

Mark Scheme Summer 2009

GCE

GCE Mathematics (8371/8374; 9371/9374)

Edexcel is one of the leading examining and awarding bodies in the UK and throughout the world. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers.

Through a network of UK and overseas offices, Edexcel's centres receive the support they need to help them deliver their education and training programmes to learners.

For further information, please call our GCE line on 0844 576 0025, our GCSE team on 0844 576 0027, or visit our website at www.edexcel.com.

If you have any subject specific questions about the content of this Mark Scheme that require the help of a subject specialist, you may find our **Ask The Expert** email service helpful.

Ask The Expert can be accessed online at the following link:

<http://www.edexcel.com/Aboutus/contact-us/>

Summer 2009

Publications Code UA021531

All the material in this publication is copyright

© Edexcel Ltd 2009

Contents

1.	6663 Core Mathematics 1	5
2.	6664 Core Mathematics 2	17
3.	6665 Core Mathematics 3	27
4.	6666 Core Mathematics 4	39
5.	6667 Further Pure Mathematics 1 (new)	47
6.	6668 Further Pure Mathematics 2 (new)	55
7.	6669 Further Pure Mathematics 3 (new)	65
8.	6674 Further Pure Mathematics 1 (legacy)	73
9.	6675 Further Pure Mathematics 2 (legacy)	81
10.	6676 Further Pure Mathematics 3 (legacy)	87
11.	6677 Mechanics 1	93
12.	6678 Mechanics 2	97
13.	6679 Mechanics 3	103
14.	6680 Mechanics 4	109
15.	6681 Mechanics 5	115
16.	6683 Statistics 1	121
17.	6684 Statistics 2	129
18.	6691 Statistics 3	137
19.	6686 Statistics 4	145
20.	6689 Decision Mathematics 1	151
21.	6690 Decision Mathematics 2	161

June 2009
6663 Core Mathematics C1
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$(3\sqrt{7})^2 = 63$	B1 (1)
(b)	$(8 + \sqrt{5})(2 - \sqrt{5}) = 16 - 5 + 2\sqrt{5} - 8\sqrt{5}$ $= 11, -6\sqrt{5}$	M1 A1, A1 (3) [4]
(a)	B1 for 63 only	
(b)	M1 for an attempt to expand <u>their</u> brackets with ≥ 3 terms correct. They may collect the $\sqrt{5}$ terms to get $16 - 5 - 6\sqrt{5}$ Allow $-\sqrt{5} \times \sqrt{5}$ or $-(\sqrt{5})^2$ or $-\sqrt{25}$ instead of the -5 These 4 values may appear in a list or table but they should have minus signs included The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule 1 st A1 for 11 from $16 - 5$ <u>or</u> $-6\sqrt{5}$ from $-8\sqrt{5} + 2\sqrt{5}$ 2 nd A1 for <u>both</u> 11 and $-6\sqrt{5}$. <u>S.C - Double sign error in expansion</u> For $16 - 5 - 2\sqrt{5} + 8\sqrt{5}$ leading to $11 + \dots$ allow <u>one</u> mark	

Question Number	Scheme	Marks
Q2	$32 = 2^5 \text{ or } 2048 = 2^{11}, \quad \sqrt{2} = 2^{1/2} \text{ or } \sqrt{2048} = (2048)^{1/2}$ $a = \frac{11}{2} \quad \left(\text{or } 5\frac{1}{2} \text{ or } 5.5 \right)$	B1, B1 B1 [3]
	<p>1st B1 for $32 = 2^5$ or $2048 = 2^{11}$ This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32 \times 2 = 2^5 \times 2 (= 2^6)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT</p> <p>2nd B1 for $2^{1/2}$ or $(2048)^{1/2}$ seen. This mark may be implied</p> <p>3rd B1 for answer as written. Need $a = \dots$ so $2^{11/2}$ is B0</p> <p>$a = \frac{11}{2} \left(\text{or } 5\frac{1}{2} \text{ or } 5.5 \right)$ with no working scores full marks.</p> <p>If $a = 5.5$ seen then award 3/3 unless it is clear that the value follows from totally incorrect work.</p> <p>Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1.</p> <p><u>Special case:</u> If $\sqrt{2} = 2^{1/2}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$, e.g. $a = 2\frac{1}{2}$, $a = 4\frac{1}{2}$, the second B1 is given by implication.</p>	


Question Number	Scheme	Marks
Q3 (a)	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3)
(b)	$\frac{2x^4}{4} + \frac{3x^{-1}}{-1} (+C)$	M1 A1
	$\frac{x^4}{2} - 3x^{-1} + C$	A1
		(3) [6]
(a)	<p>M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$</p> <p>1st A1 for $6x^2$</p> <p>2nd A1 for $-6x^{-3}$ or $-\frac{6}{x^3}$ Condone $+ -6x^{-3}$ here. Inclusion of $+c$ scores A0 here.</p>	
(b)	<p>M1 for some attempt to integrate an x term of the given y. $x^n \rightarrow x^{n+1}$</p> <p>1st A1 for both x terms correct but unsimplified- as printed or better. Ignore $+c$ here</p> <p>2nd A1 for both x terms correct and simplified and $+c$. Accept $-\frac{3}{x}$ but <u>NOT</u> $+ -3x^{-1}$</p> <p>Condone the $+c$ appearing on the first (unsimplified) line but missing on the final (simplified) line</p> <p>Apply ISW if a correct answer is seen</p> <p>If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).</p>	

Question Number	Scheme	Marks
Q6	$b^2 - 4ac$ attempted, in terms of p . $(3p)^2 - 4p = 0$ o.e. Attempt to solve for p e.g. $p(9p - 4) = 0$ Must potentially lead to $p = k, k \neq 0$ $p = \frac{4}{9}$ (Ignore $p = 0$, if seen)	M1 A1 M1 A1cso [4]
	<p>1st M1 for an attempt to substitute into $b^2 - 4ac$ or $b^2 = 4ac$ with b or c correct Condone x's in one term only. This can be inside a square root as part of the quadratic formula for example. Use of inequalities can score the M marks only</p> <p>1st A1 for any correct equation: $(3p)^2 - 4 \times 1 \times p = 0$ or better</p> <p>2nd M1 for an attempt to factorize or solve their quadratic expression in p. Method must be sufficient to lead to their $p = \frac{4}{9}$.</p> <p>Accept factors or use of quadratic formula or $(p \pm \frac{2}{9})^2 = k^2$ (o.e. eg) $(3p \pm \frac{2}{3})^2 = k^2$ or equivalent work on <u>their</u> eqn.</p> $9p^2 = 4p \Rightarrow \frac{9p^{\cancel{2}}}{\cancel{9}} = 4 \text{ which would lead to } 9p = 4 \text{ is OK for this 2}^{\text{nd}} \text{ M1}$ <p>ALT <u>Comparing coefficients</u></p> <p>M1 for $(x + \alpha)^2 = x^2 + \alpha^2 + 2\alpha x$ and A1 for a correct equation eg $3p = 2\sqrt{p}$</p> <p>M1 for forming solving leading to $\sqrt{p} = \frac{2}{3}$ or better</p> <p><u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u> If the formula is quoted accept some correct substitution leading to a partially correct expression. If the formula is not quoted only award for a fully correct expression using their values.</p>	

Question Number	Scheme	Marks
Q7	<p>(a) $(a_2 =)2k - 7$</p> <p>(b) $(a_3 =)2(2k - 7) - 7$ or $4k - 14 - 7, = 4k - 21$ (*)</p> <p>(c) $(a_4 =)2(4k - 21) - 7$ ($= 8k - 49$)</p> $\sum_{r=1}^4 a_r = k + "(2k - 7)" + (4k - 21) + "(8k - 49)"$ $k + (2k - 7) + (4k - 21) + (8k - 49) = 15k - 77 = 43 \quad k = 8$	<p>B1 (1)</p> <p>M1, A1cso (2)</p> <p>M1</p> <p>M1</p> <p>M1 A1 (4)</p> <p>[7]</p>
	<p>(b) M1 must see $2(\text{their } a_2) - 7$ or $2(2k - 7) - 7$ or $4k - 14 - 7$. Their a_2 must be a function of k. A1cso must see the $2(2k - 7) - 7$ or $4k - 14 - 7$ expression and the $4k - 21$ with no incorrect working</p> <p>(c) 1st M1 for an attempt to find a_4 using the given rule. Can be awarded for $8k - 49$ seen. Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks. 2nd M1 for attempting the sum of the 1st 4 terms. Must have "+" not just , or clear attempt to sum. Follow through their a_2 and a_4 provided they are linear functions of k. Must lead to linear expression in k. Condone use of their linear $a_3 \neq 4k - 21$ here too. 3rd M1 for forming a linear equation in k using their sum and the 43 and attempt to solve for k as far as $pk = q$ A1 for $k = 8$ only so $k = \frac{120}{15}$ is A0</p> <p><u>Answer Only</u> (e.g. trial improvement) Accept $k = 8$ <u>only if</u> $8 + 9 + 11 + 15 = 43$ is seen as well</p> <p><u>Sum</u> $a_2 + a_3 + a_4 + a_5$ or $a_2 + a_3 + a_4$</p> <p>Allow: M1 if $8k - 49$ is seen, M0 for the sum (since they are not adding the 1st 4 terms) then M1 if they use their sum along with the 43 to form a linear equation and attempt to solve but A0</p>	

Question Number	Scheme	Marks
Q8 (a)	$AB: m = \frac{2-7}{8-6}, \left(= -\frac{5}{2} \right)$ <p>Using $m_1 m_2 = -1: m_2 = \frac{2}{5}$</p> $y - 7 = \frac{2}{5}(x - 6), \quad 2x - 5y + 23 = 0 \quad (\text{o.e. with integer coefficients})$	B1 M1 M1, A1 (4)
(b)	Using $x = 0$ in the answer to (a), $y = \frac{23}{5}$ or 4.6	M1, A1ft (2)
(c)	Area of triangle = $\frac{1}{2} \times 8 \times \frac{23}{5} = \frac{92}{5}$ (o.e.) e.g. $\left(18\frac{2}{5}, 18.4, \frac{184}{10} \right)$	M1 A1 (2)
(a)	<p>B1 for an expression for the gradient of AB. Does not need the $= -2.5$</p> <p>1st M1 for use of the perpendicular gradient rule. Follow through their m</p> <p>2nd M1 for the use of (6, 7) and their changed gradient to form an equation for l.</p> <p>Can be awarded for $\frac{y-7}{x-6} = \frac{2}{5}$ o.e.</p> <p>Alternative is to use (6, 7) in $y = mx + c$ to <u>find a value</u> for c. Score when $c = \dots$ is reached.</p> <p>A1 for a correct equation in the required form and must have “= 0” and integer coefficients</p>	
(b)	M1 for using $x = 0$ in their answer to part (a) e.g. $-5y + 23 = 0$	
	A1ft for $y = \frac{23}{5}$ provided that $x = 0$ clearly seen <u>or</u> $C(0, 4.6)$. Follow through their equation in (a)	
	If $x=0, y = 4.6$ are clearly seen but C is given as (4.6,0) apply ISW and award the mark.	
	This A mark requires a simplified fraction or an exact decimal	
	Accept their 4.6 marked on diagram next to C for M1A1ft	
(c)	M1 for $\frac{1}{2} \times 8 \times y_C$ so can follow through their y coordinate of C .	
	A1 for 18.4 (o.e.) but their y coordinate of C must be positive	
	<u>Use of 2 triangles or trapezium and triangle</u>	
	Award M1 when an expression for area of OCB only is seen	
	<u>Determinant approach</u>	
	Award M1 when an expression containing $\frac{1}{2} \times 8 \times y_C$ is seen	

Question Number	Scheme	Marks
Q9 (a)	$\left[(3-4\sqrt{x})^2 = \right] 9-12\sqrt{x}-12\sqrt{x}+(-4)^2 x$ $9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$	M1 A1, A1 (3)
(b)	$f'(x) = -\frac{9}{2}x^{-\frac{3}{2}}, + \frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A1ft (3)
(c)	$f'(9) = -\frac{9}{2} \times \frac{1}{27} + \frac{16}{2} \times \frac{1}{3} = -\frac{1}{6} + \frac{16}{6} = \frac{5}{2}$	M1 A1 (2)
(a)	<p>M1 for an attempt to expand $(3-4\sqrt{x})^2$ with at least 3 terms correct- as printed or better</p> <p><u>Or</u> $9-k\sqrt{x}+16x$ ($k \neq 0$) . See also the MR rule below</p> <p>1st A1 for their coefficient of $\sqrt{x} = 16$. Condone writing $(\pm)9x^{(\pm)\frac{1}{2}}$ instead of $9x^{-\frac{1}{2}}$</p> <p>2nd A1 for $B = -24$ or their constant term = -24</p>	
(b)	<p>M1 for an attempt to differentiate an x term $x^n \rightarrow x^{n-1}$</p> <p>1st A1 for $-\frac{9}{2}x^{-\frac{3}{2}}$ <u>and</u> their constant B differentiated to zero. NB $-\frac{1}{2} \times 9x^{-\frac{3}{2}}$ is A0</p> <p>2nd A1ft follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for A, i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$</p>	
(c)	<p>M1 for some correct substitution of $x = 9$ in <u>their</u> expression for $f'(x)$ including an attempt at $(9)^{\pm\frac{k}{2}}$ (k odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or 3</p> <p>A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18}, \frac{135}{54}$ or even $\frac{67.5}{27}$</p> <p><u>Misread (MR)</u> Only allow MR of the form $\frac{(3-k\sqrt{x})^2}{\sqrt{x}}$ N.B. Leads to answer in (c) of $\frac{k^2-1}{6}$</p> <p>Score as M1A0A0, M1A1A1ft, M1A1ft</p>	

Question Number	Scheme	Marks
<p>Q10 (a)</p> <p>(b)</p> <p>(c)</p>	<p> $x(x^2 - 6x + 9)$ $= x(x - 3)(x - 3)$ </p> <p>Shape </p> <p><u>Through</u> origin (<u>not</u> touching) Touching x-axis only once Touching at $(3, 0)$, or 3 on x-axis [Must be on graph not in a table]</p> <p>Moved horizontally (either way) $(2, 0)$ and $(5, 0)$, or 2 and 5 on x-axis</p>	<p>B1</p> <p>M1 A1 (3)</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1ft (4)</p> <p>M1</p> <p>A1 (2)</p> <p>[9]</p>
<p>(a)</p> <p>S.C.</p> <p>(b)</p> <p>(c)</p>	<p>B1 for correctly taking out a factor of x</p> <p>M1 for an attempt to factorize their 3TQ e.g. $(x + p)(x + q)$ where $pq = 9$. So $(x - 3)(x + 3)$ will score M1 but A0</p> <p>A1 for a fully correct factorized expression - accept $x(x - 3)^2$ If they "solve" use ISW</p> <p>If the only correct linear factor is $(x - 3)$, perhaps from factor theorem, award B0M1A0 Do not award marks for factorising in part (b)</p> <p>For the graphs "Sharp points" will lose the 1st B1 in (b) but otherwise be generous on shape Condone $(0, 3)$ in (b) and $(0, 2)$, $(0, 5)$ in (c) if the points are marked in the correct places.</p> <p>2nd B1 for a curve that starts or terminates at $(0, 0)$ score B0</p> <p>4th B1ft for a curve that touches (not crossing or terminating) at $(a, 0)$ where their $y = x(x - a)^2$</p> <p>M1 for their graph moved horizontally (only) <u>or</u> a fully correct graph Condone a partial stretch if ignoring their values looks like a simple translation</p> <p>A1 for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)</p>	

Question Number	Scheme	Marks
Q11 (a) (b) (c)	$x = 2: y = 8 - 8 - 2 + 9 = 7$ (*) $\frac{dy}{dx} = 3x^2 - 4x - 1$ $x = 2: \frac{dy}{dx} = 12 - 8 - 1 (= 3)$ $y - 7 = 3(x - 2), \quad \underline{y = 3x + 1}$ $m = -\frac{1}{3}$ (for $-\frac{1}{m}$ with their m) $3x^2 - 4x - 1 = -\frac{1}{3}, \quad 9x^2 - 12x - 2 = 0$ or $x^2 - \frac{4}{3}x - \frac{2}{9} = 0$ (o.e.) $\left(x = \frac{12 + \sqrt{144 + 72}}{18}\right) (\sqrt{216} = \sqrt{36} \cdot \sqrt{6} = 6\sqrt{6})$ or $(3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$ $x = \frac{1}{3}(2 + \sqrt{6})$ (*)	B1 (1) M1 A1 A1ft M1, <u>A1</u> (5) B1ft M1, A1 M1 A1cso (5) [11]
(a) (b) (c) ALT	B1 there must be a clear attempt to substitute $x = 2$ leading to 7 e.g. $2^3 - 2 \times 2^2 - 2 + 9 = 7$ 1 st M1 for an attempt to differentiate with at least one of the given terms fully correct. 1 st A1 for a fully correct expression 2 nd A1ft for sub. $x = 2$ in <u>their</u> $\frac{dy}{dx}$ ($\neq y$) accept for a correct expression e.g. $3 \times (2)^2 - 4 \times 2 - 1$ 2 nd M1 for use of their “3” (provided it comes from their $\frac{dy}{dx}$ ($\neq y$) and $x=2$) to find equation of tangent. Alternative is to use (2, 7) in $y = mx + c$ to <u>find a value</u> for c . Award when $c = \dots$ is seen. No attempted use of $\frac{dy}{dx}$ in (b) scores 0/5 1 st M1 for forming an equation from their $\frac{dy}{dx}$ ($\neq y$) and their $-\frac{1}{m}$ (must be changed from m) 1 st A1 for a correct 3TQ all terms on LHS (condone missing =0) 2 nd M1 for proceeding to $x = \dots$ or $3x = \dots$ by formula or completing the square for a 3TQ. Not factorising. Condone \pm 2 nd A1 for proceeding to given answer with no incorrect working seen. Can still have \pm . <u>Verify (for M1A1M1A1)</u> 1 st M1 for attempting to square need ≥ 3 correct values in $\frac{4+6+4\sqrt{6}}{9}$, 1 st A1 for $\frac{10+4\sqrt{6}}{9}$ 2 nd M1 Dependent on 1 st M1 in this case for substituting in all terms of their $\frac{dy}{dx}$ 2 nd A1cso for cso <u>with a full comment</u> e.g. “the x co-ord of Q is ...”	

June 2009
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\int \left(2x + 3x^{\frac{1}{2}} \right) dx = \frac{2x^2}{2} + \frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ $\int_1^4 \left(2x + 3x^{\frac{1}{2}} \right) dx = \left[x^2 + 2x^{\frac{3}{2}} \right]_1^4 = (16 + 2 \times 8) - (1 + 2)$ $= 29 \quad (29 + C \text{ scores A0})$	<p>M1 A1A1</p> <p>M1</p> <p>A1 (5) [5]</p>
	<p>1st M1 for attempt to integrate $x \rightarrow kx^2$ or $x^{\frac{1}{2}} \rightarrow kx^{\frac{3}{2}}$.</p> <p>1st A1 for $\frac{2x^2}{2}$ or a simplified version.</p> <p>2nd A1 for $\frac{3x^{\frac{3}{2}}}{(\frac{3}{2})}$ or $\frac{3x\sqrt{x}}{(\frac{3}{2})}$ or a simplified version.</p> <p>Ignore + C, if seen, but two correct terms and an <u>extra non-constant</u> term scores M1A1A0.</p> <p>2nd M1 for correct use of correct limits ('top' – 'bottom'). Must be used in a 'changed function', not just the original. (The changed function may have been found by differentiation).</p> <p>Ignore 'poor notation' (e.g. missing integral signs) if the intention is clear.</p> <p><u>No working:</u> The answer 29 with no working scores M0A0A0M1A0 (1 mark).</p>	

Question Number	Scheme	Marks
Q2 (a)	<p>$(7 \times \dots \times x)$ or $(21 \times \dots \times x^2)$ The 7 or 21 can be in ‘unsimplified’ form.</p> $(2 + kx)^7 = 2^7 + 2^6 \times 7 \times kx + 2^5 \times \binom{7}{2} k^2 x^2$ $= 128; + 448kx, + 672k^2 x^2 \text{ [or } 672(kx)^2 \text{]}$ <p>(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)</p>	M1 B1; A1, A1 (4)
(b)	$6 \times 448k = 672k^2$ $k = 4 \quad (\text{Ignore } k = 0, \text{ if seen})$	M1 A1 (2) [6]
(a)	<p>The terms can be ‘listed’ rather than added. Ignore any extra terms.</p> <p>M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing.</p> <p><u>Allow</u> binomial coefficients such as $\binom{7}{1}, \binom{7}{1}, \binom{7}{2}, {}^7C_1, {}^7C_2$.</p> <p>However, $448 + kx$ or similar is M0.</p> <p>B1, A1, A1 for the <u>simplified</u> versions seen above.</p> <p><u>Alternative:</u></p> <p>Note that a factor 2^7 can be taken out first: $2^7 \left(1 + \frac{kx}{2}\right)^7$, but the mark scheme still applies.</p> <p><u>Ignoring subsequent working (isw):</u></p> <p>Is w if necessary after correct working:</p> <p>e.g. $128 + 448kx + 672k^2 x^2$ M1 B1 A1 A1 $= 4 + 14kx + 21k^2 x^2$ isw</p> <p>(Full marks are still available in part (b)).</p>	
(b)	<p>M1 for equating their coefficient of x^2 to 6 times that of $x \dots$ to get an equation in k, \dots <u>or</u> equating their coefficient of x to 6 times that of x^2, to get an equation in k.</p> <p>Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, e.g. $6 \times 448k = 672k$, but beware $k = 4$ following from this, which is A0.</p> <p><u>An equation in k alone</u> is required for this M mark, so...</p> <p>e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow k = 4$ or similar is M0 A0 (equation in coefficients only is never seen), but ...</p> <p>e.g. $6 \times 448kx = 672k^2 x^2 \Rightarrow 6 \times 448k = 672k^2 \Rightarrow k = 4$ will get M1 A1 (as coefficients rather than terms have now been considered).</p> <p>The mistake $2 \left(1 + \frac{kx}{2}\right)^7$ would give a maximum of 3 marks: M1B0A0A0, M1A1</p>	

