Question	Scheme	Marks	AOs
16 (a) (i)	Uses $\frac{dy}{dx}=-3$ at $x=2 \Rightarrow 12a+60-39=-3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a+60-39=-3 \Rightarrow 12a=-24 \Rightarrow a=-2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $10=8a+60-78+b$	M1	3.1a
	Subs $a=-2$ into $10=8a+60-78+b \Rightarrow b=44$	A1	1.1b
		(4)	
(b)	$f(x)=-2x^3+15x^2-39x+44 \Rightarrow f'(x)=-6x^2+30x-39$	B1	1.1b
	Attempts to show that $-6x^2+30x-39$ has no roots Eg. calculates $b^2-4ac=30^2-4\times-6\times-39=-36$	M1	3.1a
	States that as $f'(x)\neq 0 \Rightarrow$ hence $f(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3+15x^2-39x+44 \equiv (x-4)(-2x^2+7x-11)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
			(11 marks)

Notes

(a)(i)

M1: Attempts to use $\frac{dy}{dx}=-3$ at $x=2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a=-2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a=-2$ leading to $b=$ ...

A1: $b=44$

(b)

B1: $f'(x)=-6x^2+30x-39$ oe

M1: Correct attempt to show that "$-6x^2+30x-39$" has no roots.
This could involve an attempt at

- finding the numerical value of b^2-4ac
- finding the roots of $-6x^2+30x-39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2+30x-39$

Question	Scheme	Marks	AOs
5(a)	Attempts to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=2$	M1	1.1b
	$\frac{\mathrm{d}y}{\mathrm{d}x}=6x \Rightarrow$ gradient of tangent at *P* is 12	A1	1.1b
		(2)	
(b)	Gradient *PQ* = $\frac{3(2+h)^2-2-10}{(2+h)-2}$ oe	B1	1.1b
	$=\frac{3(2+h)^2-12}{(2+h)-2}=\frac{12h+3h^2}{h}$	M1	1.1b
	$=12+3h$	A1	2.1
		(3)	
(c)	Explains that as $h \to 0,\ 12+3h \to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
			(6 marks)

Notes

(a)

M1: Attempts to differentiate, allow $3x^2-2 \to ...x$ and substitutes $x=2$ into their answer

A1: cso $\frac{\mathrm{d}y}{\mathrm{d}x}=6x \Rightarrow$ gradient of tangent at *P* is 12

(b)

B1: Correct expression for the gradient of the chord seen or implied.

M1: Attempts $\frac{\delta y}{\delta x}$, condoning slips, and attempts to simplify the numerator. The denominator must be h

A1: cso $12+3h$

(c)

B1: Explains that as $h \to 0,\ 12+3h \to 12$ and states that the gradient of the chord tends to the gradient of the curve

Question	Scheme	Marks	AOs
8 (a)(i)	$C=\dfrac{1500}{v}+\dfrac{2v}{11}+60 \Rightarrow \dfrac{dC}{dv}=-\dfrac{1500}{v^2}+\dfrac{2}{11}$	M1 A1	3.1b 1.1b
	Sets $\dfrac{dC}{dv}=0 \Rightarrow v^2=8250$	M1	1.1b
	$\Rightarrow v=\sqrt{8250} \Rightarrow v=90.8\ (\text{km h}^{-1})$	A1	1.1b
(ii)	For substituting their $v=90.8$ in $C=\dfrac{1500}{v}+\dfrac{2v}{11}+60$	M1	3.4
	Minimum cost =awrt (£) 93	A1 ft	1.1b
		(6)	
(b)	Finds $\dfrac{d^2C}{dv^2}=+\dfrac{3000}{v^3}$ at $v=90.8$	M1	1.1b
	$\dfrac{d^2C}{dv^2}=(+0.004)>0$ hence minimum (cost)	A1 ft	2.4
		(2)	
(c)	It would be impossible to drive at this speed over the whole journey	B1	3.5b
		(1)	
			(9 marks)

Notes

(a)(i)
M1: Attempts to differentiate (deals with the powers of v correctly).

