Question	Scheme	Marks	AOs
11(a)	$h=0.5$	B1	1.1b
	$A\approx\frac{1}{2}\times\frac{1}{2}\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$	M1	1.1b
	$=2.41$	A1	1.1b
		(3)	
(b)	$\int(\ln x)^2\ \mathrm{d}x=x(\ln x)^2-\int x\times\frac{2\ln x}{x}\ \mathrm{d}x$	M1 A1	3.1a 1.1b
	$=x(\ln x)^2-2\int\ln x\ \mathrm{d}x=x(\ln x)^2-2\left(x\ln x-\int\mathrm{d}x\right)$ $=x(\ln x)^2-2\int\ln x\ \mathrm{d}x=x(\ln x)^2-2x\ln x+2x$	dM1	2.1
	$\int_2^4(\ln x)^2\ \mathrm{d}x=\left[x(\ln x)^2-2x\ln x+2x\right]_2^4$ $=4(\ln 4)^2-2\times4\ln4+2\times4-\left(2(\ln 2)^2-2\times2\ln2+2\times2\right)$ $=4(2\ln2)^2-16\ln2+8-2(\ln2)^2+4\ln2-4$	ddM1	2.1
	$=14(\ln2)^2-12\ln2+4$	A1	1.1b
		(5)	
		(8 marks)	

Notes

(a)

B1: Correct strip width. May be implied by $\frac{1}{2}\times\frac{1}{2}\{....\}$ or $\frac{1}{4}\times\{....\}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2}\times"h"\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2-\beta\int\ln x\ \mathrm{d}x$ o.e.

May be unsimplified (see scheme). Watch for candidates who know or learn $\int\ln x\ \mathrm{d}x=x\ln x-x$

who may write $\int(\ln x)^2\ \mathrm{d}x=\int(\ln x)(\ln x)\ \mathrm{d}x=\ln x(x\ln x-x)-\int\frac{x\ln x-x}{x}\ \mathrm{d}x$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2-\beta x\ln x\pm\gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm\alpha x(\ln x)^2\pm\beta x\ln x\pm\gamma x$, subtracts and applies $\ln 4=2\ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

....

It is possible to do $\int(\ln x)^2\ \mathrm{d}x$ via a substitution $u=\ln x$ but it is very similar.

M1 A1, dM1: $\int u^2\mathrm{e}^u\ \mathrm{d}u=u^2\mathrm{e}^u-\int 2u\mathrm{e}^u\ \mathrm{d}u,=u^2\mathrm{e}^u-2u\mathrm{e}^u\pm2\mathrm{e}^u$

ddM1: Applies appropriate limits and uses $\ln 4=2\ln 2$ at least once to an expression of the form $u^2\mathrm{e}^u-\beta u\mathrm{e}^u\pm\gamma\mathrm{e}^u$ Both M's must have been awarded

....

Question	Scheme	Marks	AOs
2	Time (s): 0, 5, 10, 15, 20, 25 Speed (m s^{-1}): 2, 5, 10, 18, 28, 42		
(a)	Uses an allowable method to estimate the area under the curve. E.g.	M1	3.1a
	Way 1: an attempt at the trapezium rule (see below)		
	Way 2: $\{s=\}\left(\frac{2+42}{2}\right)(25)\ \{=550\}$		
	Way 3: $42=2+25(a)\Rightarrow a=1.6\Rightarrow s=2(25)+(0.5)(1.6)(25)^2\{=550\}$		
	Way 4: $\{d=\}(2)(5)+5(5)+10(5)+18(5)+28(5)\ \{=63(5)=315\}$		
	Way 5: $\{d=\}\ 5(5)+10(5)+18(5)+28(5)+42(5)\ \{=103(5)=515\}$		
	Way 6: $\{d=\}\ \frac{315+515}{2}\ \{=415\}$		
	Way 7: $\{d=\}\left(\frac{2+5+10+18+28+42}{6}\right)(25)\ \{=437.5\}$		
	$\frac{1}{2}\times(5)\times[2+2(5+10+18+28)+42]$ or $\frac{1}{2}\times["315"+"515"]$	M1	1.1b
	$=415$ {m}	A1	1.1b
		(3)	
(b) Alt 1	**Uses a Way 1, Way 2, Way 3, Way 5, Way 6 or Way 7 method in (a)**		
	Overestimate **and** a relevant explanation e.g. • {top of} trapezia lie above the curve • Area of trapezia > area under curve • An appropriate diagram which gives reference to the extra area • Curve is convex • $\frac{d^2y}{dx^2}>0$ • Acceleration is {continually} increasing • The gradient of the curve is {continually} increasing • All the rectangles are above the curve (Way 5)	B1ft	2.4
		(1)	
(b) Alt 2	**Uses a Way 4 method in (a)**		
	Underestimate **and** a relevant explanation e.g. • All the rectangles are below the curve	B1ft	2.4
		(1)	
		(4 marks)	