Question Number	Scheme	Marks
Q3 (a)	$f(k) = -8$	B1 (1)
(b)	$f(2) = 4 \Rightarrow 4 = (6-2)(2-k) - 8$	M1
(c)	So $k = -1$	A1 (2)
	$f(x) = 3x^2 - (2+3k)x + (2k-8) = 3x^2 + x - 10$ $= (3x - 5)(x + 2)$	M1 M1A1 (3) [6]
(b)	<p>M1 for substituting $x = 2$ (<u>not</u> $x = -2$) and equating to 4 to form an equation in k. If the expression is expanded in this part, condone 'slips' for this M mark. Treat the omission of the -8 here as a 'slip' and allow the M mark.</p> <p><u>Beware:</u> Substituting $x = -2$ and equating to 0 (M0 A0) also gives $k = -1$.</p> <p><u>Alternative:</u> M1 for dividing by $(x - 2)$, to get $3x + (\text{function of } k)$, with remainder as a function of k, and equating the remainder to 4. [Should be $3x + (4 - 3k)$, remainder $-4k$].</p> <p><u>No working:</u> $k = -1$ with no working scores M0 A0.</p>	
(c)	<p>1st M1 for multiplying out <u>and</u> substituting their (constant) value of k (in either order). The multiplying-out may occur earlier. Condone, for example, sign slips, but if the 4 (from part (b)) is included in the $f(x)$ expression, this is M0. The 2nd M1 is still available.</p> <p>2nd M1 for an attempt to factorise their three term quadratic (3TQ).</p> <p>A1 The correct answer, as a <u>product of factors</u>, is required. Allow $3\left(x - \frac{5}{3}\right)(x + 2)$</p> <p>Ignore following work (such as a solution to a quadratic equation). If the 'equation' is solved but factors are never seen, the 2nd M is not scored.</p>	

Question Number	Scheme	Marks
Q4 (a)	$x = 2$ gives 2.236 (allow AWR) Accept $\sqrt{5}$ $x = 2.5$ gives 2.580 (allow AWR) Accept 2.58	B1 B1 (2)
(b)	$\left(\frac{1}{2} \times \frac{1}{2}\right), [(1.414 + 3) + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)]$ $= 6.133$ (AWRT 6.13, even following minor slips)	B1, [M1A1ft] A1 (4)
(c)	Overestimate 'Since the trapezia lie <u>above the curve</u> ', or an equivalent explanation, or sketch of (one or more) trapezia above the curve on a diagram (or on the given diagram, in which case there should be reference to this). (Note that there must be some reference to a trapezium or trapezia in the explanation or diagram).	B1 dB1 (2) [8]
(b)	B1 for $\frac{1}{2} \times \frac{1}{2}$ or equivalent. For the M mark, the first bracket must contain the 'first and last' values, and the second bracket (which must be multiplied by 2) must have no additional values. If the only mistake is to <u>omit</u> one of the values from the second bracket, this can be considered as a slip and the M mark can be allowed. Bracketing mistake: i.e. $\left(\frac{1}{2} \times \frac{1}{2}\right)(1.414 + 3) + 2(1.554 + 1.732 + 1.957 + 2.236 + 2.580)$ scores B1 M1 A0 A0 <u>unless</u> the final answer implies that the calculation has been done correctly (then full marks can be given). <u>Alternative:</u> Separate trapezia may be used, and this can be marked equivalently. $\left[\frac{1}{4}(1.414 + 1.554) + \frac{1}{4}(1.554 + 1.732) + \dots + \frac{1}{4}(2.580 + 3) \right]$	1 st A1ft for correct expression, ft their 2.236 and their 2.580
(c)	1 st B1 for 'overestimate', ignoring earlier mistakes and ignoring any reasons given. 2 nd B1 is dependent upon the 1 st B1 (overestimate).	

Question Number	Scheme	Marks
Q5 (a)	$324r^3 = 96 \quad \text{or} \quad r^3 = \frac{96}{324} \quad \text{or} \quad r^3 = \frac{8}{27}$ $r = \frac{2}{3} \quad (*)$	M1 A1cso (2)
(b)	$a\left(\frac{2}{3}\right)^2 = 324 \quad \text{or} \quad a\left(\frac{2}{3}\right)^5 = 96 \quad a = \dots, \quad 729$	M1, A1 (2)
(c)	$S_{15} = \frac{729\left(1 - \left[\frac{2}{3}\right]^{15}\right)}{1 - \frac{2}{3}}, \quad = 2182.00\dots \quad (\text{AWRT } 2180)$	M1A1ft, (3)
(d)	$S_{\infty} = \frac{729}{1 - \frac{2}{3}}, \quad = 2187$	M1, A1 (2) [9]
(a)	<p>M1 for forming an equation for r^3 based on 96 and 324 (e.g. $96r^3 = 324$ scores M1). The equation must involve multiplication/division rather than addition/subtraction.</p> <p>A1 Do not penalise solutions with working in decimals, providing these are correctly rounded or truncated to at least 2dp <u>and</u> the final answer $2/3$ is seen.</p> <p><u>Alternative:</u> (verification)</p> <p>M1 Using $r^3 = \frac{8}{27}$ and multiplying 324 by this (or multiplying by $r = \frac{2}{3}$ three times).</p> <p>A1 Obtaining 96 (cso). (A conclusion is not required). $324 \times \left(\frac{2}{3}\right)^3 = 96$ (no real evidence of calculation) is not quite enough and scores M1 A0.</p> <p>(b) M1 for the use of a correct formula or for 'working back' by dividing by $\frac{2}{3}$ (or by their r) twice from 324 (or 5 times from 96). Exceptionally, allow M1 also for using $ar^3 = 324$ or $ar^6 = 96$ instead of $ar^2 = 324$ or $ar^5 = 96$, or for dividing by r three times from 324 (or 6 times from 96)... but no other exceptions are allowed.</p> <p>(c) M1 for use of sum to 15 terms formula with values of a and r. If the wrong power is used, e.g. 14, the M mark is scored only if the correct sum formula is stated. 1st A1ft for a correct expression or correct ft their a with $r = \frac{2}{3}$. 2nd A1 for awrt 2180, even following 'minor inaccuracies'. Condone missing brackets round the $\frac{2}{3}$ for the marks in part (c). <u>Alternative:</u></p> <p>M1 for adding 15 terms and 1st A1ft for adding the 15 terms that ft from their a and $r = \frac{2}{3}$.</p> <p>(d) M1 for use of correct sum to infinity formula with their a. For this mark, if a value of r different from the given value is being used, M1 can still be allowed providing $r < 1$.</p>	

Question Number	Scheme	Marks
Q7 (i)	$\tan \theta = -1 \Rightarrow \theta = -45, 135$ $\sin \theta = \frac{2}{5} \Rightarrow \theta = 23.6, 156.4$ (AWRT: 24, 156)	B1, B1ft B1, B1ft (4)
(ii)	$4 \sin x = \frac{3 \sin x}{\cos x}$ $4 \sin x \cos x = 3 \sin x \Rightarrow \sin x(4 \cos x - 3) = 0$ Other possibilities (after squaring): $\sin^2 x(16 \sin^2 x - 7) = 0$, $(16 \cos^2 x - 9)(\cos^2 x - 1) = 0$ $x = 0, 180$ <u>seen</u> $x = 41.4, 318.6$ (AWRT: 41, 319)	M1 M1 B1, B1 B1, B1ft (6) [10]
(i)	<p>1st B1 for -45 seen (α, where $\alpha < 90$) 2nd B1 for 135 seen, <u>or ft</u> $(180 + \alpha)$ if α is negative, or $(\alpha - 180)$ if α is positive. If $\tan \theta = k$ is obtained from <u>wrong working</u>, 2nd B1ft is still available. 3rd B1 for awrt 24 (β, where $\beta < 90$) 4th B1 for awrt 156, <u>or ft</u> $(180 - \beta)$ if β is positive, or $-(180 + \beta)$ if β is negative. If $\sin \theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available.</p> <p>(ii) 1st M1 for use of $\tan x = \frac{\sin x}{\cos x}$. Condone $\frac{3 \sin x}{3 \cos x}$. 2nd M1 for correct work leading to 2 factors (may be implied). 1st B1 for 0, 2nd B1 for 180. 3rd B1 for awrt 41 (γ, where $\gamma < 180$) 4th B1 for awrt 319, <u>or ft</u> $(360 - \gamma)$. If $\cos \theta = k$ is obtained from <u>wrong working</u>, 4th B1ft is still available. N.B. Losing $\sin x = 0$ usually gives a maximum of 3 marks M1M0B0B0B1B1 <u>Alternative:</u> (squaring both sides) 1st M1 for squaring both sides and using a 'quadratic' identity. e.g. $16 \sin^2 \theta = 9(\sec^2 \theta - 1)$ 2nd M1 for reaching a factorised form. e.g. $(16 \cos^2 \theta - 9)(\cos^2 \theta - 1) = 0$ Then marks are equivalent to the main scheme. Extra solutions, if not rejected, are penalised as in the main scheme.</p> <p><u>For both parts of the question:</u> <u>Extra solutions outside required range:</u> Ignore <u>Extra solutions inside required range:</u> For each <u>pair</u> of B marks, the 2nd B mark is lost if there are two correct values and one or more extra solution(s), e.g. $\tan \theta = -1 \Rightarrow \theta = 45, -45, 135$ is B1 B0 <u>Answers in radians:</u> Loses a maximum of 2 B marks in the whole question (to be deducted at the first and second occurrence).</p>	

Question Number	Scheme	Marks
Q8 (a)	$\log_2 y = -3 \Rightarrow y = 2^{-3}$ $y = \frac{1}{8} \text{ or } 0.125$	M1 A1 (2)
(b)	$32 = 2^5 \text{ or } 16 = 2^4 \text{ or } 512 = 2^9$ <p>[or $\log_2 32 = 5 \log_2 2$ or $\log_2 16 = 4 \log_2 2$ or $\log_2 512 = 9 \log_2 2$]</p> <p>[or $\log_2 32 = \frac{\log_{10} 32}{\log_{10} 2}$ or $\log_2 16 = \frac{\log_{10} 16}{\log_{10} 2}$ or $\log_2 512 = \frac{\log_{10} 512}{\log_{10} 2}$]</p> $\log_2 32 + \log_2 16 = 9$ <p>$(\log x)^2 = \dots$ or $(\log x)(\log x) = \dots$ (May not be seen explicitly, so M1 may be implied by later work, and the base may be 10 rather than 2)</p> $\log_2 x = 3 \Rightarrow x = 2^3 = 8$ $\log_2 x = -3 \Rightarrow x = 2^{-3} = \frac{1}{8}$	M1 A1 M1 A1 A1ft (5) [7]
(a)	<p>M1 for <u>getting out of logs</u> correctly. If done by change of base, $\log_{10} y = -0.903\dots$ is insufficient for the M1, but $y = 10^{-0.903}$ scores M1. A1 for the <u>exact</u> answer, e.g. $\log_{10} y = -0.903 \Rightarrow y = 0.12502\dots$ scores M1 (implied) A0. <u>Correct answer</u> with no working scores both marks. <u>Allow</u> both marks for implicit statements such as $\log_2 0.125 = -3$.</p>	
(b)	<p>1st M1 for expressing 32 or 16 or 512 as a power of 2, or for a change of base enabling evaluation of $\log_2 32$, $\log_2 16$ or $\log_2 512$ by calculator. (Can be implied by 5, 4 or 9 respectively). 1st A1 for 9 (exact). 2nd M1 for getting $(\log_2 x)^2 = \text{constant}$. The constant can be a log or a sum of logs. If written as $\log_2 x^2$ instead of $(\log_2 x)^2$, allow the M mark <u>only</u> if subsequent work implies correct interpretation. 2nd A1 for 8 (exact). Change of base methods leading to a non-exact answer score A0. 3rd A1ft for an answer of $\frac{1}{\text{their } 8}$. An ft answer may be non-exact.</p> <p><u>Possible mistakes:</u> $\log_2(2^9) = \log_2(x^2) \Rightarrow x^2 = 2^9 \Rightarrow x = \dots$ scores M1A1(implied by 9)M0A0A0 $\log_2 512 = \log_2 x \times \log_2 x \Rightarrow x^2 = 512 \Rightarrow x = \dots$ scores M0A0(9 never seen)M1A0A0 $\log_2 48 = (\log_2 x)^2 \Rightarrow (\log_2 x)^2 = 5.585 \Rightarrow x = 5.145, x = 0.194$ scores M0A0M1A0A1ft</p> <p><u>No working</u> (or ‘trial and improvement’): $x = 8$ scores M0 A0 M1 A1 A0</p>	

Question Number	Scheme	Marks
Q9 (a)	<p>(Arc length $\Rightarrow r\theta = r \times 1 = r$. Can be awarded by implication from later work, e.g. $3rh$ or $(2rh + rh)$ in the S formula. (Requires use of $\theta = 1$).</p> <p>(Sector area $\Rightarrow \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1 = \frac{r^2}{2}$. Can be awarded by implication from later work, e.g. the correct volume formula. (Requires use of $\theta = 1$).</p> <p>Surface area = 2 sectors + 2 rectangles + curved face $(= r^2 + 3rh)$ (See notes below for what is allowed here)</p> <p>Volume = $300 = \frac{1}{2}r^2h$</p> <p>Sub for h: $S = r^2 + 3 \times \frac{600}{r} = r^2 + \frac{1800}{r}$ (*)</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>A1cso (5)</p>
(b)	<p>$\frac{dS}{dr} = 2r - \frac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$</p> <p>$\frac{dS}{dr} = 0 \Rightarrow r^3 = \dots$, $r = \sqrt[3]{900}$, or AWR 9.7 (NOT -9.7 or ± 9.7)</p>	<p>M1A1</p> <p>M1, A1 (4)</p>
(c)	<p>$\frac{d^2S}{dr^2} = \dots$ and consider sign, $\frac{d^2S}{dr^2} = 2 + \frac{3600}{r^3} > 0$ so point is a minimum</p>	<p>M1, A1ft (2)</p>
(d)	<p>$S_{\min} = (9.65\dots)^2 + \frac{1800}{9.65\dots}$</p> <p>(Using their value of r, however found, in the <u>given</u> S formula) $= 279.65\dots$ (AWRT: 280) (Dependent on full marks in part (b))</p>	<p>M1</p> <p>A1 (2)</p> <p>[13]</p>
(a)	<p>M1 for attempting a formula (with terms added) for surface area. May be incomplete or wrong and may have extra term(s), but must have an r^2 (or $r^2\theta$) term and an rh (or $rh\theta$) term.</p>	
(b)	<p><u>In parts (b), (c) and (d), ignore labelling of parts</u></p> <p>1st M1 for attempt at differentiation (one term is sufficient) $r^n \rightarrow kr^{n-1}$</p> <p>2nd M1 for setting their derivative (a 'changed function') = 0 and solving as far as $r^3 = \dots$ (depending upon their 'changed function', this could be $r = \dots$ or $r^2 = \dots$, etc., but the algebra <u>must</u> deal with a <u>negative power</u> of r and should be sound apart from possible <u>sign</u> errors, so that $r^n = \dots$ is consistent with their derivative).</p>	
(c)	<p>M1 for attempting second derivative (one term is sufficient) $r^n \rightarrow kr^{n-1}$, <u>and considering its sign</u>. Substitution of a value of r is not required. (<u>Equating it to zero is M0</u>).</p> <p>A1ft for a correct second derivative (or correct ft from their first derivative) <u>and</u> a valid reason (e.g. > 0), <u>and</u> conclusion. The actual <u>value</u> of the second derivative, if found, can be ignored. To score this mark as ft, their second derivative must indicate a minimum.</p> <p><u>Alternative:</u></p> <p>M1: Find <u>value</u> of $\frac{dS}{dr}$ on each side of their value of r and consider sign.</p> <p>A1ft: Indicate sign change of negative to positive for $\frac{dS}{dr}$, and conclude minimum.</p> <p><u>Alternative:</u></p> <p>M1: Find <u>value</u> of S on each side of their value of r and compare with their 279.65.</p> <p>A1ft: Indicate that both values are more than 279.65, and conclude minimum.</p>	

June 2009
6665 Core Mathematics C3
Mark Scheme

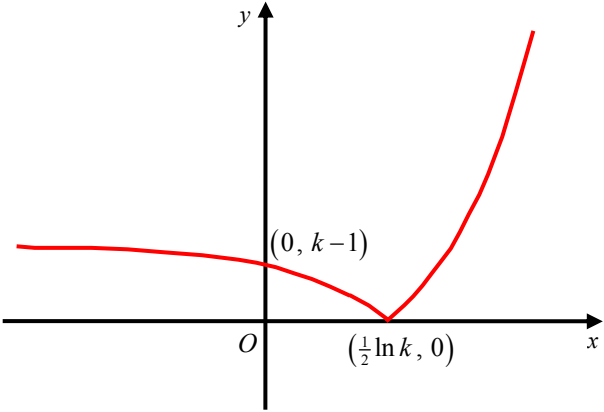
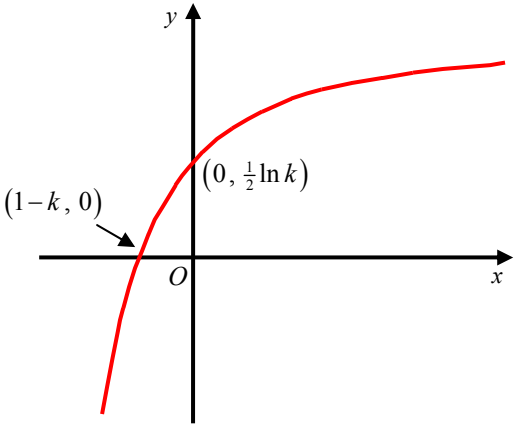
Question Number	Scheme	Marks
Q1 (a)	<p>Iterative formula: $x_{n+1} = \frac{2}{(x_n)^2} + 2$, $x_0 = 2.5$</p> <p>$x_1 = \frac{2}{(2.5)^2} + 2$</p> <p>$x_1 = 2.32$</p> <p>$x_2 = 2.371581451\dots$</p> <p>$x_3 = 2.355593575\dots$</p> <p>$x_4 = 2.360436923\dots$</p>	<p>An attempt to substitute $x_0 = 2.5$ into the iterative formula. Can be implied by $x_1 = 2.32$ or 2.320</p> <p>Both $x_1 = 2.32(0)$ and $x_2 = \text{awrt } 2.372$</p> <p>Both $x_3 = \text{awrt } 2.356$ and $x_4 = \text{awrt } 2.360$ or 2.36</p> <p>M1</p> <p>A1</p> <p>A1 cso</p> <p>(3)</p>
(b)	<p>Let $f(x) = -x^3 + 2x^2 + 2 = 0$</p> <p>$f(2.3585) = 0.00583577\dots$</p> <p>$f(2.3595) = -0.00142286\dots$</p> <p>Sign change (and $f(x)$ is continuous) therefore a root α is such that $\alpha \in (2.3585, 2.3595) \Rightarrow \alpha = 2.359$ (3 dp)</p>	<div style="border: 1px solid black; padding: 2px; width: fit-content; margin-bottom: 5px;">Choose suitable interval for x, e.g. $[2.3585, 2.3595]$ or tighter</div> <p>any one value awrt 1 sf or truncated 1 sf</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin-bottom: 5px;">both values correct, sign change and conclusion</div> <p>At a minimum, both values must be correct to 1sf or truncated 1sf, candidate states "change of sign, hence root".</p> <p>M1</p> <p>dM1</p> <p>A1</p> <p>(3)</p> <p>[6]</p>

Question Number	Scheme	Marks
Q2 (a)	$\cos^2 \theta + \sin^2 \theta = 1 \quad (\div \cos^2 \theta)$ $\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$ $1 + \tan^2 \theta = \sec^2 \theta$	M1
	$\tan^2 \theta = \sec^2 \theta - 1 \quad (\text{as required}) \quad \mathbf{AG}$	A1 cso (2)
(b)	$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2, \quad (\text{eqn } *) \quad 0 \leq \theta < 360^\circ$	
	$2(\sec^2 \theta - 1) + 4 \sec \theta + \sec^2 \theta = 2$ $2 \sec^2 \theta - 2 + 4 \sec \theta + \sec^2 \theta = 2$	M1
	$3 \sec^2 \theta + 4 \sec \theta - 4 = 0$	M1
	$(\sec \theta + 2)(3 \sec \theta - 2) = 0$ $\sec \theta = -2 \quad \text{or} \quad \sec \theta = \frac{2}{3}$	M1
	$\frac{1}{\cos \theta} = -2 \quad \text{or} \quad \frac{1}{\cos \theta} = \frac{2}{3}$ $\underline{\cos \theta = -\frac{1}{2}}; \quad \text{or} \quad \cos \theta = \frac{3}{2}$	A1;
	$\alpha = 120^\circ \quad \text{or} \quad \alpha = \text{no solutions}$	
	$\theta_1 = \underline{120^\circ}$	A1
	$\theta_2 = 240^\circ$	B1 $\sqrt{\quad}$
	$\theta = \{120^\circ, 240^\circ\}$	(6)
	<div style="border: 1px solid black; padding: 5px; display: inline-block;">Note the final A1 mark has been changed to a B1 mark.</div>	[8]