Look for an expression for $\dfrac{dC}{dv}$ in the form $\dfrac{A}{v^2}+B$

A1: $\left(\dfrac{dC}{dv}\right)=-\dfrac{1500}{v^2}+\dfrac{2}{11}$

A number of students are solving part (a) numerically or graphically. Allow these students to pick up the M1 A1 here from part (b) when they attempt the second derivative.

M1: Sets $\dfrac{dC}{dv}=0$ (which may be implied) and proceeds to an equation of the type $v^n=k, k>0$

Allow here equations of the type $\dfrac{1}{v^n}=k, k>0$

A1: $v=\sqrt{8250}$ or $5\sqrt{330}$ awrt 90.8 (km h^{-1}) .

As this is a speed withhold this mark for answers such as $v=\pm\sqrt{8250}$

* Condone $\dfrac{dC}{dv}$ appearing as $\dfrac{dy}{dx}$ or perhaps not appearing at all. Just look for the rhs.

Question	Scheme	Marks	AOs
10	Considers $\dfrac{(x+h)^3-x^3}{h}$	B1	2.1
	Expands $(x+h)^3=x^3+3x^2h+3xh^2+h^3$	M1	1.1b
	so gradient (of chord) $=\dfrac{3x^2h+3xh^2+h^3}{h}=3x^2+3xh+h^2$	A1	1.1b
	States as $h\to 0,\quad 3x^2+3xh+h^2\to 3x^2$ so derivative $=3x^2$ *	A1*	2.5
			(4 marks)

B1: Gives the correct fraction for the gradient of the chord either $\dfrac{(x+h)^3-x^3}{h}$ or $\dfrac{(x+\delta x)^3-x^3}{\delta x}$

It may also be awarded for $\dfrac{(x+h)^3-x^3}{x+h-x}$ oe. It may be seen in an expanded form

It does not have to be linked to the gradient of the chord

M1: Attempts to expand $(x+h)^3$ or $(x+\delta x)^3$ Look for two correct terms, most likely $x^3+...+h^3$
This is independent of the B1

A1: Achieves gradient (of chord) is $3x^2+3xh+h^2$ or exact un simplified equivalent such as $3x^2+2xh+xh+h^2$. Again, there is no requirement to state that this expression is the gradient of the chord

A1*: CSO. Requires correct algebra and making a link between the gradient of the chord and the gradient of the curve. See below how the link can be made. The words "gradient of the chord" do not need to be mentioned but derivative, $f'(x), \dfrac{dy}{dx}, y'$ should be. Condone invisible brackets for the expansion of $(x+h)^3$ as long as it is only seen at the side as intermediate working.

Requires either

- $f'(x)\underset{\lim h\to 0}{=}\dfrac{(x+h)^3-x^3}{h}=3x^2+3xh+h^2=3x^2$
- Gradient of chord $=3x^2+3xh+h^2$ As $h\to 0$ Gradient of chord tends to the gradient of curve so derivative is $3x^2$
- $f'(x)\underset{\lim h\to 0}{=}3x^2+3xh+h^2=3x^2$
- Gradient of **chord** $=3x^2+3xh+h^2$ when $h\to 0$ gradient of **curve** $=3x^2$
- Do not allow $h=0$ alone without limit being considered somewhere:
  so don't accept $h=0\Rightarrow f'(x)=3x^2+3x\times 0+0^2=3x^2$

....

Alternative: B1: Considers $\dfrac{(x+h)^3-(x-h)^3}{2h}$ M1: As above A1: $\dfrac{6x^2h^2+2h^3}{2h}=3x^2+h^2$

Question	Scheme	Marks	AOs
5(a)	$x^n \to x^{n-1}$	M1	1.1b
	$\left(\frac{dy}{dx}\right) = 6x - \frac{24}{x^2}$	A1 A1	1.1b 1.1b
		(3)	
(b)	Attempts $6x - \frac{24}{x^2} > 0 \Rightarrow x >$	M1	1.1b
	$x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$	A1	2.5
		(2)	
			(5 marks)

Notes

(a)

M1: $x^n \to x^{n-1}$ for any correct index of x. Score for $x^2 \to x$ or $x^{-1} \to x^{-2}$

Allow for unprocessed indices. $x^2 \to x^{2-1}$ oe

A1: Sight of either $6x$ or $-\frac{24}{x^2}$ which may be un simplified.

