2},-\dfrac{5}{2}\right)$	M1	3.1a
	Finds the length WX $\sqrt{\left(4\sqrt{3}-\dfrac{5\sqrt{3}}{2}\right)^2+\left(-4--\dfrac{5}{2}\right)^2}$	M1	1.1b
	So min distance is 3	A1	1.1b
	Alternative 2	M1	3.1a

Question	Scheme		Marks	AOs
4(i) (a)	$\frac{2+3i}{5+i}\times\frac{5-i}{5-i}$	$2+3i=k(1+i)(5+i)=...$	M1	1.1a
	$\frac{10-2i+15i+3}{25+1}$ or $\frac{13+13i}{26}$	$2+3i=k(5+i+5i-1)=...$	dM1	1.1b
	$\frac{1}{2}(1+i)$ cso	$2+3i=k(4+6i)$ therefore $\frac{2+3i}{5+i}=k(1+i)$ where $k=\frac{1}{2}$ cso	A1	2.1
			(3)	
(i)(b)	$n=4$		B1	2.2a
			(1)	
(ii)	$\lvert z\rvert=3$		B1	1.2
	$\arg(z^{10})=10\arg(z)=-\frac{5\pi}{3}\Rightarrow\arg(z)=...\left\{-\frac{\pi}{6}\right\}$ $\arg(z^{10})=10\arg(z)=\frac{\pi}{3}\Rightarrow\arg(z)=...\left\{\frac{\pi}{30}\right\}$		M1	1.1b
	$z=3\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)=...$		M1	2.1
	$z=\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ or $a=\frac{3\sqrt{3}}{2}$ and $b=-\frac{3}{2}$		A1	1.1b
			(4)	
	Alternative $a^2+b^2=9$		B1	1.2
	$10\arg z=-\frac{5\pi}{3}\Rightarrow\arg z=-\frac{5\pi}{3}\div 10$ Or e.g $10\arg(z)=\frac{\pi}{3}\Rightarrow\arg(z)=...\left\{\frac{\pi}{30}\right\}$		M1	1.1b
	Forming and solving simultaneous equations to find a value for *a* or *b* $\frac{b}{a}=\arctan\left(-\frac{\pi}{6}\right)\Rightarrow\frac{b}{a}=-\frac{\sqrt{3}}{3}\Rightarrow b=-a\frac{\sqrt{3}}{3}$ or $\frac{b}{a}=\arctan\frac{\pi}{30}\Rightarrow b=0.104...a$		M1	2.1
	$z=\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ or $a=\frac{3\sqrt{3}}{2}$ and $b=-\frac{3}{2}$		A1	1.1b
			(4)	
			(8 marks)	

Question	Scheme	Marks	AOs
7(i)		M1 A1 B1	3.1a 1.1b 1.1b
		(3)	
(ii)	$m=\tan\left(\frac{\pi}{3}\right)\left\{=\sqrt{3}\right\}$ and $y-0=m(x-2)$ leads to $y-0=\sqrt{3}(x-2)$ or $y=\sqrt{3}x-2\sqrt{3}$ $m=\tan\left(\frac{\pi}{6}\right)\left\{=\frac{\sqrt{3}}{3}\right\}$ and $y-0=m(x-(-1))$ leads to $y-0=\frac{\sqrt{3}}{3}(x-(-1))$ or $y=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}$	M1 A1 A1	3.1a 1.1b 1.1b
	$\sqrt{3}x-2\sqrt{3}=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}\Rightarrow x=\ldots$	M1	1.1b
	$y=\sqrt{3}\left(\frac{7}{2}\right)-2\sqrt{3}=\ldots$	M1	1.1b
	$\{w=\}\frac{7}{2}+\frac{3\sqrt{3}}{2}\text{i}$	A1	2.1
		(6)	
	Alternative $\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}=\frac{y}{x_{-1}}$ and $\tan\left(\frac{\pi}{3}\right)=\sqrt{3}=\frac{y}{x_2}$	M1	1.1b
	 $x_{-1}=\sqrt{3}y$ $x_2=\frac{\sqrt{3}}{3}y$	A1 A1	1.1b 1.1b

	$y\sqrt{3} = y\dfrac{\sqrt{3}}{3} + 3 \Rightarrow y = \ldots$	M1	3.1a
	Uses $x = y\sqrt{3} - 1$ or $x = \dfrac{\sqrt{3}}{3}y + 2$ with their value of *y* leading to a value for *x*	M1	1.1b
	$(w =)\dfrac{7}{2} + \dfrac{3\sqrt{3}}{2}\mathrm{i}$	A1	2.1
		(6)	
(9 marks)			
Notes:			

(i)

M1: Draws a **single** straight line through **both axes** with a negative gradient. Ignore any line joining (3, 0) and (0, -6)

A1: Draws a **single** straight line through **both axes** with a negative gradient which has a negative *y* intercept**.** Ignore any intercept marked on the axes. Ignore any line joining (3, 0) and (0, -6)

B1: Shades the area above their straight line (not a bounded region such as a triangle bounded by the axes and the line)

(ii)

M1: Finds the Cartesian equations for both loci by using the gradient as tan(argument) and correct coordinate. Must be an attempt at both equations but one correct equation scores this mark

A1: One equation correct, need not be simplified

A1: Both equations correct, need not be simplified

M1: Solve simultaneously to find either the real or imaginary component.

M1: Finds the other component to complete the process of finding *w*.

A1: Correct exact answer

Note: If leaves the answer as a coordinate this is A0. If defines $w = a + b\mathrm{i}$ and then states $a = \dfrac{7}{2}$ and $b = \dfrac{3\sqrt{3}}{2}$ this is A1

Alternative

M1: Use both arguments to form equations involving *x* and *y*

A1: (One correct triangle) value for *x* in terms of *y*

A1: (Two correct triangles), values for *x* in terms of *y*

M1: Forms and solves an equation $y\sqrt{3} = y\dfrac{\sqrt{3}}{3} + 3 \Rightarrow y = \ldots$ must be come from $x_2 = x_{-1} + 3$

M1: Uses their *y* value and $x = y\sqrt{3} - 1$ or $x = \dfrac{\sqrt{3}}{3}y + 2$ to find a value for *x*

A1: Correct exact answer

Note: If candidates use decimal instead of exact values throughout allow the method marks

$y = 1.73x - 3.46$ and $y = 0.58x + 0.58$