Question Number	Scheme	Marks
Q3	$P = 80e^{\frac{t}{5}}$	
(a)	$t = 0 \Rightarrow P = 80e^{\frac{0}{5}} = 80(1) = \underline{80}$	B1 (1)
(b)	$P = 1000 \Rightarrow 1000 = 80e^{\frac{t}{5}} \Rightarrow \frac{1000}{80} = e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{1000}{80}\right)$ $t = 12.6286\dots$	Substitutes $P = 1000$ and rearranges equation to make $e^{\frac{t}{5}}$ the subject. M1
	<div style="border: 1px solid black; padding: 2px; display: inline-block;">awrt 12.6 or 13 years</div>	A1 (2)
(c)	$\frac{dP}{dt} = 16e^{\frac{t}{5}}$	$ke^{\frac{1}{5}t}$ and $k \neq 80$. M1 A1 (2)
(d)	$50 = 16e^{\frac{t}{5}}$ $\therefore t = 5 \ln\left(\frac{50}{16}\right) \quad \{= 5.69717\dots\}$	Using $50 = \frac{dP}{dt}$ and an attempt to solve to find the value of t or $\frac{t}{5}$. M1
	$P = 80e^{\frac{1}{5}\left(5 \ln\left(\frac{50}{16}\right)\right)} \quad \text{or} \quad P = 80e^{\frac{1}{5}(5.69717\dots)}$	Substitutes their value of t back into the equation for P . dM1
	$P = \frac{80(50)}{16} = \underline{250}$	$\underline{250}$ or awrt 250 A1 (3)
		[8]

Question Number	Scheme	Marks
Q4 (i)(a)	$y = x^2 \cos 3x$ <p>Apply product rule: $\left\{ \begin{array}{l} u = x^2 \quad v = \cos 3x \\ \frac{du}{dx} = 2x \quad \frac{dv}{dx} = -3 \sin 3x \end{array} \right\}$</p> $\frac{dy}{dx} = 2x \cos 3x - 3x^2 \sin 3x$	<p>Applies $vu' + uv'$ correctly for their u, u', v, v' AND gives an expression of the form $\alpha x \cos 3x \pm \beta x^2 \sin 3x$</p> <p>M1</p> <p>Any one term correct</p> <p>A1</p> <p>Both terms correct and no further simplification to terms in $\cos \alpha x^2$ or $\sin \beta x^3$.</p> <p>A1</p> <p>(3)</p>
(b)	$y = \frac{\ln(x^2 + 1)}{x^2 + 1}$ $u = \ln(x^2 + 1) \Rightarrow \frac{du}{dx} = \frac{2x}{x^2 + 1}$ <p>Apply quotient rule: $\left\{ \begin{array}{l} u = \ln(x^2 + 1) \quad v = x^2 + 1 \\ \frac{du}{dx} = \frac{2x}{x^2 + 1} \quad \frac{dv}{dx} = 2x \end{array} \right\}$</p> $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2 + 1}\right)(x^2 + 1) - 2x \ln(x^2 + 1)}{(x^2 + 1)^2}$ $\left\{ \frac{dy}{dx} = \frac{2x - 2x \ln(x^2 + 1)}{(x^2 + 1)^2} \right\}$	$\ln(x^2 + 1) \rightarrow \frac{\text{something}}{x^2 + 1}$ <p>M1</p> $\ln(x^2 + 1) \rightarrow \frac{2x}{x^2 + 1}$ <p>A1</p> <p>Applying $\frac{vu' - uv'}{v^2}$</p> <p>M1</p> <p>Correct differentiation with correct bracketing but allow recovery.</p> <p>A1</p> <p>{Ignore subsequent working.}</p> <p>(4)</p>

Question Number	Scheme	Marks
(ii)	<p>$y = \sqrt{4x+1}, x > -\frac{1}{4}$</p> <p>At $P, y = \sqrt{4(2)+1} = \sqrt{9} = 3$</p> <p>$\frac{dy}{dx} = \frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)$</p> <p>$\frac{dy}{dx} = \frac{2}{(4x+1)^{\frac{1}{2}}}$</p> <p>At $P, \frac{dy}{dx} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$</p> <p>Hence $m(\mathbf{T}) = \frac{2}{3}$</p> <p>Either $\mathbf{T}: y - 3 = \frac{2}{3}(x - 2);$</p> <p>or $y = \frac{2}{3}x + c$ and $3 = \frac{2}{3}(2) + c \Rightarrow c = 3 - \frac{4}{3} = \frac{5}{3};$</p> <p>Either $\mathbf{T}: 3y - 9 = 2(x - 2);$</p> <p>$\mathbf{T}: 3y - 9 = 2x - 4$</p> <p>$\mathbf{T}: \underline{2x - 3y + 5 = 0}$</p> <p>or $\mathbf{T}: y = \frac{2}{3}x + \frac{5}{3}$</p> <p>$\mathbf{T}: 3y = 2x + 5$</p> <p>$\mathbf{T}: \underline{2x - 3y + 5 = 0}$</p>	<p>At $P, y = \sqrt{9}$ or 3</p> <p>$\pm k(4x+1)^{-\frac{1}{2}}$</p> <p>$2(4x+1)^{-\frac{1}{2}}$</p> <p>Substituting $x = 2$ into an equation involving $\frac{dy}{dx};$</p> <p>$y - y_1 = m(x - 2)$ or $y - y_1 = m(x - \text{their stated } x)$ with ‘their TANGENT gradient’ and their $y_1;$ or uses $y = mx + c$ with ‘their TANGENT gradient’, their x and their $y_1.$</p> <p>$2x - 3y + 5 = 0$</p> <p>Tangent must be stated in the form $ax + by + c = 0$, where a, b and c are integers.</p> <p>(6)</p> <p>[13]</p>

Question Number	Scheme	Marks
Q5 (a)		<p>Curve retains shape when $x > \frac{1}{2} \ln k$ B1</p> <p>Curve reflects through the x-axis when $x < \frac{1}{2} \ln k$ B1</p> <p>$(0, k-1)$ and $(\frac{1}{2} \ln k, 0)$ marked in the correct positions. B1</p> <p>(3)</p>
(b)		<p>Correct shape of curve. The curve should be contained in quadrants 1, 2 and 3 (Ignore asymptote) B1</p> <p>$(1-k, 0)$ and $(0, \frac{1}{2} \ln k)$ B1</p> <p>(2)</p>
(c)	<p>Range of f: $f(x) > -k$ or $y > -k$ or $(-k, \infty)$</p>	<p>Either $f(x) > -k$ or $y > -k$ or $(-k, \infty)$ or $f > -k$ or <u>Range $> -k$.</u> B1</p> <p>(1)</p>
(d)	<p>$y = e^{2x} - k \Rightarrow y + k = e^{2x}$ $\Rightarrow \ln(y + k) = 2x$ $\Rightarrow \frac{1}{2} \ln(y + k) = x$</p> <p>Hence $f^{-1}(x) = \frac{1}{2} \ln(x + k)$</p>	<p>Attempt to make x (or swapped y) the subject M1</p> <p>Makes e^{2x} the subject and takes \ln of both sides M1</p> <p>$\frac{1}{2} \ln(x + k)$ or $\ln \sqrt{x + k}$ A1 cao</p> <p>(3)</p>
(e)	<p>$f^{-1}(x)$: Domain: $x > -k$ or $(-k, \infty)$</p>	<p>Either $x > -k$ or $(-k, \infty)$ or Domain $> -k$ or x “ft one sided inequality” their part (c) RANGE answer B1 $\sqrt{}$</p> <p>(1)</p> <p>[10]</p>

Question Number	Scheme	Marks
Q6 (a)	$A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A \cos A - \sin A \sin A}$ <p>Applies $A = B$ to $\cos(A + B)$ to give the <u>underlined</u> equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$</p> <p>$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives</p> <p>$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required)</p>	<p>M1</p> <p>A1 AG (2)</p>
(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$ <p>Eliminating y correctly.</p> <p>Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.</p> $3\sin 2x = 4\left(\frac{1 - \cos 2x}{2}\right) - 2\cos 2x$ $3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$ $3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$ $3\sin 2x + 4\cos 2x = 2$ <p>Rearranges to give correct result</p>	<p>M1</p> <p>M1</p> <p>A1 AG (3)</p>
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$ $3\sin 2x + 4\cos 2x = R\cos 2x \cos \alpha + R\sin 2x \sin \alpha$ <p>Equate $\sin 2x$: $3 = R\sin \alpha$ Equate $\cos 2x$: $4 = R\cos \alpha$</p> $R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$ <p>$\tan \alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765\dots^\circ$</p> <p>Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p>

Question Number	Scheme	Marks
(d)	$3 \sin 2x + 4 \cos 2x = 2$ $5 \cos(2x - 36.87) = 2$ $\cos(2x - 36.87) = \frac{2}{5}$ $(2x - 36.87) = 66.42182\dots^\circ$ $(2x - 36.87) = 360 - 66.42182\dots^\circ$ <p>Hence, $x = 51.64591\dots^\circ, 165.22409\dots^\circ$</p>	<p>$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$ M1</p> <p>awrt 66 A1</p> <p>One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3 A1</p> <p>Both awrt 51.6 AND awrt 165.2 A1</p> <p>(4)</p> <p>[12]</p>

Question Number	Scheme	Marks
Q7	<p>$f(x) = 1 - \frac{2}{(x+4)} + \frac{x-8}{(x-2)(x+4)}$ $x \in \mathbb{R}, x \neq -4, x \neq 2.$</p> <p>(a)</p> $f(x) = \frac{(x-2)(x+4) - 2(x-2) + x-8}{(x-2)(x+4)}$ $= \frac{x^2 + 2x - 8 - 2x + 4 + x - 8}{(x-2)(x+4)}$ $= \frac{x^2 + x - 12}{[(x+4)(x-2)]}$ $= \frac{(x+4)(x-3)}{[(x+4)(x-2)]}$ $= \frac{(x-3)}{(x-2)}$ <p>(b)</p> $g(x) = \frac{e^x - 3}{e^x - 2} \quad x \in \mathbb{R}, x \neq \ln 2.$ <p>Apply quotient rule: $\left\{ \begin{array}{l} u = e^x - 3 \quad v = e^x - 2 \\ \frac{du}{dx} = e^x \quad \frac{dv}{dx} = e^x \end{array} \right\}$</p> $g'(x) = \frac{e^x(e^x - 2) - e^x(e^x - 3)}{(e^x - 2)^2}$ $= \frac{e^{2x} - 2e^x - e^{2x} + 3e^x}{(e^x - 2)^2}$ $= \frac{e^x}{(e^x - 2)^2}$	<p>An attempt to combine to one fraction M1</p> <p>Correct result of combining all three fractions A1</p> <p>Simplifies to give the correct numerator. Ignore omission of denominator A1</p> <p>An attempt to factorise the numerator. dM1</p> <p>Correct result A1 cso AG (5)</p> <p>Applying $\frac{vu' - uv'}{v^2}$ M1</p> <p>Correct differentiation A1</p> <p>Correct result A1 AG cso (3)</p>

Question Number	Scheme	Marks
(c)	$g'(x) = 1 \Rightarrow \frac{e^x}{(e^x - 2)^2} = 1$ $e^x = (e^x - 2)^2$ $e^x = e^{2x} - 2e^x - 2e^x + 4$ $\underline{e^{2x} - 5e^x + 4} = 0$ $(e^x - 4)(e^x - 1) = 0$ $e^x = 4 \text{ or } e^x = 1$ $x = \ln 4 \text{ or } x = 0$	<p>Puts their differentiated numerator equal to their denominator. M1</p> <p>$\underline{e^{2x} - 5e^x + 4}$ A1</p> <p>Attempt to factorise or solve quadratic in e^x M1</p> <p>both $x = 0, \ln 4$ A1</p> <p>(4)</p> <p>[12]</p>

Question Number	Scheme	Marks
Q8 (a)	$\sin 2x = \underline{2 \sin x \cos x}$	B1 aef (1)
(b)	$\operatorname{cosec} x - 8 \cos x = 0, \quad 0 < x < \pi$ $\frac{1}{\sin x} - 8 \cos x = 0$ $\frac{1}{\sin x} = 8 \cos x$ $1 = 8 \sin x \cos x$ $1 = 4(2 \sin x \cos x)$ $1 = 4 \sin 2x$ $\underline{\sin 2x = \frac{1}{4}}$ <p>Radians $2x = \{0.25268\dots, 2.88891\dots\}$ Degrees $2x = \{14.4775\dots, 165.5225\dots\}$</p> <p>Radians $x = \{0.12634\dots, 1.44445\dots\}$ Degrees $x = \{7.23875\dots, 82.76124\dots\}$</p>	<p>Using $\operatorname{cosec} x = \frac{1}{\sin x}$ M1</p> <p>$\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$ M1 $\underline{\sin 2x = \frac{1}{4}}$ A1</p> <p>Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π. A1 Both <u>0.13</u> and <u>1.44</u> A1 cao</p> <p>Solutions for the final two A marks must be given in x only. If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 < x < \pi$.</p> <p>(5)</p> <p>[6]</p>

June 2009
6666 Core Mathematics C4
Mark Scheme

Question Number	Scheme	Marks
Q1	$f(x) = \frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}}$ $= (4)^{-\frac{1}{2}} (1 + \dots)^{-\frac{1}{2}}$ $= \dots \left(1 + \left(-\frac{1}{2}\right) \left(\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2} \left(\frac{x}{4}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!} \left(\frac{x}{4}\right)^3 + \dots \right)$ <p style="text-align: right;">ft their $\left(\frac{x}{4}\right)$</p> $= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$ <i>Alternative</i> $f(x) = \frac{1}{\sqrt{4+x}} = (4+x)^{-\frac{1}{2}}$ $= 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right) 4^{-\frac{3}{2}}x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2} 4^{-\frac{5}{2}}x^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1.2.3} 4^{-\frac{7}{2}}x^3 + \dots$ $= \frac{1}{2} - \frac{1}{16}x + \frac{3}{256}x^2 - \frac{5}{2048}x^3 + \dots$	<p>M1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1, A1 (6)</p> <p>[6]</p> <p>M1</p> <p><u>B1</u> M1 A1</p> <p>A1, A1 (6)</p>

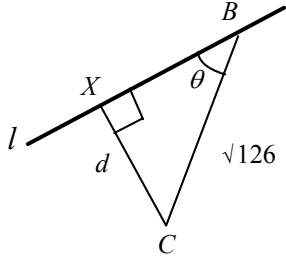
Question Number	Scheme	Marks
Q2 (a)	1.14805 awrt 1.14805	B1 (1)
(b)	$A \approx \frac{1}{2} \times \frac{3\pi}{8} (\dots)$ $= \dots (3 + 2(2.77164 + 2.12132 + 1.14805) + 0)$ $= \frac{3\pi}{16} (3 + 2(2.77164 + 2.12132 + 1.14805))$ $= \frac{3\pi}{16} \times 15.08202 \dots = 8.884$	B1 M1 A1ft A1 (4)
(c)	$\int 3 \cos\left(\frac{x}{3}\right) dx = \frac{3 \sin\left(\frac{x}{3}\right)}{\frac{1}{3}}$ $= 9 \sin\left(\frac{x}{3}\right)$	M1 A1
	$A = \left[9 \sin\left(\frac{x}{3}\right) \right]_0^{\frac{3\pi}{2}} = 9 - 0 = 9$	A1 (3)
		[8]

Question Number	Scheme	Marks
Q3 (a)	$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$ $4-2x = A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)$ <p style="text-align: center;">A method for evaluating one constant</p> $x \rightarrow -\frac{1}{2}, \quad 5 = A\left(\frac{1}{2}\right)\left(\frac{5}{2}\right) \Rightarrow A = 4$ <p style="text-align: right;">any one correct constant</p> $x \rightarrow -1, \quad 6 = B(-1)(2) \Rightarrow B = -3$ $x \rightarrow -3, \quad 10 = C(-5)(-2) \Rightarrow C = 1$ <p style="text-align: right;">all three constants correct</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 (4)</p>
(b)	<p>(i) $\int \left(\frac{4}{2x+1} - \frac{3}{x+1} + \frac{1}{x+3} \right) dx$</p> $= \frac{4}{2} \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) + C$ <p style="text-align: right;">A1 two ln terms correct</p> <p style="text-align: center;">All three ln terms correct and "+C"; ft constants</p> <p>(ii) $\left[2 \ln(2x+1) - 3 \ln(x+1) + \ln(x+3) \right]_0^2$</p> $= (2 \ln 5 - 3 \ln 3 + \ln 5) - (2 \ln 1 - 3 \ln 1 + \ln 3)$ $= 3 \ln 5 - 4 \ln 3$ $= \ln \left(\frac{5^3}{3^4} \right)$ $= \ln \left(\frac{125}{81} \right)$	<p>M1 A1ft</p> <p>A1ft (3)</p> <p>M1</p> <p>M1</p> <p>A1 (3)</p> <p>[10]</p>

Question Number	Scheme	Marks
Q4 (a)	$e^{-2x} \frac{dy}{dx} - 2ye^{-2x} = 2 + 2y \frac{dy}{dx}$ $\frac{d}{dx}(ye^{-2x}) = e^{-2x} \frac{dy}{dx} - 2ye^{-2x}$ $(e^{-2x} - 2y) \frac{dy}{dx} = 2 + 2ye^{-2x}$ $\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$	<p>A1 correct RHS</p> <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p>
(b)	<p>At P , $\frac{dy}{dx} = \frac{2 + 2e^0}{e^0 - 2} = -4$</p> <p>Using $mm' = -1$</p> $m' = \frac{1}{4}$ $y - 1 = \frac{1}{4}(x - 0)$ $x - 4y + 4 = 0$ <p>or any integer multiple</p> <p><i>Alternative for (a) differentiating implicitly with respect to y.</i></p> $e^{-2x} - 2ye^{-2x} \frac{dx}{dy} = 2 \frac{dx}{dy} + 2y$ $\frac{d}{dy}(ye^{-2x}) = e^{-2x} - 2ye^{-2x} \frac{dx}{dy}$ $(2 + 2ye^{-2x}) \frac{dx}{dy} = e^{-2x} - 2y$ $\frac{dx}{dy} = \frac{e^{-2x} - 2y}{2 + 2ye^{-2x}}$ $\frac{dy}{dx} = \frac{2 + 2ye^{-2x}}{e^{-2x} - 2y}$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1 (4)</p> <p>[9]</p> <p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1 (5)</p>

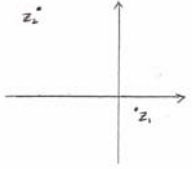
Question Number	Scheme	Marks
Q5 (a)	$\frac{dx}{dt} = -4 \sin 2t, \quad \frac{dy}{dt} = 6 \cos t$ $\frac{dy}{dx} = -\frac{6 \cos t}{4 \sin 2t} \quad \left(= -\frac{3}{4 \sin t} \right)$ <p>At $t = \frac{\pi}{3}$, $m = -\frac{3}{4 \times \frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87</p>	B1, B1 M1 A1 (4)
(b)	<p>Use of $\cos 2t = 1 - 2 \sin^2 t$</p> $\cos 2t = \frac{x}{2}, \quad \sin t = \frac{y}{6}$ $\frac{x}{2} = 1 - 2 \left(\frac{y}{6} \right)^2$ <p>Leading to $y = \sqrt{(18 - 9x)} \quad (= 3\sqrt{(2 - x)})$ cao</p> $-2 \leq x \leq 2 \quad k = 2$	M1 M1 A1 B1 (4)
(c)	$0 \leq f(x) \leq 6 \quad \text{either } 0 \leq f(x) \text{ or } f(x) \leq 6$ <p>Fully correct. Accept $0 \leq y \leq 6, [0, 6]$</p>	B1 B1 (2)
[10]		
<i>Alternatives to (a) where the parameter is eliminated</i>		
①	$y = (18 - 9x)^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}(18 - 9x)^{-\frac{1}{2}} \times (-9)$ <p>At $t = \frac{\pi}{3}, x = \cos \frac{2\pi}{3} = -1$</p> $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{\sqrt{(27)}} \times -9 = -\frac{\sqrt{3}}{2}$	B1 B1 M1 A1 (4)
②	$y^2 = 18 - 9x$ $2y \frac{dy}{dx} = -9$ <p>At $t = \frac{\pi}{3}, y = 6 \sin \frac{\pi}{3} = 3\sqrt{3}$</p> $\frac{dy}{dx} = -\frac{9}{2 \times 3\sqrt{3}} = -\frac{\sqrt{3}}{2}$	B1 B1 M1 A1 (4)

Question Number	Scheme	Marks
Q6 (a)	$\int \sqrt{5-x} dx = \int (5-x)^{\frac{1}{2}} dx = \frac{(5-x)^{\frac{3}{2}}}{-\frac{3}{2}} (+C)$ $\left(= -\frac{2}{3}(5-x)^{\frac{3}{2}} + C \right)$	M1 A1 (2)
(b)	<p>(i) $\int (x-1)\sqrt{5-x} dx = -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3} \int (5-x)^{\frac{3}{2}} dx$</p> $= \dots + \frac{2}{3} \times \frac{(5-x)^{\frac{5}{2}}}{-\frac{5}{2}} (+C)$ $= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} (+C)$ <p>(ii) $\left[-\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} \right]_1^5 = (0-0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}} \right)$</p> $= \frac{128}{15} \left(= 8 \frac{8}{15} \approx 8.53 \right) \quad \text{awrt 8.53}$	<div style="border: 1px solid black; width: 20px; height: 20px; margin-bottom: 5px;"></div> M1 A1ft M1 A1 (4) M1 A1 (2) [8]
<i>Alternatives for (b) and (c)</i>		
(b)	$u^2 = 5-x \Rightarrow 2u \frac{du}{dx} = -1 \left(\Rightarrow \frac{dx}{du} = -2u \right)$ $\int (x-1)\sqrt{5-x} dx = \int (4-u^2)u \frac{dx}{du} du = \int (4-u^2)u(-2u) du$ $= \int (2u^4 - 8u^2) du = \frac{2}{5}u^5 - \frac{8}{3}u^3 (+C)$ $= \frac{2}{5}(5-x)^{\frac{5}{2}} - \frac{8}{3}(5-x)^{\frac{3}{2}} (+C)$	<div style="border: 1px solid black; width: 20px; height: 20px; margin-bottom: 5px;"></div> M1 A1 M1 A1
(c)	$x=1 \Rightarrow u=2, \quad x=5 \Rightarrow u=0$ $\left[\frac{2}{5}u^5 - \frac{8}{3}u^3 \right]_2^0 = (0-0) - \left(\frac{64}{5} - \frac{64}{3} \right)$ $= \frac{128}{15} \left(= 8 \frac{8}{15} \approx 8.53 \right) \quad \text{awrt 8.53}$	M1 A1 (2)