Condone an additional term e.g. + 2 for this mark

The indices now must have been processed

A1: $\frac{dy}{dx} = 6x - \frac{24}{x^2}$ or exact simplified equivalent. Eg accept $\frac{dy}{dx} = 6x^1 - 24x^{-2}$

You do not need to see the $\frac{dy}{dx}$ and you should isw after a correct simplified answer.

(b)

M1: Sets an allowable $\frac{dy}{dx}$**...**0 and proceeds to x... via an allowable intermediate equation

$\frac{dy}{dx}$ must be in the form $Ax + Bx^{-2}$ where $A, B \neq 0$

and the intermediate equation must be of the form $x^p ... q$ oe

Do not be concerned by either the processing, an equality or a different inequality.

It may be implied by $x =$ awrt 1.59

A1: $x > \sqrt[3]{4}$ or $x \geqslant \sqrt[3]{4}$ oe such as $x > 4^{\frac{1}{3}}$ or $x \geqslant 2^{\frac{2}{3}}$

Question	Scheme	Marks	AOs
1	Attempts to differentiate $x^n \rightarrow x^{n-1}$ seen once	M1	1.1b
	$y = 2x^3 - 4x + 5 \Rightarrow \frac{dy}{dx} = 6x^2 - 4$	A1	1.1b
	For substituting $x = 2$ into their $\frac{dy}{dx} = 6x^2 - 4$	dM1	1.1b
	For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$	ddM1	1.1b
	$y = 20x - 27$	A1	1.1b
		(5)	
			(5 marks)

Notes

M1: Attempts to differentiate $x^n \rightarrow x^{n-1}$ seen once. Score for $x^3 \rightarrow x^2$ or $\pm 4x \rightarrow 4$ or $+5 \rightarrow 0$

A1: $\left(\frac{dy}{dx} =\right) 6x^2 - 4$ which may be unsimplified $6x^2 - 4 + C$ is A0

dM1: Substitutes $x = 2$ into their $\frac{dy}{dx}$. The first M must have been awarded.

Score for sight of embedded values, or sight of "$\frac{dy}{dx}$ at $x = 2$ is" or a correct follow through.

Note that 20 on its own is not enough as this can be done on a calculator.

ddM1: For a correct method of finding a tangent at $P(2,13)$. Score for $y - 13 = "20"(x-2)$

It is dependent upon both previous M's.

If the form $y = mx + c$ is used they must proceed as far as $c = ...$

A1: Completely correct $y = 20x - 27$ (and in this form)

Question	Scheme	Marks	AOs
14 (a)	Deduces $g(x)=ax^3+bx^2+ax$	B1	2.2a
	Uses $(2,9)\Rightarrow 9=8a+4b+2a$ $\Rightarrow 10a+4b=9$	M1 A1	2.1 1.1b
	Uses $g'(2)=0\Rightarrow 0=12a+4b+a$ $\Rightarrow 13a+4b=0$	M1 A1	2.1 1.1b
	Solves simultaneously $\Rightarrow a,b$	dM1	1.1b
	$g(x)=-3x^3+\frac{39}{4}x^2-3x$	A1	1.1b
		(7)	
(b)	Attempts $g''(x)=-18x+\frac{39}{2}$ and substitutes $x=2$	M1	1.1b
	$g''(2)=-\frac{33}{2}<0$ hence maximum	A1	2.4
		(2)	
			(9 marks)

Notes

(a)

B1: Uses the information given to deduce that $g(x)=ax^3+bx^2+ax$. (Seen or implied)

M1: Uses the fact that $(2,9)$ lies on the curve so uses $x=2, y=9$ within a cubic function

A1: For a simplified equation in just two variables. E.g. $10a+4b=9$

M1: Differentiates their cubic to a quadratic and uses the fact that $g'(2)=0$ to obtain an equation in *a* and *b*.

