Oxford Cambridge and RSA

Monday 19 October 2020 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

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You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,dx = \ln|\mathrm{f}(x)| + c$

$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,dx = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Standard deviation

$\sqrt{\dfrac{\sum(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\dfrac{\sum fx^2}{\sum f} - \bar{x}^2}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $\mathrm{P}(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 Triangle ABC has $AB = 8.5$ cm, $BC = 6.2$ cm and angle $B = 35°$.

Calculate the area of the triangle. **[2]**

2 A sequence of transformations maps the curve $y = e^x$ to the curve $y = e^{2x+3}$.

Give details of these transformations. **[3]**

3 The functions f and g are defined for all real values of x by

$f(x) = 2x^2 + 6x$ and $g(x) = 3x + 2$.

(a) Find the range of f. **[3]**

(b) Give a reason why f has no inverse. **[1]**

(c) Given that $fg(-2) = g^{-1}(a)$, where a is a constant, determine the value of a. **[4]**

(d) Determine the set of values of x for which $f(x) > g(x)$. Give your answer in set notation. **[3]**

4 A curve has equation $y = 2\ln(k - 3x) + x^2 - 3x$, where k is a positive constant.

(a) Given that the curve has a point of inflection where $x = 1$, show that $k = 6$. **[5]**

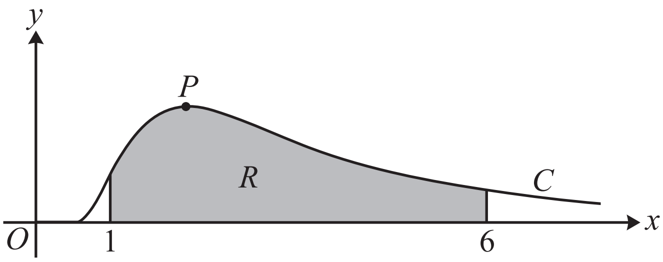
It is also given that the curve intersects the x-axis at exactly one point.

(b) Show by calculation that the x-coordinate of this point lies between 0.5 and 1.5. **[2]**

(c) Use the Newton-Raphson method, with initial value $x_0 = 1$, to find the x-coordinate of the point where the curve intersects the x-axis, giving your answer correct to 5 decimal places. Show the result of each iteration to 6 decimal places. **[3]**

(d) By choosing suitable bounds, verify that your answer to part **(c)** is correct to 5 decimal places. **[1]**

5



The diagram shows the curve C with parametric equations

$x = \frac{3}{t}$, $y = t^3 e^{-2t}$, where $t > 0$.

The maximum point on C is denoted by P.

(a) Determine the exact coordinates of P. **[4]**

The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 6$.

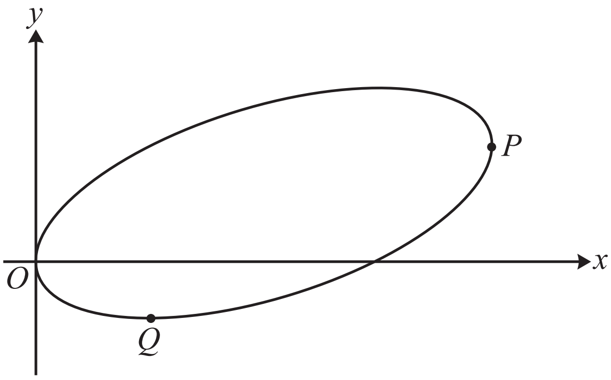
(b) Show that the area of R is given by

$$\int_a^b 3te^{-2t}\,dt,$$

where a and b are constants to be determined. **[3]**

(c) Hence determine the exact area of R. **[5]**

6 In this question you must show detailed reasoning.



The diagram shows the curve with equation $4xy = 2(x^2 + 4y^2) - 9x$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 4y - 9}{4x - 16y}$. **[3]**

At the point P on the curve the tangent to the curve is parallel to the y-axis and at the point Q on the curve the tangent to the curve is parallel to the x-axis.