Question Number	Scheme	Marks	
Q7 (a)	$\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \quad \text{or } \vec{BA} = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \quad \text{or } \mathbf{r} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ <p style="text-align: right;">accept equivalents</p>	M1 M1 A1ft (3)	
(b)	$\vec{CB} = \vec{OB} - \vec{OC} = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix} \quad \text{or } \vec{BC} = \begin{pmatrix} -1 \\ -5 \\ 10 \end{pmatrix}$ $CB = \sqrt{(1^2 + 5^2 + (-10)^2)} = \sqrt{126} \quad (= 3\sqrt{14} \approx 11.2) \quad \text{awrt } 11.2$	M1 A1 (2)	
(c)	$\vec{CB} \cdot \vec{AB} = \vec{CB} \vec{AB} \cos \theta$ $(\pm)(2 + 5 + 20) = \sqrt{126} \sqrt{9} \cos \theta$ $\cos \theta = \frac{3}{\sqrt{14}} \Rightarrow \theta \approx 36.7^\circ \quad \text{awrt } 36.7^\circ$	M1 A1 A1 (3)	
(d)	 $\frac{d}{\sqrt{126}} = \sin \theta$ $d = 3\sqrt{5} (\approx 6.7) \quad \text{awrt } 6.7$	M1 A1ft A1 (3)	
(e)	$BX^2 = BC^2 - d^2 = 126 - 45 = 81$ $! CBX = \frac{1}{2} \times BX \times d = \frac{1}{2} \times 9 \times 3\sqrt{5} = \frac{27\sqrt{5}}{2} (\approx 30.2) \quad \text{awrt } 30.1 \text{ or } 30.2$	M1 M1 A1 (3)	
[14]			
<p><i>Alternative for (e)</i></p> $! CBX = \frac{1}{2} \times d \times BC \sin \angle XCB$ $= \frac{1}{2} \times 3\sqrt{5} \times \sqrt{126} \sin(90 - 36.7)^\circ$ <p style="text-align: center;">sine of correct angle</p> $\approx 30.2 \quad \frac{27\sqrt{5}}{2}, \text{ awrt } 30.1 \text{ or } 30.2$			M1 M1 A1 (3)

Question Number	Scheme	Marks
Q8 (a)	$\int \sin^2 \theta d\theta = \frac{1}{2} \int (1 - \cos 2\theta) d\theta = \frac{1}{2} \theta - \frac{1}{4} \sin 2\theta \quad (+C)$	M1 A1 (2)
(b)	$x = \tan \theta \Rightarrow \frac{dx}{d\theta} = \sec^2 \theta$ $\pi \int y^2 dx = \pi \int y^2 \frac{dx}{d\theta} d\theta = \pi \int (2 \sin 2\theta)^2 \sec^2 \theta d\theta$ $= \pi \int \frac{(2 \times 2 \sin \theta \cos \theta)^2}{\cos^2 \theta} d\theta$ $= 16\pi \int \sin^2 \theta d\theta \qquad k = 16\pi$ $x = 0 \Rightarrow \tan \theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ $\left(V = 16\pi \int_0^{\frac{\pi}{6}} \sin^2 \theta d\theta \right)$	M1 A1 M1 A1 B1 (5)
(c)	$V = 16\pi \left[\frac{1}{2} \theta - \frac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{6}}$ $= 16\pi \left[\left(\frac{\pi}{12} - \frac{1}{4} \sin \frac{\pi}{3} \right) - (0 - 0) \right]$ $= 16\pi \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8} \right) = \frac{4}{3} \pi^2 - 2\pi \sqrt{3}$	<div style="border-left: 1px solid black; border-right: 1px solid black; padding: 0 10px;"> M1 M1 A1 (3) </div> <p style="text-align: center;">Use of correct limits</p> <p style="text-align: center;">$p = \frac{4}{3}, q = -2$</p> <p style="text-align: right;">[10]</p>

June 2009
6667 Further Pure Mathematics FP1 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	 <p>(b) $z_1 = \sqrt{2^2 + (-1)^2} = \sqrt{5}$ (or awrt 2.24)</p> <p>(c) $\alpha = \arctan\left(\frac{1}{2}\right)$ or $\arctan\left(-\frac{1}{2}\right)$ $\arg z_1 = -0.46$ or 5.82 (awrt) (answer in degrees is A0 unless followed by correct conversion)</p> <p>(d) $\frac{-8+9i}{2-i} \times \frac{2+i}{2+i}$ $= \frac{-16-8i+18i-9}{5} = -5+2i$ i.e. $a = -5$ and $b = 2$ or $-\frac{2}{3}a$</p>	<p>B1 (1)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1 A1 A1ft (3)</p> <p>[8]</p>
Notes	<p>Alternative method to part (d) $-8+9i = (2-i)(a+bi)$, and so $2a+b = -8$ and $2b-a = 9$ and attempt to solve as far as equation in one variable So $a = -5$ and $b = 2$</p> <p>(a) B1 needs both complex numbers as either points or vectors, in correct quadrants and with 'reasonably correct' relative scale</p> <p>(b) M1 Attempt at Pythagoras to find modulus of either complex number A1 condone correct answer even if negative sign not seen in (-1) term A0 for $\pm\sqrt{5}$</p> <p>(c) $\arctan 2$ is M0 unless followed by $\frac{3\pi}{2} + \arctan 2$ or $\frac{\pi}{2} - \arctan 2$ Need to be clear that $\arg z = -0.46$ or 5.82 for A1</p> <p>(d) M1 Multiply numerator and denominator by conjugate of their denominator A1 for -5 and A1 for $2i$ (should be simplified) Alternative scheme for (d) Allow slips in working for first M1</p>	<p>M1</p> <p>A1 A1cao</p>

Question Number	Scheme	Marks
Q3 (a)	$x^2 + 4 = 0 \Rightarrow x = ki, \quad x = \pm 2i$ <p>Solving 3-term quadratic</p> $x = \frac{-8 \pm \sqrt{64 - 100}}{2} = -4 + 3i \text{ and } -4 - 3i$	M1, A1 M1 A1 A1ft (5)
Notes	<p>(b) $2i + (-2i) + (-4 + 3i) + (-4 - 3i) = -8$</p> <p>Alternative method : Expands $f(x)$ as quartic and chooses \pm coefficient of x^3</p> <p>-8</p> <p>(a) Just $x = 2i$ is M1 A0 $x = \pm 2$ is M0A0 M1 for solving quadratic follows usual conventions, then A1 for a correct root (simplified as here) and A1ft for conjugate of first answer. Accept correct answers with no working here. Do not give accuracy marks for factors unless followed by roots.</p> <p>(b) M1 for adding four roots of which at least two are complex conjugates and getting a real answer. A1 for -8 following correct roots or the alternative method. If any incorrect working in part (a) this A mark will be A0</p>	M1 A1cso (2) [7] M1 A1 cso

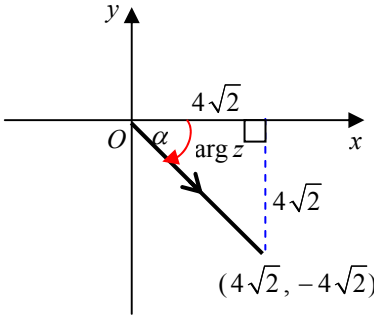
Question Number	Scheme	Marks
Q6 (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16 \times 4t^2 = 64t^2$ Or identifies that $a = 4$ and uses general coordinates $(at^2, 2at)$	B1 (1)
(b)	(4, 0)	B1 (1)
(c)	$y = 4x^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}}$ Replaces x by $4t^2$ to give gradient $[2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}]$ Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $[-t]$ $y - 8t = -t(x - 4t^2) \Rightarrow y + tx = 8t + 4t^3$ (*)	B1 M1, M1 M1 A1cso (5)
(d)	At N , $y = 0$, so $x = 8 + 4t^2$ or $\frac{8t + 4t^3}{t}$ Base $SN = (8 + 4t^2) - 4 (= 4 + 4t^2)$ Area of $\triangle PSN = \frac{1}{2}(4 + 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$ for $t > 0$ {Also Area of $\triangle PSN = \frac{1}{2}(4 + 4t^2)(-8t) = -16t(1 + t^2)$ for $t < 0$ } <i>this is not required</i> <u>Alternatives:</u> (c) $\frac{dx}{dt} = 8t$ and $\frac{dy}{dt} = 8$ B1 $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{1}{t}$ M1, then as in main scheme. (c) $2y \frac{dy}{dx} = 16$ B1 (or uses $x = \frac{y^2}{8}$ to give $\frac{dx}{dy} = \frac{2y}{8}$) $\frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} = \frac{1}{t}$ M1, then as in main scheme.	B1 B1ft M1 A1 (4) [11]
Notes	(c) Second M1 – need not be function of t Third M1 requires linear equation (not fraction) and should include the parameter t but could be given for equation of tangent (So tangent equation loses 2 marks only and could gain B1M1M0M1A0) (d) Second B1 does not require simplification and may be a constant rather than an expression in t . M1 needs correct area of triangle formula using $\frac{1}{2}$ ‘their SN ’ $\times 8t$ Or may use two triangles in which case need $(4t^2 - 4)$ and $(4t^2 + 8 - 4t^2)$ for B1ft Then Area of $\triangle PSN = \frac{1}{2}(4t^2 - 4)(8t) + \frac{1}{2}(4t^2 + 8 - 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$	

Question Number	Scheme	Marks
Q7 (a) (b) (c)	Use $4a - (-2 \times -1) = 0 \Rightarrow a = \frac{1}{2}$ Determinant: $(3 \times 4) - (-2 \times -1) = 10$ (Δ) $\mathbf{B}^{-1} = \frac{1}{10} \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$ $\frac{1}{10} \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 4(k-6) + 2(3k+12) \\ (k-6) + 3(3k+12) \end{pmatrix}$ $\begin{pmatrix} k \\ k+3 \end{pmatrix} \text{ Lies on } y = x + 3$	M1, A1 (2) M1 M1 A1cso (3) M1, A1ft A1 (3) [8]
Notes	<p><u>Alternatives:</u></p> <p>(c) $\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ x+3 \end{pmatrix}, = \begin{pmatrix} 3x-2(x+3) \\ -x+4(x+3) \end{pmatrix},$ $= \begin{pmatrix} x-6 \\ 3x+12 \end{pmatrix},$ which was of the form $(k-6, 3k+12)$</p> <p>Or $\begin{pmatrix} 3 & -2 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, = \begin{pmatrix} 3x-2y \\ -x+4y \end{pmatrix} = \begin{pmatrix} k-6 \\ 3k+12 \end{pmatrix},$ and solves simultaneous equations</p> <p>Both equations correct and eliminate one letter to get $x = k$ or $y = k + 3$ or $10x - 10y = -30$ or equivalent.</p> <p>Completely correct work (to $x = k$ and $y = k + 3$), and conclusion lies on $y = x + 3$</p> <p>(a) Allow sign slips for first M1 (b) Allow sign slip for determinant for first M1 (This mark may be awarded for 1/10 appearing in inverse matrix.) Second M1 is for correctly treating the 2 by 2 matrix, ie for $\begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$</p> <p>Watch out for determinant $(3 + 4) - (-1 + -2) = 10 - M0$ then final answer is A0</p> <p>(c) M1 for multiplying matrix by appropriate column vector A1 correct work (ft wrong determinant) A1 for conclusion</p>	M1, A1, A1 M1 A1 A1

Question Number	Scheme	Marks
Q8 (a)	$f(1) = 5 + 8 + 3 = 16$, (which is divisible by 4). (\therefore True for $n = 1$). Using the formula to write down $f(k + 1)$, $f(k + 1) = 5^{k+1} + 8(k + 1) + 3$ $f(k + 1) - f(k) = 5^{k+1} + 8(k + 1) + 3 - 5^k - 8k - 3$ $= 5(5^k) + 8k + 8 + 3 - 5^k - 8k - 3 = 4(5^k) + 8$ $f(k + 1) = 4(5^k + 2) + f(k)$, which is divisible by 4 \therefore True for $n = k + 1$ if true for $n = k$. True for $n = 1$, \therefore true for all n .	B1 M1 A1 M1 A1 A1ft A1cso (7)
(b)	For $n = 1$, $\begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^1$ (\therefore True for $n = 1$). $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$ $= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$ \therefore True for $n = k + 1$ if true for $n = k$. True for $n = 1$, \therefore true for all n	B1 M1 A1 A1 M1 A1 A1 cso (7) [14]
(a) Alternative for 2 nd M:	$f(k + 1) = 5(5^k) + 8k + 8 + 3$ M1 $= 4(5^k) + 8 + (5^k + 8k + 3)$ A1 or $= 5(5^k + 8k + 3) - 32k - 4$ $= 4(5^k + 2) + f(k)$, or $= 5f(k) - 4(8k + 1)$ which is divisible by 4 A1 (or similar methods)	
Notes Part (b) Alternative	<p>(a) B1 Correct values of 16 or 4 for $n = 1$ or for $n = 0$ (Accept “is a multiple of”) M1 Using the formula to write down $f(k + 1)$ A1 Correct expression of $f(k+1)$ (or for $f(n + 1)$) M1 Start method to connect $f(k+1)$ with $f(k)$ as shown A1 correct working toward multiples of 4, A1 ft result including $f(k + 1)$ as subject, A1cso conclusion</p> <p>(b) B1 correct statement for $n = 1$ or $n = 0$ First M1: Set up product of two appropriate matrices – product can be either way round A1 A0 for one or two slips in simplified result A1 A1 all correct simplified A0 A0 more than two slips M1: States in terms of $(k + 1)$ A1 Correct statement A1 for induction conclusion</p> <p>May write $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}^{k+1} = \begin{pmatrix} 2k+3 & -2k-2 \\ 2k+2 & -2k-1 \end{pmatrix}$. Then may or may not complete the proof.</p> <p>This can be awarded the second M (substituting $k + 1$) and following A (simplification) in part (b). The first three marks are awarded as before. Concluding that they have reached the same matrix and therefore a result will then be part of final A1 cso but also need other statements as in the first method.</p>	

June 2009
6668 Further Pure Mathematics FP2 (new)
Mark Scheme

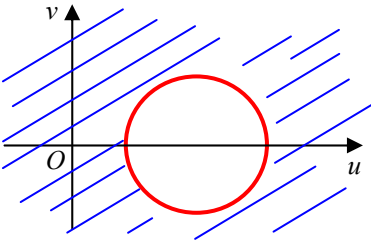
Question Number	Scheme	Marks
Q1 (a)	$\frac{1}{r(r+2)} = \frac{1}{2r} - \frac{1}{2(r+2)}$	$\frac{1}{2r} - \frac{1}{2(r+2)}$
(b)	$\sum_{r=1}^n \frac{4}{r(r+2)} = \sum_{r=1}^n \left(\frac{2}{r} - \frac{2}{r+2} \right)$ $= \left(\frac{2}{\underline{1}} - \frac{2}{\underline{3}} \right) + \left(\frac{2}{\underline{2}} - \frac{2}{\underline{4}} \right) + \dots$ $\dots + \left(\frac{2}{\underline{n-1}} - \frac{2}{\underline{n+1}} \right) + \left(\frac{2}{\underline{n}} - \frac{2}{\underline{n+2}} \right)$ $= \frac{2}{\underline{1}} + \frac{2}{\underline{2}} - \frac{2}{n+1} - \frac{2}{n+2}$ $= 3 - \frac{2}{n+1} - \frac{2}{n+2}$ $= \frac{3(n+1)(n+2) - 2(n+2) - 2(n+1)}{(n+1)(n+2)}$ $= \frac{3n^2 + 9n + 6 - 2n - 4 - 2n - 2}{(n+1)(n+2)}$ $= \frac{3n^2 + 5n}{(n+1)(n+2)}$ $= \frac{n(3n+5)}{(n+1)(n+2)}$	<p style="text-align: center;">(1)</p> <p style="text-align: center;">B1 aef</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">M1</p> <p style="text-align: center;">A1</p> <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <p style="text-align: center;">Attempt to combine to an at least 3 term fraction to a single fraction and an attempt to take out the brackets from their numerator.</p> </div> <p style="text-align: center;">Correct Result</p> <p style="text-align: center;">A1 cso AG</p> <p style="text-align: center;">(5)</p> <p style="text-align: center;">[6]</p>

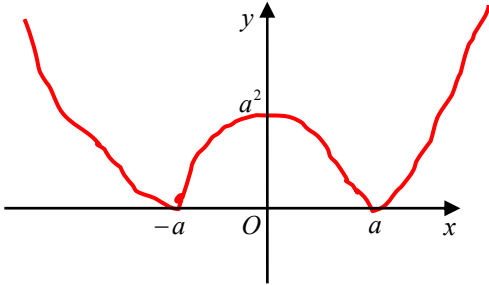
Question Number	Scheme	Marks
<p>Q2 (a)</p>	<p>$z^3 = 4\sqrt{2} - 4\sqrt{2}i$, $-\pi < \theta \leq \pi$</p>  <p>$r = \sqrt{(4\sqrt{2})^2 + (-4\sqrt{2})^2} = \sqrt{32 + 32} = \sqrt{64} = 8$</p> <p>$\theta = -\tan^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right) = -\frac{\pi}{4}$</p> <p>$z^3 = 8\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$</p> <p>So, $z = (8)^{\frac{1}{3}}\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right)$</p> <p>$\Rightarrow z = 2\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$</p> <p>Also, $z^3 = 8\left(\cos\left(\frac{7\pi}{4}\right) + i\sin\left(\frac{7\pi}{4}\right)\right)$ or $z^3 = 8\left(\cos\left(-\frac{9\pi}{4}\right) + i\sin\left(-\frac{9\pi}{4}\right)\right)$</p> <p>$\Rightarrow z = 2\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$ and $z = 2\left(\cos\left(\frac{-3\pi}{4}\right) + i\sin\left(\frac{-3\pi}{4}\right)\right)$</p> <p>Special Case 1: Award SC: M1M1A1M1A0A0 for ALL three of $2\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$, $2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ and $2\left(\cos\left(\frac{-7\pi}{12}\right) + i\sin\left(\frac{-7\pi}{12}\right)\right)$.</p> <p>Special Case 2: If r is incorrect (and not equal to 8) and candidate states the brackets () correctly then give the first accuracy mark ONLY where this is applicable.</p>	<p>A valid attempt to find the modulus and argument of $4\sqrt{2} - 4\sqrt{2}i$. M1</p> <p>Taking the cube root of the modulus and dividing the argument by 3. M1</p> <p>$2\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)$ A1</p> <p>Adding or subtracting 2π to the argument for z^3 in order to find other roots. M1</p> <p>Any one of the final two roots A1</p> <p>Both of the final two roots. A1</p> <p>[6]</p>

Question Number	Scheme	Marks
Q3	$\sin x \frac{dy}{dx} - y \cos x = \sin 2x \sin x$ $\frac{dy}{dx} - \frac{y \cos x}{\sin x} = \frac{\sin 2x \sin x}{\sin x}$ $\frac{dy}{dx} - \frac{y \cos x}{\sin x} = \sin 2x$ <p>Integrating factor = $e^{\int -\frac{\cos x}{\sin x} dx} = e^{-\ln \sin x}$</p> $= \frac{1}{\sin x}$ $\left(\frac{1}{\sin x}\right) \frac{dy}{dx} - \frac{y \cos x}{\sin^2 x} = \frac{\sin 2x}{\sin x}$ $\frac{d}{dx} \left(\frac{y}{\sin x}\right) = \sin 2x \times \frac{1}{\sin x}$ $\frac{d}{dx} \left(\frac{y}{\sin x}\right) = 2 \cos x$ $\frac{y}{\sin x} = \int 2 \cos x dx$ $\frac{y}{\sin x} = 2 \sin x + K$ $y = 2 \sin^2 x + K \sin x$	<p>An attempt to divide every term in the differential equation by $\sin x$. Can be implied.</p> <p>M1</p> <p>dM1 A1 aef</p> <p>A1 aef</p> <p>M1</p> <p>A1</p> <p>dddM1</p> <p>A1 cao</p> <p>[8]</p>