A1: For a different simplified equation in two variables E.g. $13a+4b=0$

dM1: Solves simultaneously $\Rightarrow a=...,b=...$ It is dependent upon the B and both M's

A1: $g(x)=-3x^3+\frac{39}{4}x^2-3x$

(b)

M1: Attempts $g''(x)=-18x+\frac{39}{2}$ and substitutes $x=2$. Award for second derivatives of the form $g''(x)=Ax+B$ with $x=2$ substituted in.
Alternatively attempts to find the value of their $g'(x)$ or $g(x)$ either side of $x=2$ (by substituting a value for x within 0.5 either side of 2)

A1: $g''(2)=-\frac{33}{2}<0$ hence maximum. (allow embedded values but they must refer to the sign or that it is less than zero)
If $g'(x)=-9x^2+\frac{39}{2}x-3$ or $g(x)=-3x^3+\frac{39}{4}x^2-3x$ is calculated either side of $x=2$ then the values must be correct or embedded correctly (you will need to check these) they need to compare $g'(x)>0$ to the left of $x=2$ and $g'(x)<0$ to the right of $x=2$ or $g(x)<9$ to the left and $g(x)>9$ to the right of $x=2$ hence maximum.

Note If they only sketch the cubic function $g(x)=-3x^3+\frac{39}{4}x^2-3x$ then award M1A0

Question	Scheme	Marks	AOs
2	Attempt to differentiate	M1	1.1a
	$\frac{dy}{dx} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1	1.1b
	$\Rightarrow \frac{dy}{dx} = 8$	A1ft	1.1b
		(4 marks)	

Notes:

M1: Differentiation implied by one correct term

A1: Correct differentiation

M1: Attempts to substitute $x = 5$ into their derived function

A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their $f'(5)$ so follow through slips in differentiation

Question	Scheme	Marks	AOs
3(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$\lvert AB \rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
		(4 marks)	

Notes:

(a)

M1: Attempts subtraction but may omit brackets

A1: cao (allow column vector notation)

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

A1ft: $\lvert AB \rvert = 5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Question	Scheme	Marks	AOs
6	Considers $\frac{3(x+h)^2 - 3x^2}{h}$	B1	2.1
	Expands $3(x + h)^2 = 3x^2 + 6xh + 3h^2$	M1	1.1b
	So gradient $= \frac{6xh + 3h^2}{h} = 6x + 3h$ or $\frac{6x\delta x + 3(\delta x)^2}{\delta x} = 6x + 3\delta x$	A1	1.1b
	States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $= 6x$ *	A1*	2.5
			(4 marks)

Notes:

B1: Gives correct fraction as in the scheme above or $\frac{3(x+\delta x)^2 - 3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2 = 3x^2 + 6x\delta x + 3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Uses Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

Question	Scheme	Marks	AOs
15	Finds $\frac{dy}{dx} = 8x - 6$	M1	3.1a
	Gradient of curve at P is –2	M1	1.1b
	Normal gradient is $-\frac{1}{m} = \frac{1}{2}$	M1	1.1b
	So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$	A1	1.1b
	Eliminates y between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in x	M1	3.1a
	Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}e^{\frac{7}{4}}$	M1	1.1b
	Substitutes to give value for y	M1	1.1b
	Point Q is $\left(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}\right)$	A1	1.1b
			(8 marks)

Notes:

M1: Differentiates correctly

M1: Substitutes $x = \frac{1}{2}$ to find gradient (may make a slip)