Notes for Question 2

(a)	
M1:	A low-level problem-solving mark for using an allowable method to estimate the area under the curve. E.g.
	Way 1: See scheme. Allow $\lambda(2+2(5+10+18+28)+42);\ \lambda>0$ for 1st M1
	Way 2: Uses $s=\left(\frac{u+v}{2}\right)t$ which is equivalent to finding the area of a large trapezium
	Way 3: Complete method using a uniform acceleration equation.
	Way 4: Sums rectangles lying below the curve. Condone a slip on one of the speeds.
	Way 5: Sums rectangles lying above the curve. Condone a slip on one of the speeds.
	Way 6: Average the result of Way 3 and Way 4. Equivalent to Way 1.
	Way 7: Applies (average speed) $\times$ (time)

Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1a
	$A \approx \frac{0.5}{2}\{0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)\}$	M1	1.1b
	= awrt 1.50	A1	1.1b
	For reference: **The integration on a calculator gives 1.511549071** **The full accuracy for *y* values gives 1.504726147** **The accuracy from the table gives 1.50475**		
		(3)	

(b)	3 × their (a)
	If (a) is correct, allow awrt 4.50 or awrt 4.51 even with no working.
	Only allow 4.5 if (a) is correct and working is shown e.g. 3 × 1.5
	
	If (a) is incorrect allow 3 × their (a) given to at least 3sf but do not be too concerned about the accuracy (as they may use rounded or rounded value from (a))
		B1ft	2.2a

	For reference the integration on a calculator gives 4.534647213		
		(1)	
(c)	**<u>This mark depends on the B1 having been awarded in part (b) with awrt 4.5</u>** Look for a sensible comment. Some examples: • The answer is accurate to 2 sf or one decimal place • Answer to (b) is accurate as 4.535 ≈ 4.50 • Very accurate as 4.535 to 2 sf is 4.5 • 4.51425 < 4.535 so my answer is underestimate but not too far off • It is an underestimate but quite close • It is a very good estimate • High accuracy • (Quite) accurate • It is less than 1% out • 4.535 – 4.5 = 0.035 so not far out But **not** just "it is an underestimate" or Calculates percentage error correctly using awrt 4.50 or awrt 4.51 or 4.5 (No comment is necessary in these cases although one may be given) Examples: $\left\lvert\frac{4.535-4.50}{4.535}\right\rvert\times 100 = 0.77\%$ or $\left\lvert\frac{4.535-4.51}{4.535}\right\rvert\times 100 = 0.55\%$ or $\left\lvert\frac{4.535-4.51425}{4.535}\right\rvert\times 100 = 0.46\%$ or $\left\lvert\frac{4.50}{4.535}\right\rvert\times 100 = 99\%$ In these cases don't be too concerned about accuracy e.g. allow 1sf. **This mark should be withheld if there are any contradictory statements**	B1	3.2b
		(1)	
		(5 marks)	

<u>Notes:</u>

(a)