Question Number	Scheme	Marks
Q4	$A = \frac{1}{2} \int_0^{2\pi} (a + 3 \cos \theta)^2 d\theta$ $(a + 3 \cos \theta)^2 = a^2 + 6a \cos \theta + 9 \cos^2 \theta$ $= a^2 + 6a \cos \theta + 9 \left(\frac{1 + \cos 2\theta}{2} \right)$ $A = \frac{1}{2} \int_0^{2\pi} \left(a^2 + 6a \cos \theta + \frac{9}{2} + \frac{9}{2} \cos 2\theta \right) d\theta$ $= \left(\frac{1}{2} \right) \left[a^2 \theta + 6a \sin \theta + \frac{9}{2} \theta + \frac{9}{4} \sin 2\theta \right]_0^{2\pi}$ $= \frac{1}{2} \left[(2\pi a^2 + 0 + 9\pi + 0) - (0) \right]$ $= \pi a^2 + \frac{9\pi}{2}$ <p>Hence, $\pi a^2 + \frac{9\pi}{2} = \frac{107}{2} \pi$</p> $a^2 + \frac{9}{2} = \frac{107}{2}$ $a^2 = 49$ <p>As $a > 0$, $a = 7$</p> <p>Some candidates may achieve $a = 7$ from incorrect working. Such candidates will not get full marks</p>	<p>Applies $\frac{1}{2} \int_0^{2\pi} r^2 (d\theta)$ with correct limits. Ignore $d\theta$.</p> <p>B1</p> <p>$\cos^2 \theta = \frac{\pm 1 \pm \cos 2\theta}{2}$</p> <p>M1</p> <p><u>Correct underlined expression.</u></p> <p>A1</p> <p>Integrated expression with at least 3 out of 4 terms of the form $\pm A\theta \pm B \sin \theta \pm C\theta \pm D \sin 2\theta$. Ignore the $\frac{1}{2}$. Ignore limits. $a^2 \theta + 6a \sin \theta +$ correct ft integration.</p> <p>M1*</p> <p>A1 ft</p> <p>Ignore the $\frac{1}{2}$. Ignore limits.</p> <p>$\pi a^2 + \frac{9\pi}{2}$</p> <p>A1</p> <p>Integrated expression equal to $\frac{107}{2} \pi$.</p> <p>dM1*</p> <p>$a = 7$</p> <p>A1 cso</p> <p>[8]</p>

Question Number	Scheme	Marks
<p>Q5</p> <p>(a)</p> <p>$y = \sec^2 x = (\sec x)^2$</p> <p>$\frac{dy}{dx} = 2(\sec x)^1(\sec x \tan x) = 2\sec^2 x \tan x$</p> <p>Apply product rule:</p> $\left\{ \begin{array}{l} u = 2\sec^2 x \\ \frac{du}{dx} = 4\sec^2 x \tan x \end{array} \right. \quad \left\{ \begin{array}{l} v = \tan x \\ \frac{dv}{dx} = \sec^2 x \end{array} \right.$ <p>$\frac{d^2y}{dx^2} = 4\sec^2 x \tan^2 x + 2\sec^4 x$</p> <p>$= 4\sec^2 x(\sec^2 x - 1) + 2\sec^4 x$</p> <p>Hence, $\frac{d^2y}{dx^2} = 6\sec^4 x - 4\sec^2 x$</p> <p>(b)</p> <p>$y_{\frac{\pi}{4}} = (\sqrt{2})^2 = 2, \left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 2(\sqrt{2})^2(1) = 4$</p> <p>$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 6(\sqrt{2})^4 - 4(\sqrt{2})^2 = 24 - 8 = 16$</p> <p>$\frac{d^3y}{dx^3} = 24\sec^3 x(\sec x \tan x) - 8\sec x(\sec x \tan x)$</p> <p>$= 24\sec^4 x \tan x - 8\sec^2 x \tan x$</p> <p>$\left(\frac{d^2y}{dx^2}\right)_{\frac{\pi}{4}} = 24(\sqrt{2})^4(1) - 8(\sqrt{2})^2(1) = 96 - 16 = 80$</p> <p>$\sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + \frac{16}{2}\left(x - \frac{\pi}{4}\right)^2 + \frac{80}{6}\left(x - \frac{\pi}{4}\right)^3 + \dots$</p> <p>$\left\{ \sec x \approx 2 + 4\left(x - \frac{\pi}{4}\right) + 8\left(x - \frac{\pi}{4}\right)^2 + \frac{40}{3}\left(x - \frac{\pi}{4}\right)^3 + \dots \right\}$</p>	<p>Either $2(\sec x)^1(\sec x \tan x)$ or $2\sec^2 x \tan x$</p> <p>Two terms added with one of either $A \sec^2 x \tan^2 x$ or $B \sec^4 x$ in the correct form. Correct differentiation</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Applies $\tan^2 x = \sec^2 x - 1$ leading to the correct result.</p> </div> <p>Both $y_{\frac{\pi}{4}} = 2$ and $\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = 4$</p> <p>Attempts to substitute $x = \frac{\pi}{4}$ into both terms in the expression for $\frac{d^2y}{dx^2}$.</p> <p>Two terms differentiated with either $24\sec^4 x \tan x$ or $-8\sec^2 x \tan x$ being correct</p> <p>$\left(\frac{d^3y}{dx^3}\right)_{\frac{\pi}{4}} = 80$</p> <p>Applies a Taylor expansion with at least 3 out of 4 terms fit correctly. Correct Taylor series expansion.</p>	<p>B1 aef</p> <p>M1</p> <p>A1</p> <p>A1 AG</p> <p>(4)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(6)</p> <p>[10]</p>

Question Number	Scheme	Marks
<p>Q6</p> <p>(a)</p> <p>(b)</p>	$w = \frac{z}{z+i}, z = -i$ $w(z+i) = z \Rightarrow wz + iw = z \Rightarrow iw = z - wz$ $\Rightarrow iw = z(1-w) \Rightarrow z = \frac{iw}{(1-w)}$ $ z = 3 \Rightarrow \left \frac{iw}{1-w} \right = 3$ $\left\{ \begin{array}{l} iw = 3 1-w \Rightarrow w = 3 w-1 \Rightarrow w ^2 = 9 w-1 ^2 \\ \Rightarrow u+iv ^2 = 9 u+iv-1 ^2 \end{array} \right\}$ $\Rightarrow u^2 + v^2 = 9[(u-1)^2 + v^2]$ $\left\{ \begin{array}{l} \Rightarrow u^2 + v^2 = 9u^2 - 18u + 9 + 9v^2 \\ \Rightarrow 0 = 8u^2 - 18u + 8v^2 + 9 \end{array} \right\}$ $\Rightarrow 0 = u^2 - \frac{9}{4}u + v^2 + \frac{9}{8}$ $\Rightarrow \left(u - \frac{9}{8}\right)^2 - \frac{81}{64} + v^2 + \frac{9}{8} = 0$ $\Rightarrow \left(u - \frac{9}{8}\right)^2 + v^2 = \frac{9}{64}$ <p>{Circle} centre $\left(\frac{9}{8}, 0\right)$, radius $\frac{3}{8}$</p> 	<p>Complete method of rearranging to make z the subject.</p> $z = \frac{iw}{(1-w)}$ <p>Putting z in terms of their $w = 3$</p> <p>Applies $w = u + iv$, and uses Pythagoras correctly to get an equation in terms of u and v without any i's.</p> <p>Correct equation.</p> <p>Simplifies down to $u^2 + v^2 \pm \alpha u \pm \beta v \pm \delta = 0$.</p> <p>One of centre or radius correct. Both centre and radius correct.</p> <p>Circle indicated on the Argand diagram in the correct position in follow through quadrants. Ignore plotted coordinates.</p> <p>Region outside a circle indicated only.</p> <p>M1</p> <p>A1 aef</p> <p>dM1</p> <p>ddM1</p> <p>A1</p> <p>dddM1</p> <p>A1</p> <p>A1</p> <p>B1ft</p> <p>B1</p> <p>(8)</p> <p>(2)</p> <p>[10]</p>

Question Number	Scheme	Marks
<p>Q7</p> <p>(a)</p> <p>(b)</p> <p>(c)</p>	<p>$y = x^2 - a^2 , a > 1$</p>  <p>$x^2 - a^2 = a^2 - x, a > 1$</p> <p>$\{ x > a\}, \quad x^2 - a^2 = a^2 - x$</p> <p>$\Rightarrow x^2 + x - 2a^2 = 0$</p> <p>$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(1)(-2a^2)}}{2}$</p> <p>$\Rightarrow x = \frac{-1 \pm \sqrt{1 + 8a^2}}{2}$</p> <p>$\{ x < a\}, \quad -x^2 + a^2 = a^2 - x$</p> <p>$\{\Rightarrow x^2 - x = 0 \Rightarrow x(x - 1) = 0\}$</p> <p>$\Rightarrow x = 0, 1$</p> <p>$x^2 - a^2 > a^2 - x, a > 1$</p> <p>$x < \frac{-1 - \sqrt{1 + 8a^2}}{2} \quad \{\text{or}\} \quad x > \frac{-1 + \sqrt{1 + 8a^2}}{2}$</p> <p>$\{\text{or}\} \quad 0 < x < 1$</p>	<p>Correct Shape. Ignore cusps. Correct coordinates.</p> <p>B1 B1</p> <p>(2)</p> <p>M1 aef</p> <p>Applies the quadratic formula or completes the square in order to find the roots.</p> <p>Both correct “simplified down” solutions.</p> <p>M1 aef</p> <p>B1 A1</p> <p>(6)</p> <p>B1 ft B1 ft</p> <p>For $\{ x < a\}$, Lowest $< x <$ Highest $0 < x < 1$</p> <p>M1 A1</p> <p>(4)</p> <p>[12]</p>

Question Number	Scheme	Marks
<p>Q8</p> <p>(a)</p>	$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 2e^{-t}, \quad x = 0, \frac{dx}{dt} = 2 \text{ at } t = 0.$ <p>AE, $m^2 + 5m + 6 = 0 \Rightarrow (m + 3)(m + 2) = 0$ $\Rightarrow m = -3, -2.$</p> <p>So, $x_{CF} = Ae^{-3t} + Be^{-2t}$</p> $\left\{ x = ke^{-t} \Rightarrow \frac{dx}{dt} = -ke^{-t} \Rightarrow \frac{d^2x}{dt^2} = ke^{-t} \right\}$ <p>$\Rightarrow ke^{-t} + 5(-ke^{-t}) + 6ke^{-t} = 2e^{-t} \Rightarrow 2ke^{-t} = 2e^{-t}$ $\Rightarrow k = 1$</p> <p>$\{ \text{So, } x_{PI} = e^{-t} \}$</p> <p>So, $x = Ae^{-3t} + Be^{-2t} + e^{-t}$</p> $\frac{dx}{dt} = -3Ae^{-3t} - 2Be^{-2t} - e^{-t}$ <p>$t = 0, x = 0 \Rightarrow 0 = A + B + 1$ $t = 0, \frac{dx}{dt} = 2 \Rightarrow 2 = -3A - 2B - 1$</p> $\begin{cases} 2A + 2B = -2 \\ -3A - 2B = 3 \end{cases}$ <p>$\Rightarrow A = -1, B = 0$</p> <p>So, $x = -e^{-3t} + e^{-t}$</p>	<p>$Ae^{m_1t} + Be^{m_2t}$, where $m_1 \neq m_2$. $Ae^{-3t} + Be^{-2t}$</p> <p>M1 A1</p> <p>Substitutes ke^{-t} into the differential equation given in the question. Finds $k = 1$.</p> <p>M1 A1</p> <p>their x_{CF} + their x_{PI}</p> <p>M1*</p> <p>Finds $\frac{dx}{dt}$ by differentiating their x_{CF} and their x_{PI}</p> <p>dM1*</p> <p>Applies $t = 0, x = 0$ to x and $t = 0, \frac{dx}{dt} = 2$ to $\frac{dx}{dt}$ to form simultaneous equations.</p> <p>ddM1*</p> <p>$x = -e^{-3t} + e^{-t}$</p> <p>A1 cao (8)</p>

Question Number	Scheme	Marks
(b)	$x = -e^{-3t} + e^{-t}$ $\frac{dx}{dt} = 3e^{-3t} - e^{-t} = 0$ $3 - e^{2t} = 0$ $\Rightarrow t = \frac{1}{2} \ln 3$ <p>So, $x = -e^{-\frac{3}{2} \ln 3} + e^{-\frac{1}{2} \ln 3} = -e^{\ln 3^{-\frac{3}{2}}} + e^{\ln 3^{-\frac{1}{2}}}$</p> $x = -3^{-\frac{3}{2}} + 3^{-\frac{1}{2}}$ $= -\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$ $\frac{d^2x}{dt^2} = -9e^{-3t} + e^{-t}$ <p>At $t = \frac{1}{2} \ln 3$, $\frac{d^2x}{dt^2} = -9e^{-\frac{3}{2} \ln 3} + e^{-\frac{1}{2} \ln 3}$</p> $= -9(3)^{-\frac{3}{2}} + 3^{-\frac{1}{2}} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = -\frac{3}{\sqrt{3}} + \frac{1}{\sqrt{3}}$ <p>As $\frac{d^2x}{dt^2} = -\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} = \left\{ -\frac{2}{\sqrt{3}} \right\} < 0$ then x is maximum.</p>	<p>Differentiates their x to give $\frac{dx}{dt}$ and puts $\frac{dx}{dt}$ equal to 0. M1</p> <p>A credible attempt to solve. $t = \frac{1}{2} \ln 3$ or $t = \ln \sqrt{3}$ or awrt 0.55 dM1* A1</p> <p>Substitutes their t back into x and an attempt to eliminate out the ln's. ddM1</p> <p>uses exact values to give $\frac{2\sqrt{3}}{9}$ A1 AG</p> <p>Finds $\frac{d^2x}{dt^2}$ and substitutes their t into $\frac{d^2x}{dt^2}$ dM1*</p> <p>$-\frac{9}{3\sqrt{3}} + \frac{1}{\sqrt{3}} < 0$ and maximum conclusion. A1</p> <p>(7)</p> <p>[15]</p>

June 2009
6669 Further Pure Mathematics FP3 (new)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{7}{\cosh x} - \frac{\sinh x}{\cosh x} = 5 \Rightarrow \frac{14}{e^x + e^{-x}} - \frac{(e^x - e^{-x})}{e^x + e^{-x}} = 5$ $\therefore 14 - (e^x - e^{-x}) = 5(e^x + e^{-x}) \Rightarrow 6e^x - 14 + 4e^{-x} = 0$ $\therefore 3e^{2x} - 7e^x + 2 = 0 \Rightarrow (3e^x - 1)(e^x - 2) = 0$ $\therefore e^x = \frac{1}{3} \text{ or } 2$ $x = \ln\left(\frac{1}{3}\right) \text{ or } \ln 2$	M1 A1 M1 A1 B1ft [5]
Alternative (i)	Write $7 - \sinh x = 5 \cosh x$, then use exponential substitution $7 - \frac{1}{2}(e^x - e^{-x}) = \frac{5}{2}(e^x + e^{-x})$ Then proceed as method above.	M1
Alternative (ii)	$(7 \operatorname{sech} x - 5)^2 = \tanh^2 x = 1 - \operatorname{sech}^2 x$ $50 \operatorname{sech}^2 x - 70 \operatorname{sech} x + 24 = 0$ $2(5 \operatorname{sech} x - 3)(5 \operatorname{sech} x - 4) = 0$ $\operatorname{sech} x = \frac{3}{5} \text{ or } \operatorname{sech} x = \frac{4}{5}$ $x = \ln\left(\frac{1}{3}\right) \text{ or } \ln 2$	M1 A1 M1 A1 B1ft
Q2 (a)	$\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$	M1 A1 A1 (3)
(b)	$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0 + 5 = 5$	M1 A1 ft (2)
(c)	Area of triangle $OBC = \frac{1}{2} 5\mathbf{j} + 5\mathbf{k} = \frac{5}{2} \sqrt{2}$	M1 A1 (2)
(d)	Volume of tetrahedron $= \frac{1}{6} \times 5 = \frac{5}{6}$	B1 ft (1) [8]

Question Number	Scheme	Marks
Q3 (a)	$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \therefore (6-\lambda)(7-\lambda)(2-\lambda) + 3(7-\lambda) = 0$ <p>$(7-\lambda) = 0$ verifies $\lambda = 7$ is an eigenvalue (can be seen anywhere)</p> <p>$\therefore (7-\lambda)\{12-8\lambda+\lambda^2+3\} = 0 \quad \therefore (7-\lambda)\{\lambda^2-8\lambda+15\} = 0$</p> <p>$\therefore (7-\lambda)(\lambda-5)(\lambda-3) = 0$ and 3 and 5 are the other two eigenvalues</p> <p>(b) Set $\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ or $\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$</p> <p>Solve $-x + y - z = 0$ and $3x - y - 5z = 0$ to obtain $x = 3z$ or $y = 4z$ and a second equation which can contain 3 variables</p> <p>Obtain eigenvector as $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ (or multiple)</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>(5)</p> <p>M1</p> <p>M1 A1</p> <p>A1</p> <p>(4)</p> <p>[9]</p>

Question Number	Scheme	Marks
Q4 (a)	$\frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}} \times \frac{1}{\sqrt{1+(\sqrt{x})^2}}$ $\therefore \frac{dy}{dx} = \frac{\frac{1}{2} x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}} \right)$	B1, M1 A1 (3)
(b)	$\therefore \int_{\frac{1}{4}}^4 \frac{1}{\sqrt{x(x+1)}} dx = [2 \operatorname{ar sinh} \sqrt{x}]_{\frac{1}{4}}^4$ $= [2 \operatorname{ar sinh} 2 - 2 \operatorname{ar sinh}(\frac{1}{2})]$ $= [2 \ln(2 + \sqrt{5})] - [2 \ln(\frac{1}{2} + \sqrt{\frac{5}{4}})]$ $2 \ln \frac{(2 + \sqrt{5})}{(\frac{1}{2} + \sqrt{\frac{5}{4}})} = 2 \ln \frac{2(2 + \sqrt{5})}{(1 + \sqrt{5})} = 2 \ln \frac{2(\sqrt{5} + 2)(\sqrt{5} - 1)}{(\sqrt{5} + 1)(\sqrt{5} - 1)} = 2 \ln \frac{(3 + \sqrt{5})}{2}$ $= \ln \frac{(3 + \sqrt{5})(3 + \sqrt{5})}{4} = \ln \frac{14 + 6\sqrt{5}}{4} = \ln \left(\frac{7}{2} + \frac{3\sqrt{5}}{2} \right)$	M1 M1 M1 M1 A1 A1 (6) [9]
Alternative (i) for part (a)	<p>Use $\sinh y = \sqrt{x}$ and state $\cosh y \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$</p> $\therefore \frac{dy}{dx} = \frac{\frac{1}{2} x^{-\frac{1}{2}}}{\sqrt{1 + \sinh^2 y}} = \frac{\frac{1}{2} x^{-\frac{1}{2}}}{\sqrt{1 + (\sqrt{x})^2}}$ $\therefore \frac{dy}{dx} = \frac{\frac{1}{2} x^{-\frac{1}{2}}}{\sqrt{1+x}} \quad \left(= \frac{1}{2\sqrt{x(1+x)}} \right)$	B1 M1 A1 (3)
Alternative (i) for part (b) Alternative (ii) for part (b)	<p>Use $x = \tan^2 \theta$, $\frac{dx}{d\theta} = 2 \tan \theta \sec^2 \theta$ to give $2 \int \sec \theta d\theta = [2 \ln(\sec \theta + \tan \theta)]$</p> $= [2 \ln(\sec \theta + \tan \theta)]_{\tan \theta = \frac{1}{2}}^{\tan \theta = 2}$ <p>i.e. use of limits then proceed as before from line 3 of scheme</p> <p>Use $\int \frac{1}{\sqrt{[(x + \frac{1}{2})^2 - \frac{1}{4}]}} dx = \operatorname{arcosh} \frac{x + \frac{1}{2}}{\frac{1}{2}}$</p> $= [\operatorname{arcosh} 9 - \operatorname{arcosh}(\frac{3}{2})]$ $= [\ln(9 + \sqrt{80})] - [\ln(\frac{3}{2} + \frac{1}{2}\sqrt{5})]$ $= \ln \frac{(9 + \sqrt{80})}{(\frac{3}{2} + \frac{1}{2}\sqrt{5})} = \ln \frac{2(9 + \sqrt{80})(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})}$ $= \ln \frac{2(9 + 4\sqrt{5})(3 - \sqrt{5})}{(3 + \sqrt{5})(3 - \sqrt{5})} = \ln \left(\frac{7}{2} + \frac{3\sqrt{5}}{2} \right)$	M1 M1 M1 M1 M1 A1 A1 (6) [9]

Question Number	Scheme	Marks
Q5 (a) (b) (c)	$-(25-x^2)^{\frac{1}{2}} \quad (+c)$ $I_n = \int x^{n-1} \cdot \frac{x}{\sqrt{(25-x^2)}} dx = -x^{n-1}\sqrt{25-x^2} + \int (n-1)x^{n-2}\sqrt{(25-x^2)} dx$ $I_n = \left[-x^{n-1}\sqrt{25-x^2} \right]_0^5 + \int_0^5 \frac{(n-1)x^{n-2}(25-x^2)}{\sqrt{(25-x^2)}} dx$ $I_n = 0 + 25(n-1) I_{n-2} - (n-1) I_n$ $\therefore nI_n = 25(n-1)I_{n-2} \text{ and so } I_n = \frac{25(n-1)}{n} I_{n-2} \quad *$ $I_0 = \int_0^5 \frac{1}{\sqrt{(25-x^2)}} dx = \left[\arcsin\left(\frac{x}{5}\right) \right]_0^5 = \frac{\pi}{2}$ $I_4 = \frac{25 \times 3}{4} \times \frac{25 \times 1}{2} I_0 = \frac{1875}{16} \pi$	M1A1 (2) M1 A1ft M1 M1 A1 (5) M1 A1 M1 A1 (4) [11]
Alternative for (b)	Using substitution $x = 5\sin\theta$ $I_n = 5^n \int_0^{\frac{\pi}{2}} \sin^n \theta d\theta = \left[-5^n \sin^{n-1} \theta \cos \theta \right]_0^{\frac{\pi}{2}} + 5^n (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} \theta \cos^2 \theta d\theta$ $= \left[-5^n \sin^{n-1} \theta \cos \theta \right]_0^{\frac{\pi}{2}} + 5^n (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} \theta (1 - \sin^2 \theta) d\theta$ $I_n = 0 + 25(n-1) I_{n-2} - (n-1) I_n$ $\therefore nI_n = 25(n-1)I_{n-2} \text{ and so } I_n = \frac{25(n-1)}{n} I_{n-2} \quad *$ <p>(need to see that $I_{n-2} = 5^{n-2} \int_0^{\frac{\pi}{2}} \sin^{n-2} \theta d\theta$ for final A1)</p>	M1A1 M1 M1 A1 (5)