M1: Uses negative reciprocal gradient

A1: Correct equation for normal

M1: Attempts to eliminate y to find an equation in x

M1: Attempts to solve their equation using exp

M1: Uses their x value to find y

A1: Any correct exact form

Question	Scheme	Marks	AOs
16(a)	Sets $2xy + \frac{\pi x^2}{2} = 250$	B1	2.1
	Obtain $y = \frac{250 - \frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P = 2x + 2y + \pi x$ with their y substituted	M1	2.1
	$P = 2x + \frac{250}{x} - \frac{\pi x^2}{2x} + \pi x = 2x + \frac{250}{x} + \frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x > 0$ and $y > 0$ (distance) $\Rightarrow \frac{250 - \frac{\pi x^2}{2}}{2x} > 0$ or $250 - \frac{\pi x^2}{2} > 0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0 < x < \sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{dP}{dx}$; $x^{-1} \to x^{-2}$	M1	3.4
	$\frac{dP}{dx} = 2 - \frac{250}{x^2} + \frac{\pi}{2}$	A1	1.1b
	Sets $\frac{dP}{dx} = 0$ and proceeds to $x =$	M1	1.1b
	Substitutes their x into $P = 2x + \frac{250}{x} + \frac{\pi x}{2}$ to give perimeter = 59.8 M	A1	1.1b
		(4)	
		(10 marks)	

Question Number	Scheme		Marks
9. (a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$	$\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360} \times \pi x^2$ simplified or un-simplified	M1
		$\frac{\pi x^2}{3}$	A1
			[2]
	Parts (b) and (c) may be marked together		
(b)	$\{A =\}\ \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	Attempt to sum 3 areas (at least one correct)	M1
		Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow \underline{y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
(c)	$\{P =\}\ x + x\theta + y + 2x + y\ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	Correct expression in x and y for their θ measured in rads	B1ft
	$\ldots 2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$	Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$		
	$\Rightarrow \underline{P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
	Parts (d) and (e) should be marked together		
(d)	$\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$	$\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$	M1
		Correct differentiation (need not be simplified).	A1;
		Their $P' = 0$	M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ (= 16.63392808...)	$\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..$ (m)	awrt 120	A1
			[5]
(e)	$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow$ Minimum	Finds P'' **and** considers sign.	M1
		$\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
			[2]
			15

Question Number	Scheme	Marks
5.(a)	$y = 27x^{0.5} - 2x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$	M1A1A1
		[3]
(b)	Sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1
	$\frac{27}{2}x^{-0.5} - 4x = 0 \Rightarrow x^{1.5} = \frac{27}{8} \Rightarrow x = \frac{9}{4}$	dM1,A1
	$x = \frac{9}{4} \Rightarrow y = \frac{243}{8}$	dM1A1
		[5] **(8 marks)**

(a)

M1 Uses $x^n \to x^{n-1}$ at least once. So sight of either index $x^{-0.5} / x^{-\frac{1}{2}}$ or $x = x^1$

A1 Either term correct (may be unsimplified). Eg. $2\times 2x^1$ is acceptable. The indices must be tidied up however so don't allow $2\times 2x^{2-1}$

A1 $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$ or exact equivalent such as $\frac{\mathrm{d}y}{\mathrm{d}x} = 13.5\times\frac{1}{\sqrt{x}} - 4x$.
It must be all tidied up for this mark so do not allow $2\times 2x$

(b)

M1 States or sets their $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ This may be implied by subsequent working.

dM1 Dependent upon the previous M and correct indices in (a). It is awarded for correct index work leading to $x^{\pm 1.5} = k$ Also allow squaring $27x^{-0.5} = 8x \Rightarrow \frac{27^2}{x} = 64x^2 \Rightarrow x^3 =$

A1 $x = \frac{9}{4}$ or exact equivalent. A correct answer following a correct derivative can imply the previous mark provided you have not seen incorrect work.

dM1 Dependent upon the first M1 in (b). For substituting their value of x into y to find the maximum point. There is no need to check this with a calculator. (y appearing from an x found from $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ is fine.)

A1 $y = \frac{243}{8}$ or exact equivalent (30.375) . You do not need to see the coordinates for this award.
Ignore any other solutions outside the range. If extra solutions are given within the range withhold only this final mark.