Question Number	Scheme	Marks
9. (a)	x: 4, 5, 6, 7, 8, 9; y: e^2, $e^{\sqrt{5}}$, $e^{\sqrt{6}}$, $e^{\sqrt{7}}$, $e^{\sqrt{8}}$, e^3 (values: 7.389056..., 9.356469..., 11.582435..., 14.094030..., 16.918828..., 20.085536...)	M1
	$\frac{1}{2}\times 1\times\{...........................\}$	B1 oe
	$\frac{1}{2}\times 1\times\left\{e^2+e^3+2\left(e^{\sqrt{5}}+e^{\sqrt{6}}+e^{\sqrt{7}}+e^{\sqrt{8}}\right)\right\}$ $\quad\{=\tfrac{1}{2}(27.47459302...+103.903526...)\}$	M1
	$= 65.6890595... = 65.69$ (2 dp)	A1
	Special case (s.c.) Uses h = 5/4 with 5 ordinates giving answer 65.76 – award M0B0M1A1(s.c.) *See note below*	**[4]**
(b)	$\left\{u=\sqrt{x}\Rightarrow\right\}\ \frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $\frac{dx}{du}=2u$	B1
	$\left\{\int e^{\sqrt{x}}\,dx\right\} = \int e^u\, 2u\, du$	M1 A1
	$= \{2\}\left(ue^u - \int e^u\, du\right)$	M1
	$= \{2\}\left(ue^u - e^u\right)$	A1
	$\left[2\left(ue^u - e^u\right)\right]_2^3 = 2\left(3e^3-e^3\right)-2\left(2e^2-e^2\right)$	ddM1
	$4e^3-2e^2$ or $2e^2(2e-1)$ etc.	A1
		[7] **11**

The first row of the scheme is printed as this table:

x	4	5	6	7	8	9
y	e^2	$e^{\sqrt{5}}$	$e^{\sqrt{6}}$	$e^{\sqrt{7}}$	$e^{\sqrt{8}}$	e^3
	7.389056...	9.356469...	11.582435...	14.094030...	16.918828...	20.085536...

Question Number	Scheme	Marks
12(a)	0.9242 exactly	B1 **(1)**
(b)	Strip width =0.5	B1
	Area $\approx \frac{0.5}{2}\left((2+1.2958+2\times(1.3041+'0.9242'+0.9089)\right)$	M1
	=2.393	A1 **(3)**
(c)	$\int \frac{x^2 \ln x}{3} - 2x + 4 \, dx$	
	$= \frac{x^3}{9}\ln x - \int \frac{x^3}{9} \times \frac{1}{x} dx, \quad -x^2 + 4x$	M1A1, B1
	$= \frac{x^3}{9}\ln x - \frac{x^3}{27} \quad \left(-x^2 + 4x\right)$	A1
	Area $= \left[\frac{x^3}{9}\ln x - \frac{x^3}{27} - x^2 + 4x\right]_1^3 = (3\ln 3 - 1 - 9 + 12) - \left(-\frac{1}{27} - 1 + 4\right)$	dM1
	$= \ln 27 - \frac{26}{27}$	A1 **(6)**
(d)	% error = $\pm \frac{\lvert real - approx \rvert}{real} \times 100$ =Accept awrt $\pm 2.6\%$	M1A1 **(2)**
(e)	Increase the number of 'strips'	B1 **(1)** **(13 marks)**

(a)
B1 0.9242 exactly either in the table or within the trapezium rule in part (b)

(b)
B1 Uses a strip width of 0.5 or equivalent.
M1 Uses the correct form of the trapezium rule, a form of which appears in the formula booklet.