Question Number	Scheme	Marks
Q6 (a)	$\frac{x^2}{a^2} - \frac{(mx+c)^2}{b^2} = 1 \quad \text{and so} \quad b^2x^2 - a^2(mx+c)^2 = a^2b^2$ $\therefore (b^2 - a^2m^2)x^2 - 2a^2mcx - a^2(c^2 + b^2) = 0$ $\text{Or } (a^2m^2 - b^2)x^2 + 2a^2mcx + a^2(c^2 + b^2) = 0 \quad *$	M1
(b)	$(2a^2mc)^2 = 4(a^2m^2 - b^2) \times a^2(c^2 + b^2)$ $4a^4m^2c^2 = -4a^2(b^2c^2 + b^4 - a^2m^2c^2 - a^2m^2b^2)$ $c^2 = a^2m^2 - b^2 \quad \text{or} \quad a^2m^2 = b^2 + c^2 \quad *$	A1 (2)
(c)	<p>Substitute (1, 4) into $y = mx + c$ to give $4 = m + c$ and</p> <p>Substitute $a = 5$ and $b = 4$ into $c^2 = a^2m^2 - b^2$ to give $c^2 = 25m^2 - 16$</p> <p>Solve simultaneous equations to eliminate m or $c : (4 - m)^2 = 25m^2 - 16$</p> <p>To obtain $24m^2 + 8m - 32 = 0$</p> <p>Solve to obtain $8(3m + 4)(m - 1) = 0 \dots m = \dots$</p> $m = 1 \text{ or } -\frac{4}{3}$ <p>Substitute to get $c = 3$ or $\frac{16}{3}$</p> <p>Lines are $y = x + 3$ and $3y + 4x = 16$</p>	B1 M1 A1 M1 A1 M1 A1
		(7) [11]

Question Number	Scheme	Marks
Q7 (a)	<p>If the lines meet, $-1+3\lambda = -4+3\mu$ and $2+4\lambda = 2\mu$</p> <p>Solve to give $\lambda = 0$ ($\mu = 1$ but this need not be seen).</p> <p>Also $1-\lambda = \alpha$ and so $\alpha = 1$.</p> <p>(b) $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix} = -6\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ is perpendicular to both lines and hence to the plane</p> <p>The plane has equation $\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$, which is $-6x + 2y - 3z = -14$, i.e. $-6x + 2y - 3z + 14 = 0$.</p>	<p>M1</p> <p>M1 A1</p> <p>B1</p> <p>(4)</p> <p>M1 A1</p> <p>M1</p> <p>A1 o.a.e.</p> <p>(4)</p>
OR (b)	<p>Alternative scheme</p> <p>Use $(1, -1, 2)$ and $(\alpha, -4, 0)$ in equation $ax+by+cz+d=0$</p> <p>And third point so three equations, and attempt to solve</p> <p>Obtain $6x - 2y + 3z =$ $(6x - 2y + 3z) - 14 = 0$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>A1 o.a.e.</p> <p>(4)</p>
(c)	<p>$(\mathbf{a}_1 - \mathbf{a}_2) = \mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$</p> <p>Use formula $\frac{(\mathbf{a}_1 - \mathbf{a}_2) \cdot \mathbf{n}}{ \mathbf{n} } = \frac{(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \cdot (-6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})}{\sqrt{(36+4+9)}} = \left(\frac{-6}{7}\right)$</p> <p>Distance is $\frac{6}{7}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>(3)</p> <p>[11]</p>

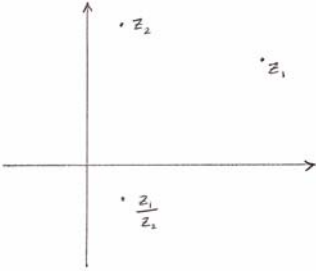
Question Number	Scheme	Marks
Q8 (a)	$\frac{dx}{d\theta} = -3\sin\theta, \quad \frac{dy}{d\theta} = 5\cos\theta$ <p>so $S = 2\pi \int 5\sin\theta \sqrt{(-3\sin\theta)^2 + (5\cos\theta)^2} d\theta$</p> $\therefore S = 2\pi \int 5\sin\theta \sqrt{9 - 9\cos^2\theta + 25\cos^2\theta} d\theta$ <p>Let $c = \cos\theta$, $\frac{dc}{d\theta} = -\sin\theta$, limits 0 and $\frac{\pi}{2}$ become 1 and 0</p> <p>So $S = k\pi \int_0^{\alpha} \sqrt{16c^2 + 9} dc$, where $k = 10$, and α is 1</p>	B1 M1 M1 M1 A1, A1 (6)
	(b)	<p>Let $c = \frac{3}{4}\sinh u$. Then $\frac{dc}{du} = \frac{3}{4}\cosh u$</p> <p>So $S = k\pi \int_?^? \sqrt{9\sinh^2 u + 9} \frac{3}{4}\cosh u du$</p> $= k\pi \int_?^? \frac{9}{4}\cosh^2 u du = k\pi \int_?^? \frac{9}{8}(\cosh 2u + 1) du$ $= k\pi \left[\frac{9}{16}\sinh 2u + \frac{9}{8}u \right]_0^d$ $= \frac{45\pi}{4} \left[\frac{20}{9} + \ln 3 \right] \quad \text{i.e. } \underline{117}$

June 2009
6674 Further Pure Mathematics FP1 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$x = -i$ is a root (Scored here or in (b)) Factor $(x+i)(x-i) = x^2 + 1$ $x^4 + 6x^3 + 26x^2 + 6x + 25 = (x^2 + 1)(x^2 + 6x + 25)$	1B1 2B1 1M1 1A1 (4)
(b)	$x = \frac{-6 \pm \sqrt{36 - 100}}{2}$ Solving quadratic: $x = -3 \pm 4i$	1M1 1A1 (2) [6]
(a)	1B1 CAO, $x = -i$, maybe seen in (b) 2B1 $x^2 + 1$ CAO 1M1 Getting the three term quadratic 1A1 CAO for correct quadratic	
(b)	1M1 Solving a three term quadratic to $x = \text{complex}$, correct formula used 1A1 CAO	

Question Number	Scheme	Marks
Q2	$m^2 + 6m + 10 = 0 \quad m = \frac{-6 \pm \sqrt{36 - 40}}{2} = -3 \pm i$ <p>C.F. $(x =) e^{-3t} (A \cos t + B \sin t)$</p> <p>P.I. $x = ke^{-4t}$</p> $\frac{dx}{dt} = -4ke^{-4t} \quad \frac{d^2x}{dt^2} = 16ke^{-4t}$ $16k - 24k + 10k = 1 \quad k = \frac{1}{2}$ <p>General solution: $x = e^{-3t} (A \cos t + B \sin t) + \frac{1}{2} e^{-4t}$</p>	<p>1B1</p> <p>1M1 1A1ft</p> <p>2B1</p> <p>2M1</p> <p>3M1 2A1</p> <p>3A1ft=3B1ft</p> <p>[8]</p>
	<p>1B1 CAO (may be implied)</p> <p>1M1 Correct 'shape' $e^{at} (A \cos bt + B \sin bt)$ accept alternative (single) variable here.</p> <p>No complex</p> <p>1A1ft condone their variables</p> <p>2B1 CAO</p> <p>2M1 Attempt at both, accept ke^{-at} ($a > 0$) derivatives here.</p> <p>3M1 Linear in k, to k =</p> <p>2A1 CAO</p> <p>3A1ft = 3B1ft but must be $x = f(t)$.</p>	

Question Number	Scheme	Marks
Q3 (a)	$r(r+2)(r+4) = r^3 + 6r^2 + 8r, \text{ so use } \sum r^3 + 6\sum r^2 + 8\sum r$ $= \frac{1}{4}n^2(n+1)^2 + 6\left(\frac{1}{6}n(n+1)(2n+1)\right) + 8\left(\frac{1}{2}n(n+1)\right)$ $= \frac{1}{4}n(n+1)\{n(n+1) + 4(2n+1) + 16\}$ $= \frac{1}{4}n(n+1)\{n^2 + 9n + 20\} = \frac{1}{4}n(n+1)(n+4)(n+5) \quad (*)$	1M1 1A1 2M1 2A1 3A1 (5)
(b)	$\sum_{21}^{30} = \sum_1^{30} - \sum_1^{20}$ $= \frac{1}{4}(30 \times 31 \times 34 \times 35) - \frac{1}{4}(20 \times 21 \times 24 \times 25) = 213675$	1M1 1A1 (2) [7]
(a)	<p>Alternative (induction):</p> $\frac{1}{4}k(k+1)(k+4)(k+5) + (k+1)(k+3)(k+5)$ <p style="text-align: right;">1M1 (Adding on (k+1)th term)</p> $= \frac{1}{4}(k+1)(k+5)(k^2 + 4k + 4k + 12)$ <p style="text-align: right;">2M1 Quadratic factor seen</p> $= \frac{1}{4}(k+1)(k+2)(k+5)(k+6)$ <p style="text-align: right;">1A1 cso</p> <p>Check for k = 1: Term = 15, Sum = $\frac{60}{4} = 15$</p> <p style="text-align: right;">1B1 cao</p> <p>Induction argument + conclusion 2A1 cao</p> <p>Q3 Notes</p> <p>(a) 1M1 Expand in terms of $\sum r^3, \sum r^2, \sum r$ 1A1 Correct substitution in correct expansion. 2M1 Factorisation, 3 term quadratic factor seen 2A1 a correct quadratic factor 3A1 cso</p> <p>(b) 1M1 allowed for $\sum_{21}^{30} = \sum_1^{30} - \sum_1^{19}$ or $\sum_{21}^{30} = \sum_1^{30} - \sum_1^{21}$ but must be used. 1A1 cao</p>	

Question Number	Scheme	Marks
Q4 (a) (b) (c) (d)	$z_2 = \frac{z_1}{1-i} = \frac{5+2pi}{1-i} \times \frac{1+i}{1+i}$ $\frac{(5-2p)+i(5+2p)}{2} = \left(\frac{5-2p}{2}\right) + i\left(\frac{5+2p}{2}\right)$ $\frac{5+2p}{5-2p} = 4 \qquad 5+2p = 20-8p \qquad p = \frac{3}{2}$ $ z_2 = \sqrt{1^2 + 4^2} = \sqrt{17} = 4.12$  <p style="text-align: center;">$\frac{z_1}{z_2}$</p> <p>For z_2</p> <p>For z_1 and z_2 ($z_1 = 5 + 3i$ and $z_2 = 1 + 4i$)</p>	1 M1 1A1,2A1 (3) 1M1 1A1ft (2) 1M1 1A1 (2) 1B1 2B1ft (2) [9]
(a) (c)	<p>Alternative:</p> $5 + 2pi = (1-i)(a+bi) \quad \text{and equate real and imaginary parts} \quad \text{M1}$ $(a+b=5 \text{ and } b-a=2p)$ <p>Alternative:</p> $ z_2 = \frac{ z_1 }{\sqrt{2}} = \frac{\sqrt{25+(2p)^2}}{\sqrt{2}} \quad \text{and substitute value for p.} \quad \text{M1}$ <p>Q4 Notes</p> <p>(a) 1M1 A correct method leading to coordinate 1A1 cao 2A1 cao</p> <p>(b) 1M1 linear equation in p, their Im/Re = 4 1A1ft from their (a)</p> <p>(c) 1M1 Pythagoras 1A1 cao (awrt 4.12)</p> <p>(d) 1B1 cao 2B1ft If points unlabelled withhold this mark, relative positions plausible</p>	

Question Number	Scheme	Marks
<p>Q5 (a)</p> <p>(b)</p> <p>(c)</p>	<p>$f(0.8) = \sin 1.6 - \ln 2.4 \quad (= 0.1241\dots)$</p> <p>$f(0.9) = \sin 1.8 - \ln 2.7 \quad (= -0.0194\dots)$</p> <p>Values correct (to 1 s.f.). Change of sign \Rightarrow Root</p> <p>$f'(x) = 2 \cos 2x, -\frac{1}{x}$</p> <p>$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.9 - \frac{-0.0194}{-1.5655}, = 0.888$</p> <p>$\frac{0.1241}{k} = \frac{0.0194}{0.1-k}$ (where root is approx. $0.8 + k$)</p> <p>$k = 0.086\dots$ $\alpha \approx 0.886$ (Allow awrt)</p>	<p>1M1</p> <p>1A1 (2)</p> <p>1B1, 2B1</p> <p>1M1 1A1 2A1 (5)</p> <p>M1 1A1ft</p> <p>2A1 (3)</p> <p>[10]</p>
	<p>Alternative for (c)</p> <p>$\frac{0.9 - \alpha}{0.0194} = \frac{\alpha - 0.8}{0.1241}$ M1 A1</p> <p>$0.11169 - 0.1241\alpha = 0.0194\alpha - 0.01552$</p> <p>$0.12721 = 0.1435\alpha$</p> <p>$\alpha \approx 0.886$ A1</p> <p>Q5 Notes</p> <p>(a) 1M1 Both evaluated 1A1 cao including conclusion statement</p> <p>(b) 1B1 $2 \cos 2x$ cao 2B1 $-1/x$ cao 1M1 Substituting values 1A1 cao 3 dp rounded or truncated 2A1 cao 0.888 gets both A marks</p> <p>(c) 1M1 Accept sign errors here, accept $f(0.8)$ and $f(0.9)$ 1A1ft their values from (a), signs correct. 2A1 cao 0.886 gets both A marks</p>	

Question Number	Scheme	Marks
<p>Q6 (a)</p> <p>(b)</p>	<p>Integrating factor $e^{\int \cot x dx} = e^{\ln(\sin x)} = \sin x$</p> <p>$y \sin x = \int \sin^2 x dx$ or $\frac{d}{dx}(y \sin x) = \sin^2 x$</p> <p>$\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \frac{x}{2} - \frac{\sin 2x}{4} (+C)$</p> <p>$y = \frac{2x - \sin 2x + C}{4 \sin x}$ (or equiv.)</p> <p>$y = 1$ at $x = \frac{\pi}{2}$: $1 = \frac{\pi + C}{4}$ $C = 4 - \pi$</p> <p>At $x = \frac{\pi}{4}$, $y = \frac{\frac{\pi}{2} - 1 + 4 - \pi}{4} = \frac{\sqrt{2}}{4} \left(3 - \frac{\pi}{2} \right) = \frac{(6 - \pi)\sqrt{2}}{8}$ (*)</p>	<p>1M1</p> <p>2M1 1A1</p> <p>3M1 2A1</p> <p>3A1 (6)</p> <p>1M1 1A1</p> <p>2M1 2A1 (4)</p> <p>[10]</p>
<p>(a)</p> <p>(b)</p>	<p>Alternative (special case):</p> <p>Multiply by $\sin x$ and integrate 'by inspection' M2</p> <p>Achieve $y \sin x = \int \sin^2 x dx$ or $\frac{d}{dx}(y \sin x) = \sin^2 x$ A1</p> <p>Note that other C values are possible,</p> <p>e.g. from $y = \frac{2x - \sin 2x}{4 \sin x} + \frac{C}{\sin x}$</p> <p>Q6 Notes</p> <p>(a) 1M1 Integrating factor found, condone sign error 2M1 One side correct 1A1 cao both sides correct 3M1 'RHS' in a form that can be integrated 2A1 'RHS' integrated cao 3A1 cao to $y =$, general solution</p> <p>(b) 1M1 Substitute to find their C 1A1 their C cao 2M1 substitute to find y 2A1 cso</p>	

Question Number	Scheme	Marks
<p>Q7 (a)</p> <p>(b)</p> $x + 2 = \frac{1}{x - 2} \quad x^2 - 4 = 1 \quad x = \sqrt{5}$ $x + 2 = \frac{1}{2 - x} \quad 4 - x^2 = 1 \quad x = \sqrt{3}$ $x < -\sqrt{3}, \quad \sqrt{3} < x < \sqrt{5}$	<p>Line, positive grad., intercepts (0, 2), (-2, 0) Curve, branch $x > 2$ Curve, branch $x < 2$</p> <p>Curve intercept $\left(0, \frac{1}{2}\right)$</p> <p>Asymptotes $x = 2$ and $y = 0$</p>	<p>1B1 2B1 3B1</p> <p>4B1</p> <p>1M1 1A1(6)</p> <p>1M1 1A1</p> <p>2M1 2A1</p> <p>1B1ft, 2B1ft (6) [12]</p>
	<p>Special case (a) for $y = \left \frac{1}{x + 2} \right$ allow 2B1 if both branches correct</p> <p>Q7 Notes</p> <p>(a) 1B1 cao intercepts clear 2B1 cao 3B1 cao 4B1 cao 1/2 indicated 1M1 One stated 1A1 both stated</p> <p>(b) 1M1 condone inequality here, seeking one critical value 1A1 finding 1st critical value, exact, but ignore signs 2M1 condone inequality here, seeking second critical value 2A1 finding 2nd critical value, exact, but ignore signs 1B1ft ft their values penalise \leq once only at first occurrence 2B1ft ft their values condone $x \neq 2$.</p>	

Question Number	Scheme	Marks
Q8 (a)	$r \sin \theta = \sin \theta + \sin \theta \cos \theta$ $\frac{d(r \sin \theta)}{d\theta} = \cos \theta + \cos 2\theta = \cos \theta + \cos^2 \theta - \sin^2 \theta$ $2 \cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \quad r = \frac{3}{2} \quad (*)$	1M1 1A1 2M1 2A1 (4)
(b)	$\frac{1}{2} \int r^2 d\theta = \frac{1}{2} \int (1 + 2 \cos \theta + \cos^2 \theta) d\theta$ $\int (1 + 2 \cos \theta + \cos^2 \theta) d\theta = \left[\theta + 2 \sin \theta + \frac{\sin 2\theta}{4} + \frac{\theta}{2} \right]$ $\left[\frac{3\theta}{2} + 2 \sin \theta + \frac{\sin 2\theta}{4} \right]_0^{\pi/3} = \frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} \quad \left(= \frac{\pi}{2} + \frac{9\sqrt{3}}{8} \right)$ $AH = r \sin \theta = \frac{3}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}, \quad PH = 2 - r \cos \theta = 2 - \frac{3}{2} \times \frac{1}{2} = \frac{5}{4}$ $\frac{1}{2} \left(2 + \frac{5}{4} \right) \frac{3\sqrt{3}}{4} \quad \left(= \frac{39\sqrt{3}}{32} \right)$ <p>Area of trapezium OAHP:</p> $\frac{39\sqrt{3}}{32} - \left(\frac{\pi}{4} + \frac{9\sqrt{3}}{16} \right) = \frac{21\sqrt{3}}{32} - \frac{\pi}{4}$ <p>Area of R:</p>	1 M1 2M1 1A1 3M1 1B1, 2B1 4M1 5M1 2A1 (9) [13]
Q8 Notes	<p>(a) 1M1 Finding $r \sin \theta$ 1A1 cao</p> <p>2M1 putting $\frac{d(r \sin \theta)}{d\theta} = 0$ to $\theta =$, accept substitution of θ to show derivative = 0 2A1 cso</p> <p>(b) 1M1 $\frac{1}{2} \int r^2 d\theta$ in terms of θ, expanded. 2M1 integrating, at least 1 trig term correctly handled 1A1 cao 3M1 substituting correct limits 1B1 $3\sqrt{3}/4$ cao careful, may be on diagram 2B1 $5/4$ or $3/4$ cao careful, may be on diagram</p> <p>4M1 Trapezium or $\left(\frac{1}{2} \times \frac{3}{4} \times \frac{3\sqrt{3}}{4} \right) + \left(\frac{5}{4} \times \frac{3\sqrt{3}}{4} \right) = \frac{9\sqrt{3}}{32} + \frac{15\sqrt{3}}{32} = \frac{39\sqrt{3}}{32}$</p> <p>5M1 Subtracting their integral and their trapezium 2A1 cao</p>	

June 2009
6675 Further Pure Mathematics FP2 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1	$\frac{dy}{dx} = 2 \times \operatorname{arsinh} 2x \times \frac{2}{\sqrt{(4x^2 + 1)}}$ $\text{At } x = \frac{1}{2}, \frac{dy}{dx} = \frac{4}{\sqrt{2}} \operatorname{arsinh} 1$ $= 2\sqrt{2} \ln(\sqrt{2} + 1)$ <p><i>Alternative</i></p> $\sinh y^{\frac{1}{2}} = 2x$ $\frac{1}{2} y^{-\frac{1}{2}} \cosh y^{\frac{1}{2}} \frac{dy}{dx} = 2$ $\sqrt{(1 + \sinh^2 y^{\frac{1}{2}})} \frac{dy}{dx} = 4y^{\frac{1}{2}}$ $\text{At } x = \frac{1}{2}, \sinh y^{\frac{1}{2}} = 1$ $\sqrt{(1 + 1)} \frac{dy}{dx} = 4 \operatorname{arsinh} 1$ $\frac{dy}{dx} = \frac{4}{\sqrt{2}} \operatorname{arsinh} 1$ $= 2\sqrt{2} \ln(\sqrt{2} + 1)$	<p>M1 A1</p> <p>M1 A1ft</p> <p>A1 (5)</p> <p>[5]</p> <p>M1 A1</p> <p>M1</p> <p>A1ft</p> <p>A1 (5)</p>
Q2 (a)	$b^2 = a^2(1 - e^2) \Rightarrow 8 = a^2 \left(1 - \frac{1}{2}\right) \Rightarrow a = 4$	<p>M1 A1 (2)</p>
(b)	<p>At S, $x = ae = 2\sqrt{2}$; at D, $y = 2\sqrt{2}$ two coordinates</p> <p>(SDS'D' is a square)</p> $A = 4 \times \frac{1}{2} \times 2\sqrt{2} \times 2\sqrt{2} = 16$	<p>B1</p> <p>M1 A1 (3)</p> <p>[5]</p>