(b) Show that the distance PQ is $k\sqrt{5}$, where k is a rational number to be determined. **[8]**

Section B: Mechanics

Answer **all** the questions.

7 A particle P moves with constant acceleration $(-4\mathbf{i}+2\mathbf{j})\,\mathrm{m\,s^{-2}}$. At time $t=0$ seconds, P is moving with velocity $(7\mathbf{i}+6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Determine the speed of P when $t=3$. **[4]**

(b) Determine the change in displacement of P between $t=0$ and $t=3$. **[2]**

8 A car is travelling on a straight horizontal road. The velocity of the car, $v\,\mathrm{m\,s^{-1}}$, at time t seconds as it travels past three points, P, Q and R, is modelled by the equation

$v = at^2 + bt + c,$

where a, b and c are constants.

The car passes P at time $t=0$ with velocity $8\,\mathrm{m\,s^{-1}}$.

(a) State the value of c. **[1]**

The car passes Q at time $t=5$ and at that instant its deceleration is $0.12\,\mathrm{m\,s^{-2}}$. The car passes R at time $t=18$ with velocity $2.96\,\mathrm{m\,s^{-1}}$.

(b) Determine the values of a and b. **[4]**

(c) Find, to the nearest metre, the distance between points P and R. **[2]**

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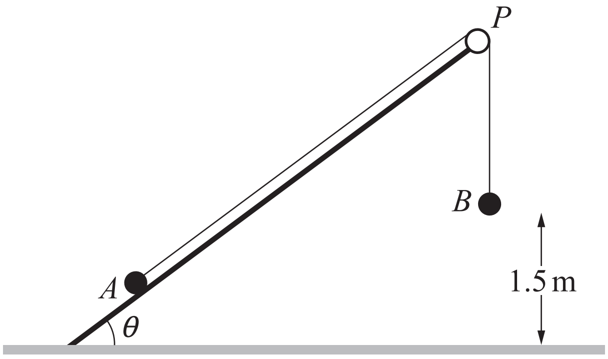

One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 2.5 kg. Particle A is in contact with a rough plane inclined at θ to the horizontal, where $\cos\theta = \frac{4}{5}$. The string is taut and passes over a small smooth pulley P at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane. Particle B hangs freely below P at a distance 1.5 m above horizontal ground, as shown in the diagram.

The coefficient of friction between A and the plane is μ. The system is released from rest and in the subsequent motion B hits the ground before A reaches P. The speed of B at the instant that it hits the ground is $1.2\,\mathrm{m\,s^{-1}}$.

(a) For the motion before B hits the ground, show that the acceleration of B is $0.48\,\mathrm{m\,s^{-2}}$. **[1]**

(b) For the motion before B hits the ground, show that the tension in the string is 23.3 N. **[3]**

(c) Determine the value of μ. **[5]**

After B hits the ground, A continues to travel up the plane before coming to instantaneous rest before it reaches P.

(d) Determine the distance that A travels from the instant that B hits the ground until A comes to instantaneous rest. **[4]**

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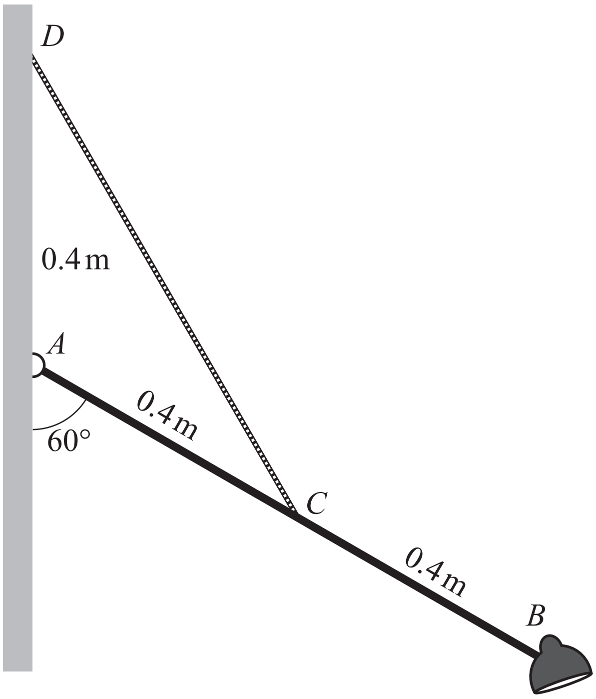