Note: This question requires differentiation in (a) and minimal working in (b). A correct answer without any differentiation will not score any marks.

Allow (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$ (b) $0 = \frac{27}{2}x^{-0.5} - 4x \Rightarrow x = \frac{9}{4}, y = \frac{243}{8}$ for all marks

Whereas (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{27}{2}x^{-0.5} - 4x$ (b) $x = \frac{9}{4}, y = \frac{243}{8}$ scores (a) 3 (b) 0 marks

Question Number	Scheme	Marks
16.	$\frac{dy}{dx}=3ax^2+2bx+2$	B1
	Sub $x=1, y=4 \Rightarrow y=ax^3+bx^2+2x-5$ or $x=1$ into $ax^3+bx^2+2x-5=12x-8$	M1
	Sub $x=1, \frac{dy}{dx}=12 \Rightarrow 3a+2b+2=12$	M1
	Solves simultaneously $a+b=7, 3a+2b=10 \Rightarrow a=-4, b=11$	dM1A1
		[5]
		(5 marks)

B1 States or uses $\frac{dy}{dx}=3ax^2+2bx+2$

M1 Attempts to substitute $x=1, y=4$ in $y=f(x)$ $\Rightarrow a+b+2-5=4$

This also can be scored by to substituting $x=1$ into $ax^3+bx^2+2x-5=12x-8 \Rightarrow a+b+2-5=12-8$

M1 Attempts to substitute $x=1, \frac{dy}{dx}=12$ in their $\frac{dy}{dx}=3ax^2+2bx+2$

dM1 Solves simultaneously to find both *a* and *b*. Both M's must have been awarded. Allow from a graphical calculator. Sight of both values is sufficient.

A1 $a=-4, b=11$

Question Number	Scheme	Marks
4(a)(i)	$\frac{dy}{dx} = 6x^{0.5} - 24x^{-1.5}$	M1A1A1
(ii)	$\frac{d^2y}{dx^2} = 3x^{-0.5} + 36x^{-2.5}$	M1A1
		(5)
(b)	$\frac{dy}{dx} = 0 \Rightarrow 6x^{0.5} - 24x^{-1.5} = 0$	M1
	$x^2 = 4 \Rightarrow x = 2$	dM1, A1
	Substitutes their $x = 2$ into $y = 4x\sqrt{x} + \frac{48}{\sqrt{x}} - \sqrt{8} \Rightarrow y = 30\sqrt{2}$	M1,A1
		(5)
(c)	Substitutes their $x = 2$ into their $\frac{d^2y}{dx^2} = 3x^{-0.5} + 36x^{-2.5}$	M1
	Statement +reason. ie $\frac{d^2y}{dx^2} > 0 \Rightarrow$ minimum	A1 cso
		(2)
		(12 marks)

Question Number	Scheme		Marks
1(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{3x^2}{3}-2\times 2x+3$	M1: $x^n \to x^{n-1}$ or $5 \to 0$ A1: **Any 3** of the 4 terms differentiated correctly - this could be 2 terms correct and $5 \to 0$ (allow simplified or un-simplified for this mark, including $3x^0$ for 3)	M1A1
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)x^2-4x+3$	Cao. All 3 terms correct and simplified and on the same line and no + 0. (Do not allow $1x^2$ for x^2 or x^1 for x or $3x^0$ for 3). Condone poor notation e.g. omission of dy/dx = … or if they use y = …	A1
	Candidates who multiply by 3 before differentiating: e.g. $\left(\frac{x^3}{3}-2x^2+3x+5\right)\times 3 = x^3-6x^2+9x+15 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-12x+9$ Scores M1A0A0 but could recover in (a) if they then divide by 3 **If they persist with** $\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2-12x+9$ **in (b), allow full recovery in (b)**		
			(3)
(b)	$x^2-4x+3=0 \Rightarrow x=1,\ 3$	M1: Attempt to solve their 3TQ from part (a) as far as x = … (see general guidance for solving a 3TQ). If no working is shown and the roots are incorrect for their 3TQ, score M0 here but the second method mark below is still available. A1: Correct values (may be implied by their inequalities e.g. a correct quadratic followed by just $x > 1$ and $x > 3$ could score M1A1 here)	M1A1
	$x<\text{"1"},\quad x>\text{"3"}$	**Chooses outside region** (x < their lower limit x > their upper limit). Do not award simply for diagram or table.	M1
	$x<1,\quad x>3$ Correct answer. Allow the correct regions separated by a comma or written separately and allow other notation e.g. $(-\infty,1)\cup(3,\infty)$. Do not allow $1>x>3$ or $x<1$ **and** $x>3$ (These score M1A0). ISW if possible e.g. $x>3,\quad x<1$ followed by $1>x>3$ can score M1A1. $x>3,\quad x>1$ followed by $x>3$ (or) $x<1$ can score M1A1. **Fully correct answer with no working scores both marks. Answers that are otherwise correct but use** $\le, \ge$ **lose final mark as would** $[-\infty,1]\cup[3,\infty]$.		A1
			(4)
			(7 marks)