Question Number	Scheme	Marks
Q3 (a)	$\int_0^1 (1-x)^n \cosh x \, dx = \left[(1-x)^n \sinh x \right]_0^1 + \int_0^1 n(1-x)^{n-1} \sinh x \, dx$ $= \int_0^1 n(1-x)^{n-1} \sinh x \, dx$ $= \left[n(1-x)^{n-1} \cosh x \right]_0^1 + \int_0^1 n(n-1)(1-x)^{n-2} \cosh x \, dx$ $= -n + n(n-1) \int_0^1 (1-x)^{n-2} \cosh x \, dx$ $I_n = n(n-1)I_{n-2} - n \quad *$	M1 A1 M1 M1 A1 (5)
(b)	$I_0 = \int_0^1 \cosh x \, dx = \left[\sinh x \right]_0^1 = \sinh 1 \left(= \frac{1}{2}(e - e^{-1}) \right)$ $I_2 = 2I_0 - 2$ $I_4 = 12I_2 - 4 = 24I_0 - 28$ $= 12e - \frac{12}{e} - 28$	B1 M1 M1 A1 (4) [9]
Q4 (a)	$\frac{dy}{dx} = 15 \cosh x - 17 \sinh x + 6$ $\frac{dy}{dx} = 0 \Rightarrow 15 \left(\frac{e^x + e^{-x}}{2} \right) - 17 \left(\frac{e^x - e^{-x}}{2} \right) + 6 = 0$ $e^{2x} - 6e^x - 16 = 0$ $(e^x - 8)(e^x + 2) = 0$ $x = 3 \ln 2$	B1 M1 M1 A1 M1 A1 (6)
(b)	$\frac{d^2y}{dx^2} = 15 \sinh x - 17 \cosh x$ $= -e^x - 16e^{-x} < 0 \quad (\text{for any real } x)$ $\Rightarrow \text{maximum}$ <p style="text-align: center;">Accept equivalent arguments or a sketch</p>	M1 M1 A1 (3) [9]

Question Number	Scheme	Marks
Q5	<p>Use of $S = 2\pi \int y \left(\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right)^{\frac{1}{2}} dt$</p> <p>$\int y \sqrt{(\dot{x}^2 + \dot{y}^2)} dt = \int 3t^2 \sqrt{(36t^4 + 36t^2)} dt$ $= \int 18t^3 \sqrt{(t^2 + 1)} dt$</p> <p>Let $u^2 = t^2 + 1$, $u \frac{du}{dt} = t$</p> <p>$\int t^3 \sqrt{(t^2 + 1)} dt = \int (u^2 - 1)u^2 du$ $= \left(\frac{u^5}{5} - \frac{u^3}{3} \right)$</p> <p>$\left[\left(\frac{u^5}{5} - \frac{u^3}{3} \right) \right]_1^{\sqrt{2}} = \frac{1}{15} (2\sqrt{2} - (-2))$ using correct limits</p> <p>Leading to $A = \frac{24\pi}{5} (\sqrt{2} + 1)$ * cso</p> <p><i>Alternative substitutions</i></p> <p>① Let $u = t^2 + 1$, $\frac{du}{dt} = 2t$</p> <p>$\int t^3 \sqrt{(t^2 + 1)} dt = \frac{1}{2} \int (u - 1)u^{\frac{1}{2}} du$ $= \frac{1}{2} \int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du = \frac{1}{2} \left(\frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} \right)$</p> <p>Using the limits $u = 1$ and $u = 2$</p> <p>Leading to $A = \frac{24\pi}{5} (\sqrt{2} + 1)$ * cso</p> <p>② Let $t = \sinh u$, $\frac{dt}{du} = \cosh u$</p> <p>$\int t^3 \sqrt{(t^2 + 1)} dt = \int \sinh^3 u \cosh^2 u du$ $= \int (\cosh^4 u - \cosh^2 u) \sinh u du = \frac{\cosh^5 u}{5} - \frac{\cosh^3 u}{3}$</p> <p>Using the limits $\cosh u = 1$ and $\cosh u = \sqrt{2}$</p> <p>Leading to $A = \frac{24\pi}{5} (\sqrt{2} + 1)$ * cso</p>	<p>B1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (9)</p> <p>[9]</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p>

Question Number	Scheme	Marks
Q6	$u = \cosh \theta \Rightarrow \frac{du}{d\theta} = \sinh \theta$ $I = \int \frac{u+1}{\sinh^2 \theta (u-1)^2} du$ $= \int \frac{u+1}{(u^2-1)(u-1)^2} du$ $= \int \frac{1}{(u-1)^3} du$ $= -\frac{1}{2(u-1)^2}$ <p>At $\theta = \ln 4$, $u = \frac{4 + \frac{1}{4}}{2} = \frac{17}{8}$; at $\theta = \ln 2$, $u = \frac{2 + \frac{1}{2}}{2} = \frac{5}{4}$ both</p> $\left[-\frac{1}{2(u-1)^2} \right]_{\frac{5}{4}}^{\frac{17}{8}} = 8 - \frac{32}{81} = \frac{616}{81}$	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1 (10)</p> <p>[10]</p>

Question Number	Scheme	Marks
Q7 (a)	$\frac{dy}{dx} = \frac{\cos x}{\sin x} (= \cot x)$ $\frac{dy}{dx} = \tan \psi = \cot x$ $\tan \psi = \tan\left(\frac{\pi}{2} - x\right) \Rightarrow \psi = \frac{\pi}{2} - x \quad *$	B1 M1 A1 (3)
(b)	$s = \int \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{1}{2}} dx = \int (1 + \cot^2 x)^{\frac{1}{2}} dx$ $= \int \operatorname{cosec} x dx$ $= -\ln(\operatorname{cosec} x + \cot x) (+C)$ $= -\ln(\sec \psi + \tan \psi) (+C)$ $\left(0, \frac{\pi}{4}\right) \Rightarrow 0 = -\ln(\sqrt{2} + 1) + C$ $s = \ln\left(\frac{\sqrt{2} + 1}{\sec \psi + \tan \psi}\right) \quad *$	M1 A1 A1 M1 M1 A1 (6)
(c)	$\frac{ds}{d\psi} = -\sec \psi$ $\psi = \frac{\pi}{6} \Rightarrow \rho = \left \frac{ds}{d\psi}\right = \frac{2}{\sqrt{3}}$	M1 M1 A1 (3)
[12]		
<i>Alternative to (c)</i>		
$\psi = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{3}$		
	$\text{At } x = \frac{\pi}{3}; \quad \frac{dy}{dx} = \cot x = \frac{1}{\sqrt{3}}, \quad \frac{d^2y}{dx^2} = -\operatorname{cosec}^2 x = -\frac{4}{3}$ $\rho = \frac{\left \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} \right }{\left \frac{d^2y}{dx^2} \right } = \frac{\left(1 + \frac{1}{3}\right)^{\frac{3}{2}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}}$	both M1 M1 A1 (3)

Question Number	Scheme	Marks
Q8 (a)	$\frac{dx}{dp} = 2ap, \frac{dy}{dp} = 2a; \frac{dy}{dx} = \frac{1}{p}$ $y - 2ap = -p(x - ap^2)$ $y + px = 2ap + ap^3 \quad *$	M1 A1 M1 A1 (4)
(b)	Eliminating x between $y^2 = 4ax$ and $y + px = 2ap + ap^3$ $y + \frac{py^2}{4a} = 2ap + ap^3$ $py^2 + 4ay - 8a^2p - 4a^2p^3 = 0$ $(y - 2ap)(py + 4a + 2ap^2) = 0$ At Q , $y = -\frac{4a + 2ap^2}{p} = -2a\left(\frac{2 + p^2}{p}\right) \quad *$	M1 A1 M1 A1 A1 (5)
(c)	At Q , $x = a\left(\frac{2 + p^2}{p}\right)^2$ $PQ^2 = \left(ap^2 - a\left(\frac{2 + p^2}{p}\right)^2 \right)^2 + \left(2ap + 2a\left(\frac{2 + p^2}{p}\right) \right)^2$ $= \frac{16a^2(p^2 + 1)^3}{p^4}$ $\frac{d}{dx}(PQ^2) = 16a^2 \left(\frac{6(p^2 + 1)^2 p^5 - (p^2 + 1)^3 \cdot 4p^3}{p^8} \right)$ $\frac{d}{dx}(PQ^2) = 0 \Rightarrow \frac{2(p^2 + 1)^2(p^2 - 2)}{p^5} = 0$ $p = (\pm)\sqrt{2}$ $PQ^2 = \frac{16a^2 \times 27}{4}$ $PQ_{\min} = 6\sqrt{3a} \quad *$	M1 A1 M1 M1 A1 M1 A1 (7)
		[16]

June 2009
6676 Further Pure Mathematics FP3 (legacy)
Mark Scheme

Question Number	Scheme	Marks
Q1	<p>At</p> $x = 0.1, y_1 = 0.1(0 \times 0 + 3) + 0 = 0.3$ $x = 0.2, y_2 = 0.1(0.1 \times 0.3^2 + 3) + 0.3$ $= 0.3009 + 0.3$ $= 0.6009$ $x = 0.3, y_3 = 0.1(0.2 \times 0.6009^2 + 3) + 0.6009$ $= 0.307221616\dots + 0.6009$ $= 0.908(121616\dots) \quad \text{Allow awrt } 0.908$	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">[5]</p>
Q2	<p>(a) $\mathbf{b} \times \mathbf{c} = 0\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$</p> <p>(b)</p> $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} = 0 + 5 + 0 = 5$ <p>(c) Area of triangle $OBC = \frac{1}{2} 5\mathbf{j} + 5\mathbf{k} = \frac{5}{2}\sqrt{2}$ oe</p> <p>(d) Volume of tetrahedron $= \frac{1}{6} \times 5 = \frac{5}{6}$</p>	<p>M1 A1 A1</p> <p style="text-align: right;">(3)</p> <p>M1 A1ft</p> <p style="text-align: right;">(2)</p> <p>M1 A1</p> <p style="text-align: right;">(2)</p> <p>B1 ft</p> <p style="text-align: right;">(1)</p> <p style="text-align: right;">[8]</p>

Question Number	Scheme	Marks
Q3 (a)	$\begin{vmatrix} 6-\lambda & 1 & -1 \\ 0 & 7-\lambda & 0 \\ 3 & -1 & 2-\lambda \end{vmatrix} = 0$ $\therefore (6-\lambda)((7-\lambda)(2-\lambda)-0)-1\times 0-1(0-3(7-\lambda))=0$ $\therefore (6-\lambda)(7-\lambda)(2-\lambda)+3(7-\lambda)=0$ $(7-\lambda)=0 \text{ verifies } \lambda=7 \text{ is an eigenvalue}$ <p>They may show $\lambda=7$ in the determinant (e.g. $-1(0-0)-1(0-0)-1(0-0)$)</p> $\therefore (7-\lambda)\{12-8\lambda+\lambda^2+3\}=0$ $\therefore (7-\lambda)\{\lambda^2-8\lambda+15\}=0$ <p>(NB $\therefore \lambda^3-15\lambda^2+71\lambda-105=0$)</p> $\therefore (7-\lambda)(\lambda-5)(\lambda-3)=0 \text{ and } 3 \text{ and } 5 \text{ are the other two eigenvalues}$	M1 M1 A1 M1 A1 (5)
(b)	$\begin{pmatrix} 6 & 1 & -1 \\ 0 & 7 & 0 \\ 3 & -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ or } \begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & 0 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $-x+y-z=0$ $(0=0)$ $3x-y-5z=0$ <p>Solves to obtain $x=3z$ and $y=4z$ ($3y=4x$) or equivalent</p> <p>Obtains eigenvector as $\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ (or multiple)</p>	M1 M1 A1 A1 (4) [9]

Question Number	Scheme	Marks
Q4 (a)	$\frac{d^2y}{dx^2} + 2 \times 2 + 1 = 1, \text{ and so } \frac{d^2y}{dx^2} = -4 \text{ at } x = 0.$ $y''' + \{(1 + y^2)y'' + 2y(y')(y')\} + y' = 2e^{2x}$ $y''' + (1+1)(-4) + 2 \times 1(2)(2) + 2 = 2, \text{ i.e. } y''' = 0$	B1 M1 {M1 A1} A1 B1 cso (6)
(b)	$y = 1, +2x(+\dots)$ $-\frac{4x^2}{2} + \frac{0x^3}{6} + \frac{40x^4}{24}$ $(-2x^2 + \frac{5x^4}{3})$	B1, B1 M1 A1 (4) [10]
Q5 (a)	$\cos 6\theta = \operatorname{Re}[(\cos \theta + i \sin \theta)^6]$ $(\cos \theta + i \sin \theta)^6 = c^6 + 6c^5is + 15c^4i^2s^2 + 20c^3i^3s^3 + 15c^2i^4s^4 + 6ci^5s^5 + i^6s^6$ $\cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$ $= c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$ $\cos 6\theta = c^6 - 15c^4 + 15c^6 + 15c^2(1-2c^2+c^4) - (1-3c^2+3c^4-c^6)$ $\cos 6\theta = 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1 *$	M1 M1 A1 M1 A1 (5)
(b)	$\cos 6\theta = \cos 2\theta \rightarrow 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1 = 2 \cos^2 \theta - 1$ $32 \cos^6 \theta - 48 \cos^4 \theta + 16 \cos^2 \theta = 0$ $16 \cos^2 \theta (2 \cos^4 \theta - 3 \cos^2 \theta + 1) = 0$ $(2 \cos^2 \theta - 1)(\cos^2 \theta - 1) = 0$ $\therefore \cos^2 \theta = 0, \frac{1}{2} \text{ or } 1 \text{ so } \cos \theta = 0, \pm \frac{1}{\sqrt{2}} \text{ or } \pm 1$ <p>Uses arccos on at least 3 different values</p> $\therefore \theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4} \text{ and } \pi$ <p>Decimals: Allow 0, 0.785, 1.57, 2.36, 3.14 (awrt) 3 correct solutions A1, all correct A1</p>	M1 A1 M1 M1 A1, A1 (6) [11]

Question Number	Scheme	Marks
Q6 (a)	<p>When $n = 1$ LHS = RHS = $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$. Result true for $n = 1$</p> <p>Assume result true for $n = k$ i.e. $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}^k = \begin{pmatrix} \cos k\theta & -\sin k\theta \\ \sin k\theta & \cos k\theta \end{pmatrix}$</p> <p>And multiply both sides by $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$</p> <p>Then $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}^{k+1} = \begin{pmatrix} \cos k\theta & -\sin k\theta \\ \sin k\theta & \cos k\theta \end{pmatrix} \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$</p> <p>$= \begin{pmatrix} \cos k\theta \cos \theta - \sin k\theta \sin \theta & -\cos k\theta \sin \theta - \sin k\theta \cos \theta \\ \sin k\theta \cos \theta + \cos k\theta \sin \theta & -\sin k\theta \sin \theta + \cos k\theta \cos \theta \end{pmatrix}$</p> <p>i.e. $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}^{k+1} = \begin{pmatrix} \cos(k+1)\theta & -\sin(k+1)\theta \\ \sin(k+1)\theta & \cos(k+1)\theta \end{pmatrix}$</p> <p>Conclude, that by induction result is true for all positive integers</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1 cso (5)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1 cso (5)</p> <p>[10]</p>
(b)	<p>When $n = 1$, $f(n) = 7 \times 5 - 3 = 32$, which is divisible by 16, so result true for $n = 1$</p> <p>Consider $f(k+1) - f(k) = (4k+7)5^{k+1} - (4k+3)5^k$</p> <p>$= 5^k(20k+35-4k-3)$</p> <p>$= 5^k(16k+32)$, which is divisible by 16</p> <p>If $f(k)$ is divisible by 16, then this implies $f(k+1)$ is also divisible by 16 Thus by induction $f(n)$ is divisible by 16 for all positive integers n.</p>	<p>B1 cso (5)</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1 cso (5)</p>

Question Number	Scheme	Marks
Q7 (a)	<p>If the lines meet, $-1+3\lambda = -4+3\mu$ and $2+4\lambda = 2\mu$</p> <p>Solve to give $\lambda = 0(\mu = 1)$</p> <p>Also $1-\lambda = \alpha$ and so $\alpha = 1$.</p>	M1 M1 A1 B1 (4)
(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 3 & 4 \\ 0 & 3 & 2 \end{vmatrix} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix}$ $\mathbf{r} \cdot \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ -3 \end{pmatrix} \cdot (e.g. \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}) = -14$ <p>Hence $-6x + 2y - 3z + 14 = 0$</p>	M1 A1 M1 A1 (4)
(c)	$\pm(\mathbf{a}_1 - \mathbf{a}_2) = \pm(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})$ $\frac{ \mathbf{a}_1 - \mathbf{a}_2 \cdot \mathbf{n} }{ \mathbf{n} } = \frac{ (\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \cdot (-6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) }{ -6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} } = \frac{ -6 - 6 + 6 }{\sqrt{6^2 + 2^2 + 3^2}}$ <p>Distance is $\frac{6}{7}$</p>	<div style="border: 1px solid black; width: 20px; height: 20px; display: inline-block; vertical-align: middle;"></div> M1 M1 A1 cso (3) [11]

Question Number	Scheme	Marks
Q8 (a)	$\sqrt{\{(x-3)^2 + y^2\}} = 2\sqrt{\{x^2 + (y-4)^2\}} \text{ or } (x-3)^2 + y^2 = 4\{x^2 + (y-4)^2\}$ $3x^2 + 3y^2 + 6x - 32y + 55 = 0$ $(x+1)^2 + (y - \frac{16}{3})^2 = \frac{100}{9}$ <p style="text-align: center;">Centre is $(-1, 16/3)$ and radius is $10/3$</p>	M1 A1 M1 A1,A1,A1 CSO (6)
	(b) $w = \frac{12}{z} \rightarrow z = \frac{12}{w}, \text{ and so } \left \frac{12}{w} - 3 \right = 2 \left \frac{12}{w} - 4i \right \quad \text{substituting for } z$ $ 3w - 12 = 2 4iw - 12 \quad \text{multiplication by } w \text{ or equivalent}$ $ w - 4 = \frac{8}{3} w + 3i \quad \text{obtains the locus of Q in the required form}$ <p style="text-align: center;">A2 if completely correct deduct 1 for each error on their a, k or b</p>	M1 M1 M1, A2, 1, 0 (5) [11]

June 2009
6677 Mechanics M1
Mark Scheme

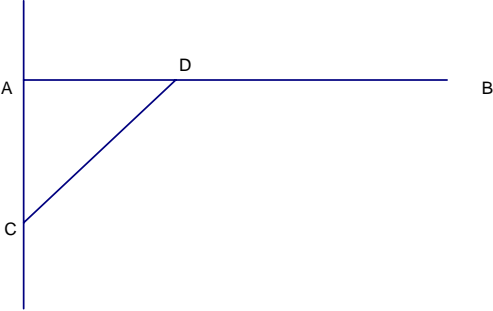
Question Number	Scheme	Marks
Q1	$45 = 2u + \frac{1}{2}a2^2 \Rightarrow 45 = 2u + 2a$ $165 = 6u + \frac{1}{2}a6^2 \Rightarrow 165 = 6u + 18a$ <p style="text-align: center;">eliminating either u or a</p> $u = 20 \text{ and } a = 2.5$	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 A1</p> <p style="text-align: right;">[7]</p>
Q2 (a) (b)	$\tan \theta = \frac{p}{2p} \Rightarrow \theta = 26.6^\circ$ $\mathbf{R} = (\mathbf{i} - 3\mathbf{j}) + (p\mathbf{i} + 2p\mathbf{j}) = (1 + p)\mathbf{i} + (-3 + 2p)\mathbf{j}$ <p>\mathbf{R} is parallel to $\mathbf{i} \Rightarrow (-3 + 2p) = 0$</p> $\Rightarrow p = \frac{3}{2}$	<p>M1 A1 (2)</p> <p>M1 A1</p> <p>DM1</p> <p>A1 (4)</p> <p style="text-align: right;">[6]</p>
Q3 (a) (b)	<p>For A:</p> $-\frac{7mu}{2} = 2m(v_A - 2u)$ $v_A = \frac{u}{4}$ <p>For B:</p> $\frac{7mu}{2} = m(v_B - -3u)$ $v_B = \frac{u}{2}$ <p>OR CLM:</p> $4mu - 3mu = 2m\frac{u}{4} + mv_B$ $v_B = \frac{u}{2}$	<p>M1 A1</p> <p>A1 (3)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>OR</p> <p>M1 A1</p> <p>A1 (3)</p> <p style="text-align: right;">[6]</p>