The diagram shows a wall-mounted light. It consists of a rod *AB* of mass 0.25 kg and length 0.8 m which is freely hinged to a vertical wall at *A*, and a lamp of mass 0.5 kg fixed at *B*. The system is held in equilibrium by a chain *CD* whose end *C* is attached to the midpoint of *AB*. The end *D* is fixed to the wall a distance 0.4 m vertically above *A*. The rod *AB* makes an angle of 60° with the downward vertical.

The chain is modelled as a light inextensible string, the rod is modelled as uniform and the lamp is modelled as a particle.

(a) By taking moments about *A*, determine the tension in the chain. **[4]**

(b) **(i)** Determine the magnitude of the force exerted on the rod at *A*. **[4]**

(ii) Calculate the direction of the force exerted on the rod at *A*. **[2]**

(c) Suggest one improvement that could be made to the model to make it more realistic. **[1]**

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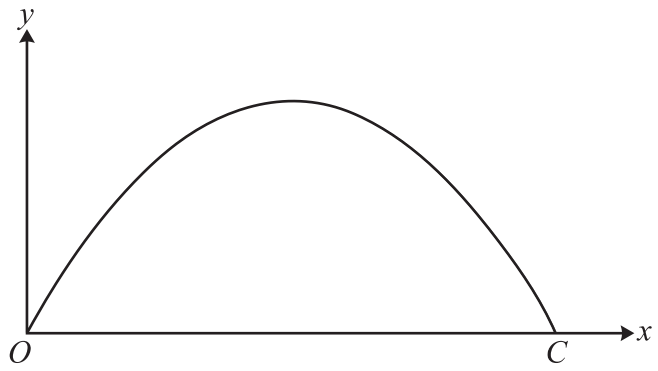
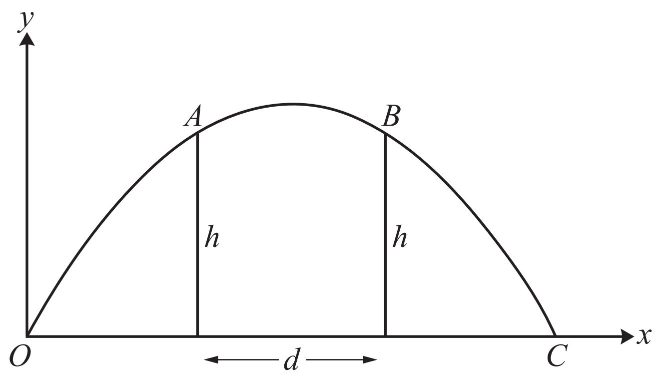


A particle P moves freely under gravity in the plane of a fixed horizontal axis Ox, which lies on horizontal ground, and a fixed vertical axis Oy. P is projected from O with a velocity whose components along Ox and Oy are U and V, respectively. P returns to the ground at a point C.

(a) Determine, in terms of U, V and g, the distance OC. **[4]**



P passes through two points A and B, each at a height h above the ground and a distance d apart, as shown in the diagram.

(b) Write down the horizontal and vertical components of the velocity of P at A. **[2]**

(c) Hence determine an expression for d in terms of U, V, g and h. **[3]**

(d) Given that the direction of motion of P as it passes through A is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{1}{2}$, determine an expression for V in terms of g, d and h. **[4]**

END OF QUESTION PAPER

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