Look for $\frac{\dots}{2}\left((2+1.2958+2\times(1.3041+\text{their } 0.9242+0.9089)\right)$

Accept for this the sum of four trapezia

A1 Awrt 2.393 (3dp)

Question Number	Scheme	Marks
13 (a)	awrt 0.3799 – may be seen in the table	B1 **(1)**
(b)	Area = $\frac{1}{2}\times\left(\frac{e^2-e}{2}\right)(1+2\times'0.3799'+0)$ [The +0 is not required]	B1M1
	$= \text{awrt } 2.055$	A1 **(3)**
(c) Way 1	$\int(\ln x)^2\,dx=\int 1\times(\ln x)^2\,dx=x(\ln x)^2-\int x\times\frac{2\ln x}{x}\,dx$	M1
	$=x(\ln x)^2-\int 2\ln x\,dx$	A1
	$=x(\ln x)^2-2x\ln x+\int 2\,dx$	dM1
	$=x(\ln x)^2-2x\ln x+2x(+c)$	A1* (4)
(c) Way 2	$\int(\ln x)^2\,dx=\int(\ln x)\times(\ln x)\,dx=\ln x(x\ln x-x)-\int\frac{1}{x}\times(x\ln x-x)\,dx$	M1
	$=\ln x(x\ln x-x)-\int \ln x-1\,dx$	A1
	$=\ln x(x\ln x-x)-(x\ln x-x-x)$	dM1
	$=x(\ln x)^2-2x\ln x+2x(+c)$	A1* (4)
(c) Way3	Use $u=\ln x$ substitution to get to	
	$\int u^2e^u\,du=u^2e^u-\int 2ue^u\,du$	M1
	$=u^2e^u-\left[2ue^u-\int 2e^u\,du\right]$	A1
	$=u^2e^u-2ue^u+2e^u+k$	
	$=x(\ln x)^2-2x\ln x+2x(+c)$	dM1A1 (4)
(d)	Volume = $\int\pi y^2\,dx=\int\pi(2-\ln x)^2\,dx$	B1
	$\int(2-\ln x)^2\,dx=\int 4-4\ln x+(\ln x)^2\,dx$	M1
	Correct integration of at least two of their three terms (see notes)	M1
	$=4x-4(x\ln x-x)+x(\ln x)^2-2x\ln x+2x\ (+c)$	A1
	Volume $=\pi\left[4x-4(x\ln x-x)+x(\ln x)^2-2x\ln x+2x\right]_e^{e^2}$	ddM1
	$=2\pi e^2-5\pi e$	
	$=\pi e(2e-5)$	A1
		(6) **(14 marks)**

Question Number	Scheme	Marks
7.(a)	$\dfrac{dy}{dx}=\dfrac{(x^2+1)\times\dfrac{6x}{x^2+1}-6x\times\ln(x^2+1)}{(x^2+1)^2}$ Or $\dfrac{6x\left(1-\ln(x^2+1)\right)}{(x^2+1)^2}$	M1A1 [2]
(b)	$\dfrac{dy}{dx}=0 \Rightarrow \cancel{(x^2+1)}3\dfrac{2x}{\cancel{(x^2+1)}}-3\ln(x^2+1)(2x)=0$	M1
	$\ln(x^2+1)=1$ *so* $x=\sqrt{e-1}$	M1A1
	$y=\dfrac{3}{e}$	ddM1A1 [5]
(c)	$\dfrac{3}{2}\ln 2$ or 1.0397	B1 [1]
(d)	$\dfrac{1}{2}\times 1\times\underline{\{..........................\}}$	B1 oe
	$\dfrac{1}{2}\times 1\times\underline{\left\{0+\dfrac{3}{10}\ln 10+2\left(\dfrac{3}{2}\ln 2+\dfrac{3}{5}\ln 5\right)\right\}}$	M1
	$\left\{=\tfrac{1}{2}(0.6907755279..+4.010767..)\right\}$	
	$=2.351$ (awrt 4 sf)	A1 [3]
		(11 marks)

(a)

M1 Applies the Quotient rule, a form of which appears in the formula book, to $\dfrac{3\ln(x^2+1)}{(x^2+1)}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln\left(x^2+1\right)$, $v=x^2+1$, $u'=..$, $v'=...$ followed by their $\dfrac{vu'-uv'}{v^2}$, then only accept answers of the form

$\dfrac{dy}{dx}=\dfrac{(x^2+1)A\dfrac{x}{x^2+1}-Bx\ln(x^2+1)}{(x^2+1)^2}$ *or* $\dfrac{Ax-Bx\ln(x^2+1)}{(x^2+1)^2}$. Condone invisible brackets for the M.