Question	Scheme	Marks
12 (a)	$\mathrm{f}(x)=\dfrac{x^3-9x^2-81x}{27}=0 \quad \Rightarrow x(x^2-9x-81)=0$	M1
	$x=\dfrac{9\pm\sqrt{81+324}}{2}$	dM1
	$x=\dfrac{9\pm\sqrt{405}}{2}$ or $x=\dfrac{9\pm9\sqrt{5}}{2}$	A1 A1 **[4]**
(b)	Differentiates (usual rules), correctly and sets = 0 $\mathrm{f}'(x)=3x^2-18x-81=0$	M1, A1
	Solves $\mathrm{f}'(x)=0$ (or multiple)$\Rightarrow$ x = 9 and -3	dM1 A1
	Substitutes one of their values for x into f(x)	ddM1
	x = 9 y = -27 and x =-3 y = 5	A1 **[6]**
(c)	a = 9	B1 **[1]**
		11 marks
	Notes	

(a)

M1: Attempts to solve f(x) = 0, by taking out a factor of (/cancelling by) x and obtaining a quadratic factor.

Allow on $x\left(\dfrac{x^2}{27}-\dfrac{9x}{27}-\dfrac{81}{27}\right)=0$ or just the numerator $x(x^2-9x-81)=0$

This is implied by sight of $x^2-9x-81=0$

dM1: Uses formula or completion of square method to find at least one value for x , for **their** three term quadratic. Factorisation is M0. Note that their 3 term quadratic equation may be $\dfrac{1}{27}x^2-\dfrac{1}{3}x-3=0$

A1: One correct solution – need not be fully simplified. So allow $x=\dfrac{9+\sqrt{405}}{2}$ but not $x=\dfrac{9+\sqrt{81+324}}{2}$

A1: Two correct solutions – need not be simplified or attributed correctly to A or B.

Special case: If a candidate takes out a common factor of x and uses a calculator to write down the exact surd answers to the quadratic they have used (a limited) amount of algebra. Decimals would not be awarded for this SC. We will therefore score this SC M1 M1 A0 A0 for 2 out of 4. $x(x^2-9x-81)=0\Rightarrow x=\dfrac{9\pm9\sqrt{5}}{2}$ Just writing down the answers with no working scores 0 marks

(b)

M1: Differentiates f(x) to a 3 term quadratic

You may see confusion over the 27 but score for $\mathrm{f}'(x)$ being a 3 term quadratic

A1: Differentiates correctly and sets correct derivative = 0

$3x^2-18x-81=0$ or any multiple thereof. For example it may be common to see $\dfrac{3x^2}{27}-\dfrac{18x}{27}-\dfrac{81}{27}=0$

dM1: Solves quadratic to give two solutions. It is dependent upon the previous M.

Allow any appropriate method including the use of a calculator.