Alternatively applies the product rule with $u=3\ln(x^2+1)$, $v=\left(x^2+1\right)^{-1}$

If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms. If the rule is not quoted nor implied by their working, meaning that terms are written out $u=3\ln(x^2+1)$, $v=\left(x^2+1\right)^{-1}$, $u'=..$, $v'=...$ followed by their $vu'+uv'$, then only accept answers of the form

$\left(x^2+1\right)^{-1}\times A\dfrac{x}{x^2+1}+\left(x^2+1\right)^{-2}\times Bx\ln(x^2+1)$.

Condone invisible brackets for the M.

A1 Any fully correct (unsimplified) form of $f'(x)$. Remember to isw.

Question Number	Scheme	Notes	Marks
7 (a)	Strip width = 1	May be implied by their trapezium rule.	B1
	Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{15}}+2\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)\right)$ $\approx \frac{1}{2}(0.33...+0.25...+2(0.30...+0.27...))$	M1: Correct structure for the y values. Look for (y at $x = 2$) + (y at $x = 5$) + 2(sum of other y values). A1: Correct numerical expression. If decimals are used, look for awrt 1dp initially, however a correct final answer would imply this mark.	M1 A1
	Awrt 0.875		A1
			(4)
	May use separate trapezia:		
	Area $\approx \frac{1}{2}\left(\frac{1}{\sqrt{9}}+\frac{1}{\sqrt{11}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{13}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{11}}+\frac{1}{\sqrt{15}}\right)$		
	B1: Strip width = 1 M1: Correct structure for the y values as above A1: Correct expression as described above A1: Awrt 0.875		
(b)	$\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x = (2x+5)^{\frac{1}{2}}$	M1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x = k(2x+5)^{\frac{1}{2}}$ A1: $\int \frac{1}{\sqrt{2x+5}}\mathrm{d}x = (2x+5)^{\frac{1}{2}}$	M1A1
	$\int_2^5 \frac{1}{\sqrt{2x+5}}\mathrm{d}x = (2(5)+5)^{\frac{1}{2}} - (2(2)+5)^{\frac{1}{2}}$	Substitutes 5 and 2 and subtracts the right way round. May be implied by the correct exact answer but not by a decimal answer **only** e.g. 0.8729… and not by work in decimals e.g. 3.872… −3 unless the substitution of 5 and 2 is explicitly seen.	dM1
	$= \sqrt{15} - \sqrt{9}\ (= \sqrt{15} - 3)$	$\sqrt{15} - \sqrt{9}$ or $\sqrt{15} - 3$	A1
			(4)

Qu	Scheme	Marks
5.(a)	$\frac{7\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{7\pi\sqrt{2}}{8}$ AND $\frac{9\pi}{4\sqrt{2}}$ or equivalent e.g. $\frac{9\pi\sqrt{2}}{8}$	B1 **(1)**
(b)	$\underline{\frac{1}{2}\times\frac{\pi}{4}\times\{..........................\}}$	B1 oe
	$\frac{1}{2}\times h\times\underline{\left\{0+0+2\left("\frac{7\pi}{4\sqrt{2}}"+2\pi+"\frac{9\pi}{4\sqrt{2}}"\right)\right\}}$	M1
	$= 11.91$ (only)	A1 **(3)**
(c)	$\int x\cos x\,dx \quad = [x\sin x]-\int\sin x\,dx$	M1
	$= x\sin x+\cos x\ (+c)$	dM1 A1 **(3)**
(d)	$[x\sin x+\cos x]_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \quad = \frac{5\pi}{2}+\frac{3\pi}{2}=4\pi$	M1 A1 **(2)**
		(9 marks)

(a)

B1: Both correct (as above) Must be exact and not decimal

(b)

B1: See $\frac{1}{2}\times\frac{\pi}{4}\times$ as part of trapezium rule or $h=\frac{\pi}{4}$ stated or used. This can be scored if '*h*' is in an unsimplified form.

M1: Correct structure of the bracket in the trapezium rule.

You may not see the zero's Eg $2\left("\frac{7\pi}{4\sqrt{2}}"+2\pi+"\frac{9\pi}{4\sqrt{2}}"\right)$

A1: 11.91 only. This may be a result of using the decimal equivalents. Sight of 11.91 will score all 3 marks

(c)

M1: For a correct attempt at integration by parts to give an expression of the form $[\pm x\sin x]-\int\pm\sin x\,dx$

If you see such an expression you would only withhold the mark if there is evidence of an incorrect formula (seen or implied) Eg $\int udv = uv+\int vdu$

dM1: For $\pm\ x\sin x\ \pm\ \cos x$ following line one

A1: cso

Allow all three marks for candidate who just writes down the correct answer with no working

Watch for candidates who write down methods like this.

D	I
x	$\cos x$
1	$\sin x$
0	$-\cos x$

and then write $x\sin x-(-\cos x)$

This is a commonly taught algorithm (They differentiate down the lh column and integrate on the rh column. The answer is found by $D1\times I2-D2\times I3$ where $D2$ is the second entry in the D column. This can score full marks for the answer $x\sin x+\cos x$ but also pick up methods for slips.

If they attempt $D1\times I2+D2\times I3$ it is M0 as they are implying an incorrect formula. Ask your TL if unclear.

(d)

M1: Using limits $\frac{5\pi}{2}$ and $\frac{3\pi}{2}$ correctly in their answer to part (c) - substituting (seen correctly in all terms or implied in all terms) and subtracting either way around

A1: 4π or equivalent single term. CSO. It must have been derived from $x\sin x+\cos x$

Question Number	Scheme	Marks
8 (a)	Strip Width =1	B1
	Area $\approx \frac{1}{2}(0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027))$ $\left(=\frac{1}{2}\times 4.7547\right)$	M1
	Awrt =2.377	A1
		(3)
(b)	$\text{Volume}=(\pi)\int_2^7 \frac{x}{x^2+1}\mathrm{d}x=(\pi)\left[\frac{1}{2}\ln\left(x^2+1\right)\right]_2^7$	M1A1
	$=\frac{(\pi)}{2}(\ln 50-\ln 5)$	dM1
	$=\frac{\pi}{2}\ln 10$	A1
		(4)
		(7 marks)

(a)

B1: Strip width = 1 which may be implied by the $\frac{1}{2}$ in the trapezium rule

M1: For a correct attempt at using the trapezium rule.

Look for $\frac{1}{2}h((y \text{ at } x=2)+(y \text{ at } x=7)+2(\text{sum of other } y \text{ values}))$. Must be correct with no missing values and no extra values. (May be implied by a correct answer)

A1: Awrt = 2.377

Note: $h = 5/6$ gives Area 1.981125 and $h = 5$ gives 11.88675 and will probably just score the M1

Note that $\frac{1}{2}\times 1\times 0.6325+0.3742+2\times(0.5477+0.4851+0.4385+0.4027)$ scores B1 only unless the missing brackets are implied by a correct answer.

(b)

M1: Attempts to find $C\int \frac{x}{x^2+1}\mathrm{d}x$ to give an expression of the form $D\left[\ln k\left(x^2+1\right)\right]$

A1: $\text{Volume}=(\pi)\int \frac{x}{x^2+1}\mathrm{d}x=\frac{(\pi)}{2}\left[\ln\left(x^2+1\right)\right]$. Correct expression **with or without** π. **Ignore any limits.**

Do not allow the brackets around the $x^2 + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting $x = 7$ and $x = 2$ and subtracting either way round. Following correct work, this mark may be implied by awrt 3.62.