Condone $\dfrac{x^2}{9}-\dfrac{2x}{3}-3=0\Rightarrow(x-9)(x+3)=0$

A1 : Gives **both** 9 and −3

ddM1: Substitute at least one of their values of x (obtained from a solution of $\mathrm{f}'(x)=0$) into f(x) to give y = .

A1: Gives both −27 and 5 (arising from x values of 9 and −3) (Do not require coordinates).

Again they do not need to be attributed correctly to C or D

(c)

B1: For a = 9 only (no ft)

Question	Scheme	Marks
15 (a)	$200 = \pi r^2 + \pi rh + 2rh$	M1 A1
	$(h =)\dfrac{200-\pi r^2}{\pi r + 2r}$ or $(rh =)\dfrac{200-\pi r^2}{\pi + 2}$	dM1
	$V = \dfrac{1}{2}\pi r^2 h =$	M1
	$\Rightarrow V = \dfrac{\pi r^2(200-\pi r^2)}{2(2r+\pi r)} = \dfrac{\pi r(200-\pi r^2)}{4+2\pi}$ *	A1 cso * **[5]**
(b)	$\dfrac{\mathrm{d}V}{\mathrm{d}r} = \dfrac{200\pi - 3\pi^2 r^2}{4+2\pi}$ Accept awrt $\dfrac{\mathrm{d}V}{\mathrm{d}r} = 61.1 - 2.9r^2$	M1 A1
	$\dfrac{200\pi - 3\pi^2 r^2}{4+2\pi} = 0$ or $200\pi - 3\pi^2 r^2 = 0$ leading to $r^2 =$	dM1
	$r = \sqrt{\dfrac{200}{3\pi}}$ or answers which round to 4.6	dM1 A1
	$V = 188$	B1 **[6]**
(c)	$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = \dfrac{-6\pi^2 r}{4+2\pi}$, and sign considered Accept $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = \text{awrt} -5.8r$	M1
	$\left.\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}\right\rvert_{r=..} = -27 < 0$ and therefore maximum	A1 **[2]**
		13 marks

Question Number	Scheme	Marks
15(a)	Uses Volume = 60 000 $\quad 60000=\pi r^2 h \Rightarrow h=\dfrac{60000}{\pi r^2}$	M1
	Subs in $S=\pi r^2+2\pi rh \Rightarrow S=\pi r^2+2\pi r\times\dfrac{60000}{\pi r^2}$	M1
	$\Rightarrow S=\pi r^2+\dfrac{120000}{r}$	A1*
		(3)
(b)	$\dfrac{dS}{dr}=2\pi r-\dfrac{120000}{r^2}$	M1A1
	$\Rightarrow \dfrac{dS}{dr}=0\Rightarrow r^3=\dfrac{120000}{2\pi}\Rightarrow r=awrt\ 27\,(\text{cm})$	dM1A1
	$\Rightarrow S=\pi\times\text{"}26.7\text{"}^2+\dfrac{120000}{\text{"}26.7\text{"}}=awrt\ 6730\,(\text{cm}^2)$	dM1 A1
		(6)
(c)	$\dfrac{d^2S}{dr^2}=2\pi+\left.\dfrac{240000}{r^3}\right\rvert_{r=26.7}=awrt\,19>0\Rightarrow$ Minimum	M1A1
		(2) **(11 marks)**

(a)

M1 Uses $60000=\pi r^2 h\Rightarrow h=..$ Alternatively uses $60000=\pi r^2 h\Rightarrow \pi rh=..$
Condone errors on the number of zeros but the formula must be correct

M1 Score for the attempt to substitute any $h=..$ or $\pi rh=..$ from a dimensionally correct formula for V $\left(\text{Eg. } 60000=\frac{1}{3}\pi r^2 h\Rightarrow h=..\right)$ into $S=k\pi r^2+2\pi rh$ where $k=1$ or 2 to get S in terms of r
Allow if S is called something else such as A.

A1* Completes proof with no errors (or omissions) $S=\pi r^2+\dfrac{120000}{r}$.
Allow from $S=\pi r^2+\dfrac{2V}{r}$ if quoted. $S=$ must be somewhere in the proof