A1: $V=\frac{\pi}{2}\ln 10$ or exact equivalent e.g. $\pi\ln\sqrt{10}$ and isw once the correct answer is seen. Allow $V=\frac{\pi}{2}\ln|10|$

By substitution 1:

M1: Uses $u=x^2$ Attempts to find $C\int \frac{x}{x^2+1}\mathrm{d}x$ to give an expression of the form $D[\ln k(u+1)]$

A1: $\text{Volume}=(\pi)\int \frac{x}{u+1}\frac{1}{2x}\mathrm{d}u=\frac{(\pi)}{2}[\ln(u+1)]$. Correct expression **with or without** π. **Ignore any limits.**

Do not allow the brackets around the $u + 1$ to be missing unless their presence is implied by later work.

dM1: Dependent upon previous M. It is for substituting $x = 7^2$ and $x = 2^2$ and subtracting either way round or changing back to x and substituting $x = 7$ and $x = 2$ and subtracting either way round.

Question Number	Scheme	Marks
6 (a)	0.41576	B1
		(1)
(b)	Strip width $=\dfrac{\pi}{4}$	B1
	Area $\approx \dfrac{1}{2}\times\dfrac{\pi}{4}\times\left\{0+2\left(0.76679+0.41576+0.15940\right)+0\right\}$ Or separate trapezia: $\dfrac{1}{2}\times\dfrac{\pi}{4}\times\{0+0.766792\}+\dfrac{1}{2}\times\dfrac{\pi}{4}\times\{0.766792+0.41576\}+$ $\dfrac{1}{2}\times\dfrac{\pi}{4}\times\{0.41576+0.15940\}+\dfrac{1}{2}\times\dfrac{\pi}{4}\times\{0.15940+0\}$	M1
	1.0540	A1
		(3)
(c)	Uses $vu'+uv'$: $\dfrac{dy}{dx}=2e^{-x}\times\dfrac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2e^{-x}(\sin x)^{\frac{1}{2}}$ **or** Uses $\dfrac{vu'-uv'}{v^2}$: $\dfrac{dy}{dx}=\dfrac{e^{x}\times 2\times\frac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2e^{x}(\sin x)^{\frac{1}{2}}}{e^{2x}}$	M1A1A1
		(3)
(d)	$\dfrac{dy}{dx}=0\Rightarrow 2e^{-x}\times\dfrac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2e^{-x}(\sin x)^{\frac{1}{2}}=0$	
	$\cos x=2\sin x$	M1
	$\tan x=\dfrac{1}{2}\Rightarrow x=0.464$	dM1A1
		(3)
		(10 marks)

(a)

B1: awrt 0.41576

(Note that degrees gives 0.068835….and scores B0)

(b)

B1: Strip width $=\dfrac{\pi}{4}$ or awrt 0.785. This may be implied by seeing $\dfrac{1}{2}\times\dfrac{\pi}{4}\times\{...\}$ or $\dfrac{\pi}{8}\times\{...\}$ within the trapezium formula

M1: Correct structure for the trapezium formula. Do not condone missing brackets unless they are implied by subsequent work. (Allow the 0's to be omitted in the brackets)

A1: awrt 1.0540 (Not 1.054) (note that this mark is still available even if (a) is not given to the required accuracy)

(Note that degrees gives 0.78149…)

(c)

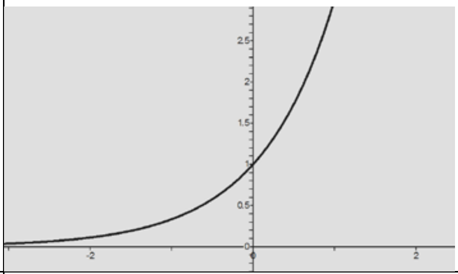
M1: Uses *vu*'+*uv*' with $u/v=2e^{-x}, u/v=(\sin x)^{0.5}$ If the rule is quoted it must be correct.

It may be implied by, for example, $u=2e^{-x}, v=\sqrt{\sin x}$ followed by their $u'=..,v'=..$and $vu'+uv'$

If it is not quoted nor implied then look for an expression of the form f(x) ± g(x) where f(x) or g(x) is of the form $Ae^{-x}\sqrt{\sin x}$ or $Ae^{-x}(\sin x)^{-0.5}\cos x$ with A non-zero.

A1: Either term of the derivative correct

A1: Completely correct derivative $\dfrac{dy}{dx}=2e^{-x}\times\dfrac{1}{2}(\sin x)^{-\frac{1}{2}}(\cos x)-2e^{-x}(\sin x)^{\frac{1}{2}}$. Allow un-simplified and allow …+–… for … – …

Question	Scheme		Marks
9 (a)		Shape **or** y intercept at 1	B1
		Fully correct shape and intercept	B1
			[2]
(b)	State $h = 2$, or use of $\frac{1}{2}\times 2$;		B1
	$\underline{\{0.0625+16+2(\ 0.25+1+4\ )\}}$	For structure of $\{\ldots\ldots\ldots\ldots\ldots\}$;	M1A1
	$\frac{1}{2}\times 2\times\underline{\{26.5625\}} =$ *awrt* 26.56	Exact answer $= 425/16$	A1cao
			[4]
(c)(i)	$4\times(b) =$ *awrt* 106	Exact answer $= 425/4$	M1A1ft
(ii)	$24+(b) =$ *awrt* 50.6	Exact answer $= 809/16$	M1A1ft
			[4] **(10 marks)**

(a)

B1 Score for either

- a correct shape for the curve. It must lie only in quadrants 1 and 2 and have a positive and increasing gradient from left to right. The gradient must be approximately 0 at the left hand end. Condone the curve appearing to be a straight line on the rhs. See Practice/Qualification items for clarification. Do not be concerned if it does not appear to be asymptotic to the x-axis at the LHS
- intercept at (0,1). Allow 1 being marked on the y - axis. Condone (1,0) on the correct axis.

B1 Fully correct. As a guide the gradient of the curve must appear to be 0 at the lh end and it must reach a level that is more than half way below the level of the intercept at (0,1). Allow $x = 0, y = 1$ in the text, it does not need to be on the sketch. Do not condone (1,0) even on the correct axis for this mark.

(b)

B1 For using a strip width of 2. This may appear in a trapezium rule as $\frac{1}{2}\times 2$ or 1 or equivalent

M1 Scored for the correct $\{\ldots\ldots\}$ outer bracket structure. It needs to contain first y value plus last y value and the inner bracket to be multiplied by 2 and to be the summation of the remaining y values in the table with no additional values. If the only mistake is a copying error or is to omit one value from inner bracket this may be regarded as a slip and the M mark can be allowed (An extra repeated term forfeits the M mark however). M0 if values used in brackets are x values instead of y values

A1 For the correct bracket $\{\ldots\ldots\}$

A1 For awrt 26.56. Accept 425/16

NB: Separate trapezia may be used: B1 for h =1, M1 for 1/2 $h(a + b)$ used 3 or 4 times (and A1 if it is all correct) Then A1 as before.

Note: As h =1 the expression $1\times(16 + 0.0625) + 2(0.25 + 1 + 4)$ will scores B1 M1 A1 with awrt 26.56 scoring the final A1.

(c)(i)

M1 For an attempt at finding $4\times(b)$. Also allow repeating the trapezium rule with each value $\times 4$

A1ft For either awrt 106 or ft on the answer to $4\times(b)$ You may see $425/4$ following 425/16 in (b)

(c)(ii)

M1 For an attempt at $24+(b)$ or $[3x]_{-4}^{4}+(b)$ Also allow repeating the trapezium rule with each value $+3$

A1ft For either awrt 50.6 or ft on the answer to 24+(b) You may see 809/16