Question Number	Scheme	Marks
Q4	$0.5g\sin\theta - F = 0.5a$ $F = \frac{1}{3}R \text{ seen}$ $R = 0.5g\cos\theta$ <p>Use of $\sin\theta = \frac{4}{5}$ or $\cos\theta = \frac{3}{5}$ or decimal equiv or decimal angle e.g 53.1° or 53°</p> $a = \frac{3g}{5} \text{ or } 5.88 \text{ m s}^{-2} \text{ or } 5.9 \text{ m s}^{-2}$	M1 A1 A1 B1 M1 A1 B1 DM1 A1 [9]
Q5	$F = P\cos 50^\circ$ $F = 0.2R \text{ seen or implied.}$ $P\sin 50^\circ + R = 15g$ <p>Eliminating R; Solving for P ; $P = 37 \text{ (2 SF)}$</p>	M1 A1 B1 M1 A1 A1 DM1;D M1; A1 [9]
Q6	<p>(a) For whole system: $1200 - 400 - 200 = 1000a$</p> $a = 0.6 \text{ m s}^{-2}$ <p>(b) For trailer: $T - 200 = 200 \times 0.6$</p> $T = 320 \text{ N}$ <p>OR: For car: $1200 - 400 - T = 800 \times 0.6$</p> $T = 320 \text{ N}$ <p>(c) For trailer: $200 + 100 = 200f$ or $-200f$</p> $f = 1.5 \text{ m s}^{-2} \text{ (-1.5)}$ <p>For car: $400 + F - 100 = 800f$ or $-800f$</p> $F = 900$ <p>(N.B. For both: $400 + 200 + F = 1000f$)</p>	M1 A1 A1 (3) M1 A1 ft A1 OR: M1 A1 ft A1 (3) M1 A1 A1 M1 A2 A1 (7) [13]

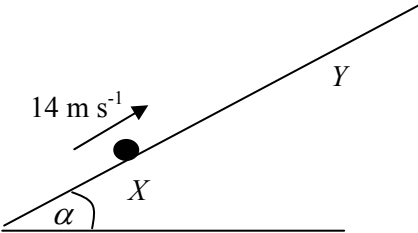
Question Number	Scheme	Marks
Q7 (a)	$M(Q), 50g(1.4 - x) + 20g \times 0.7 = T_p \times 1.4$ $T_p = 588 - 350x \quad \text{Printed answer}$	M1 A1 A1 (3)
(b)	$M(P), 50gx + 20g \times 0.7 = T_Q \times 1.4 \quad \text{or} \quad R(\uparrow), T_p + T_Q = 70g$ $T_Q = 98 + 350x$	M1 A1 A1 (3)
(c)	$\text{Since } 0 < x < 1.4, \quad 98 < T_p < 588 \text{ and } 98 < T_Q < 588$	M1 A1 A1 (3)
(d)	$98 + 350x = 3(588 - 350x)$ $x = 1.19$	M1 DM1 A1 (3) [12]
Q8 (a)	$ \mathbf{v} = \sqrt{1.2^2 + (-0.9)^2} = 1.5 \text{ m s}^{-1}$	M1 A1 (2)
(b)	$(\mathbf{r}_H =) 100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j}) \text{ m}$	M1 A1 (2)
(c)	$(\mathbf{r}_K =) 9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j}) \text{ m}$	M1 A1
(d)	$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j} \text{ m} \quad \text{Printed Answer}$ <p>Meet when $\overrightarrow{HK} = \mathbf{0}$</p> $(9 - 0.45t) = 0 \quad \text{and} \quad (2.7t - 54) = 0$ $t = 20 \text{ from both equations}$ $\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j}) \text{ m}$	M1 A1 A1 DM1 A1 cso (5) [13]

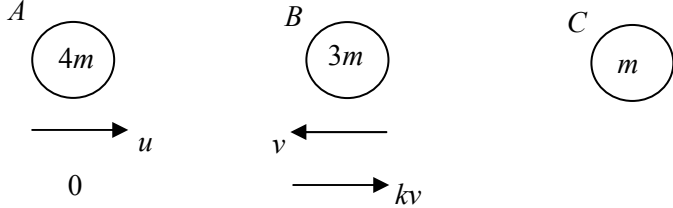
June 2009
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
Q1	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $5\mathbf{i} - 3\mathbf{j} = \frac{1}{4}\mathbf{v} - \frac{1}{4}(3\mathbf{i} + 7\mathbf{j})$ $\mathbf{v} = 23\mathbf{i} - 5\mathbf{j}$ $ \mathbf{v} = \sqrt{23^2 + 5^2} = 23.5$	<p>M1A1</p> <p>A1</p> <p>M1A1</p> <p style="text-align: right;">[5]</p>
Q2	<p>(a)</p> $\frac{dv}{dt} = 8 - 2t$ $8 - 2t = 0$ $\text{Max } v = 8 \times 4 - 4^2 = 16 \text{ (ms}^{-1}\text{)}$ <p>(b)</p> $\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+c)$ <p>($t=0$, displacement = 0 $\Rightarrow c=0$)</p> $4T^2 - \frac{1}{3}T^3 = 0$ $T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$ $T = 12 \text{ (seconds)}$	<p>M1</p> <p>M1</p> <p>M1A1</p> <p style="text-align: right;">(4)</p> <p>M1A1</p> <p>DM1</p> <p>DM1</p> <p>A1</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">[9]</p>
Q3	<p>(a) Constant $v \Rightarrow$ driving force = resistance $\Rightarrow F=120 \text{ (N)}$ $\Rightarrow P=120 \times 10 = 1200\text{W}$</p> <p>(b) Resolving parallel to the slope, zero acceleration: $\frac{P}{v} = 120 + 300g \sin \theta (= 330)$ $\Rightarrow v = \frac{1200}{330} = 3.6 \text{ (ms}^{-1}\text{)}$</p>	<p>M1</p> <p>M1</p> <p style="text-align: right;">(2)</p> <p>M1A1A1</p> <p>A1</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;">[6]</p>

Question Number	Scheme	Marks
<p>Q4 (a)</p>  <p>(b)</p>	<p>Taking moments about A:</p> $3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$ $T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$ $\leftarrow \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$ $\uparrow \pm V + \frac{T}{\sqrt{2}} = 3g \quad (\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7N)$ $\Rightarrow R = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$ at angle $\tan^{-1} \frac{1}{3} = 18.4^\circ$ (0.322 radians) below the line of BA 161.6° (2.82 radians) below the line of AB (108.4° or 1.89 radians to upward vertical)	<p>M1A1A1</p> <p>A1</p> <p>(4)</p> <p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>M1A1</p> <p>(7) [11]</p>
<p>Q5 (a)</p> <p>(b)</p>	<p>Ratio of areas triangle:sign:rectangle = 1 : 5 : 6 (1800:9000:10800) Centre of mass of the triangle is 20cm down from AD (seen or implied)</p> $\Rightarrow 6 \times 45 - 1 \times 20 = 5 \times \bar{y}$ $\bar{y} = 50cm$ Distance of centre of mass from AB is 60cm	<p>B1</p> <p>B1</p> <p>M1A1</p> <p>A1</p> <p>(5)</p> <p>B1</p> Required angle is $\tan^{-1} \frac{60}{50}$ (their values) $= 50.2^\circ$ (0.876 rads)

Question Number	Scheme	Marks
Q6 (a)	$\rightarrow x = u \cos \alpha t = 10$	M1A1
	$\uparrow y = u \sin \alpha t - \frac{1}{2}gt^2 = 2$	M1A1
	$\Rightarrow t = \frac{10}{u \cos \alpha}$	
	$2 = u \sin \alpha \times \frac{10}{u \cos \alpha} - \frac{g}{2} \times \frac{100}{u^2 \cos^2 \alpha}$	M1
	$= 10 \tan \alpha - \frac{50g}{u^2 \cos^2 \alpha}$ (given answer)	A1
		(6)
(b)	$2 = 10 \times 1 - \frac{100g \times 2}{2u^2 \times 1}$	M1A1
	$u^2 = \frac{100g}{8}, u = \sqrt{\frac{100g}{8}} = 11.1 \text{ (m s}^{-1}\text{)}$	A1
	$\frac{1}{2}mu^2 = m \times 9.8 \times 2 + \frac{1}{2}mv^2$	M1A1
	$v = 9.1 \text{ms}^{-1}$	A1
		(6) [12]

Question Number	Scheme	Marks
Q7 (a)	 <p>KE at X = $\frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 14^2$</p> <p>GPE at Y = $mgd \sin \alpha \left(= 2 \times g \times d \times \frac{7}{25} \right)$</p> <p>Normal reaction $R = mg \cos \alpha$</p> <p>Friction = $\mu \times R = \frac{1}{8} \times 2g \times \frac{24}{25}$</p> <p>Work Energy: $\frac{1}{2}mv^2 - mgd \sin \alpha = \mu \times R \times d$ or equivalent</p> $196 = \frac{14gd}{25} + \frac{6gd}{25} = \frac{20gd}{25}$ $d = 25 \text{ m}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1A1</p> <p>A1</p> <p>(7)</p>
(b)	<p>Work Energy</p> <p>First time at X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2$</p> <p>Work done = $\mu \times R \times 2d = \frac{1}{8} \times 2g \times \frac{24}{25} \times 2d$</p> <p>Return to X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2 - \frac{1}{8} \times 2g \times \frac{24}{25} \times 50$</p> $v = 8.9 \text{ ms}^{-1} \quad (\text{accept } 8.85 \text{ ms}^{-1})$ <p>OR: Resolve parallel to XY to find the acceleration and use of $v^2 = u^2 + 2as$</p> $2a = 2g \sin \alpha - F_{\max} = 2g \times \frac{7}{25} - \frac{6g}{25} = \frac{8g}{25}$ $v^2 = (0+)^2 + 2 \times a \times s = 8g; v = 8.9 \quad (\text{accept } 8.85 \text{ ms}^{-1})$	<p>M1A1</p> <p>DM1A1</p> <p>(4)</p> <p>M1A1</p> <p>DM1;A1</p> <p>[11]</p>

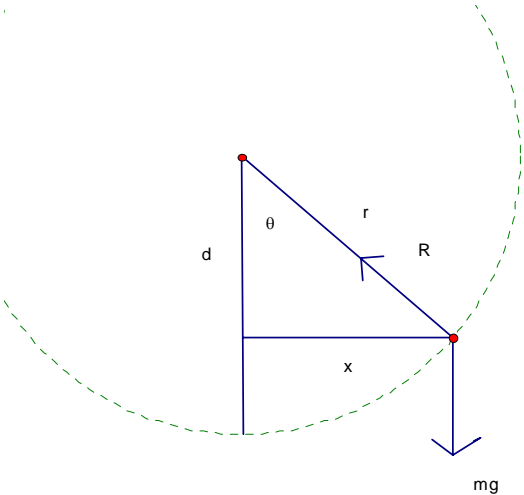
Question Number	Scheme	Marks
Q8 (a)	<div style="text-align: center; margin-bottom: 20px;">  </div> <p>Conservation of momentum: $4mu - 3mv = 3mkv$</p> <p>Impact law: $kv = \frac{3}{4}(u + v)$</p> <p>Eliminate k:</p> $4mu - 3mv = 3m \times \frac{3}{4}(u + v)$ $u = 3v \text{ (Answer given)}$ <p>(b) $kv = \frac{3}{4}(3v + v), k = 3$</p> <p>(c) Impact law: $(kv + 2v)e = v_C - v_B$ ($5ve = v_C - v_B$) Conservation of momentum: $3 \times kv - 1 \times 2v = 3v_B + v_C$ ($7v = 3v_B + v_C$) Eliminate v_C: $v_B = \frac{v}{4}(7 - 5e) > 0$ hence no further collision with A.</p>	<p>M1A1</p> <p>M1A1</p> <p>DM1</p> <p>A1</p> <p>(6)</p> <p>M1,A1</p> <p>(2)</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p>(4)</p> <p>[12]</p>

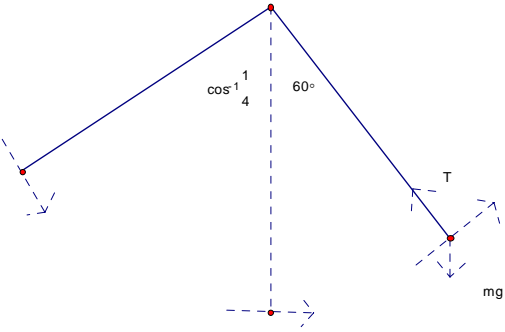
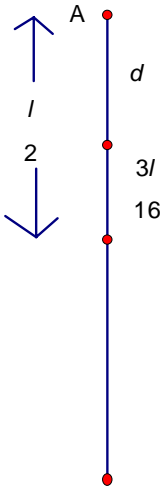
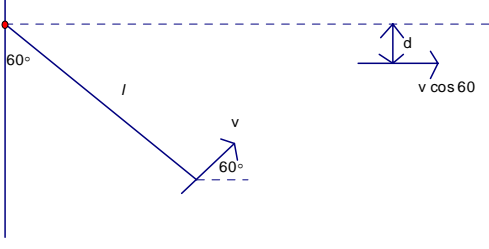
June 2009
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks												
Q1 (a)	<p style="margin-left: 20px;">Resolving vertically: $2T \cos \theta = W$</p> <p style="margin-left: 20px;">Hooke's Law: $T = \frac{80 \times 3.5}{4}$ $W = 84\text{N}$</p>	M1A2,1,0 M1A1 A1												
(b)	<p>$EPE = 2 \times \frac{80 \times 3.5^2}{2 \times 4}, = 245$ (or awrt 245)</p> <p>(alternative $\frac{80 \times 7^2}{16} = 245$)</p>	M1A1ft,A1												
Q2 (a)	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 20%;">Object</td> <td style="width: 20%;">Mass</td> <td style="width: 60%;">c of m above base</td> </tr> <tr> <td>Cone</td> <td>m</td> <td>$2h+3h$</td> </tr> <tr> <td>Base</td> <td>$3m$</td> <td>h</td> </tr> <tr> <td>Marker</td> <td>$4m$</td> <td>d</td> </tr> </table> <p>$m \times 5h + 3m \times h = 4m \times d$</p> <p>$d = 2h$</p>	Object	Mass	c of m above base	Cone	m	$2h+3h$	Base	$3m$	h	Marker	$4m$	d	B1(ratio masses) B1(distances)
Object	Mass	c of m above base												
Cone	m	$2h+3h$												
Base	$3m$	h												
Marker	$4m$	d												
(b)	<p style="margin-left: 20px;">$\frac{r}{d} = \frac{1}{12}$</p> <p style="margin-left: 20px;">$6r = h$</p>	M1A1ft A1 M1A1ft A1												

[9]

[8]

Question Number	Scheme	Marks
<p>Q3 (a)</p> <p>(b)</p>	 $\leftrightarrow R \sin \theta = mx\omega^2$ $R \times \frac{x}{r} = mx \times \frac{3g}{2r}$ $R = \frac{3mg}{2}$ $\downarrow R \cos \theta = mg$ $\frac{3mg}{2} \times \frac{d}{r} = mg$ $d = \frac{2}{3}r$	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>[8]</p>
<p>Q4 (a)</p> <p>(b)</p>	$\text{Volume} = \int_{\frac{1}{4}}^1 \pi y^2 dx = \int_{\frac{1}{4}}^1 \pi \frac{1}{x^4} dx$ $= \left[\pi \times \frac{-1}{3x^3} \right]_{\frac{1}{4}}^1$ $= \pi \left(\frac{-1}{3} + \frac{64}{3} \right) = 21\pi \quad *$ $21\pi \bar{x} = \rho \int \pi y^2 x dx = \rho \int \pi \frac{1}{x^4} x dx$ $21\pi \bar{x} = \pi \left[\frac{-1}{2x^2} \right]_{\frac{1}{4}}^1$ $\bar{x} = \frac{1}{21} \left(\frac{-1}{2} + \frac{16}{2} \right) = \frac{5}{14} \quad \text{or awrt } 0.36$ <p>$\bar{y} = 0$ by symmetry</p>	<p>M1A1</p> <p>A1ft</p> <p>A1</p> <p>M1A1</p> <p>A1ft</p> <p>A1</p> <p>B1</p> <p>[9]</p>

Question Number	Scheme	Marks
Q5 (a)	 <p>Energy: $\left(\frac{1}{2}mv^2 + \right)mgl\left(\cos\theta - \frac{1}{4}\right) = \frac{1}{2}mv^2$ Resolving: $T - mg \cos\theta = \frac{mv^2}{l}$ Eliminate v^2: $T = mg \cos\theta + \frac{1}{l}\left(2mgl\left(\cos\theta - \frac{1}{4}\right)\right)$ $T = 3mg \cos\theta - \frac{mg}{2} *$ </p>	M1A1 M1A1
(b)	 <p>$\theta = 60^\circ \Rightarrow mv^2 = 2mgl\left(\frac{1}{2} - \frac{1}{4}\right)$ $\Rightarrow v^2 = \frac{gl}{2}$</p> <p>vertical motion under gravity: $\uparrow 0 = (v \cos 30^\circ)^2 - 2gs$ $0 = \frac{gl}{2} \times \frac{3}{4} - 2gs \Rightarrow s = \frac{3l}{16}$</p> <p>Distance below A = $\frac{l}{2} - \frac{3l}{16} = \frac{5l}{16}$</p>	M1 M1 A1 M1A1
Alternative for end of (b) using energy	 <p>$\frac{1}{2}mv^2 - mgl \cos 60 = \frac{1}{2}m(v \cos 60)^2 - mgd$ $\frac{gl}{4} - \frac{gl}{2} = \frac{gl}{4} \times \frac{1}{4} - gd$ $d = \frac{1 - 4 + 8}{16}l = \frac{5l}{16}$</p>	M1A1 M1 A1

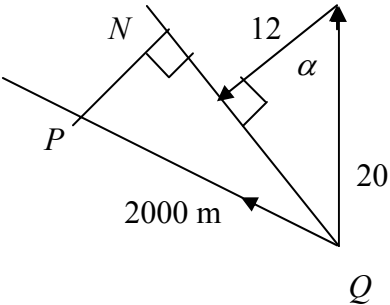
[11]

Question Number	Scheme	Marks
Q6 (a)	<p>At max v, driving force = resistance</p> $\text{Driving force} = \frac{80}{v}$ $\Rightarrow \frac{80}{20} = k \times 20^2 \Rightarrow k = \frac{1}{100}$ $F = ma \Rightarrow 100a = \frac{80}{v} - kv^2 \quad \left(= \frac{8000 - v^3}{100v} \right)$ $* \Rightarrow v \frac{dv}{dx} = \frac{8000 - v^3}{10000v} *$ <p>(b)</p> $\int_4^8 \frac{10000v^2}{8000 - v^3} dv = \int_0^D 1 dx$ $D = \left[-\frac{10000}{3} \ln 8000 - v^3 \right]_4^8$ $= \left(-\frac{10000}{3} \ln \frac{7488}{7936} \right) = 193.7 \dots \approx 194 \text{ m (accept 190)}$ <p>(c)</p> $\frac{dv}{dt} = \frac{8000 - v^3}{10000v} \Rightarrow \int_0^T 1 dt = \int_4^8 \frac{10000v}{8000 - v^3} dv$ $\Rightarrow T \approx \frac{1}{2} \times 2 \times 10000 \times \left\{ \frac{4}{7936} + \frac{2 \times 6}{7784} + \frac{8}{7488} \right\}$ $\Rightarrow T (= 31.1409 \dots) \approx 31$	<p>B1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>[14]</p>

Question Number	Scheme	Marks
<p>Q7 (a)</p>	<div style="text-align: center;"> </div> <p>Hooke's law: Equilibrium $\Rightarrow \frac{16(d-2)}{2} = \frac{12(4-d)}{1}$ $\Rightarrow d = 3.2$ so extensions are 1.2m and 0.8m.</p> <p>(b) If the particle is displaced distance x towards B then $-m\ddot{x} = \frac{16(1.2+x)}{2} - \frac{12(0.8-x)}{1} (= 20x)$ $\Rightarrow \ddot{x} = -40x$ or $\ddot{x} = -\frac{20}{m}$ (\Rightarrow SHM)</p> <p>(c) $T = \frac{2\pi}{\sqrt{40}}$ $a = \frac{\sqrt{10}}{\text{their } \omega}$ $x = a \sin \omega t$ their a, their ω $\frac{1}{4} = \frac{1}{2} \sin \sqrt{40}t$ $\sqrt{40}t = \frac{\pi}{6} (\Rightarrow t = \frac{\pi}{6\sqrt{40}})$</p> <p>Proportion $\frac{4t}{T} = \frac{4\pi}{6\sqrt{40}} \times \frac{\sqrt{40}}{2\pi} = \frac{1}{3}$</p>	<p>M1A1A1</p> <p>A1 A1</p> <p>M1A1ft A1ft</p> <p>A1</p> <p>B1ft</p> <p>B1ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1A1</p> <p style="text-align: right;">[16]</p>

June 2009
6680 Mechanics M4
Mark Scheme

Question Number	Scheme	Marks
Q1	CLM along plane: $v \cos 30^\circ = u \cos 45^\circ$ $v = u \sqrt{\frac{2}{3}}$ Fraction of KE Lost = $\frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \frac{\frac{1}{2}mu^2 - \frac{1}{2}m\frac{2}{3}u^2}{\frac{1}{2}mu^2} = \frac{1}{3}$	M1 A1 A1 M1 M1 A1 [6]
Q2	$-mg - mkv^2 = ma$ $-(g + kv^2) = v \frac{dv}{dx}$ $\pm \int_0^x dx = \int_{\sqrt{\frac{g}{k}}}^{\frac{1}{2}\sqrt{\frac{g}{k}}} \frac{-v dv}{(g + kv^2)}$ $X = \frac{1}{2k} \left[\ln(g + kv^2) \right]_{\frac{1}{2}\sqrt{\frac{g}{k}}}^{\sqrt{\frac{g}{k}}}$ $= \frac{1}{2k} \left(\ln 2g - \ln \frac{5g}{4} \right)$ $= \frac{1}{2k} \ln \frac{8}{5}$	M1 A1 M1 DM1 A1 (both previous) M1 A1 M1 A1 [9]

Question Number	Scheme	Marks
Q3 (a)	 <p style="text-align: center;">$\cos \alpha = \frac{12}{20}$</p> <p style="text-align: center;">Bearing is $180^\circ + \alpha = 233^\circ$ (nearest degree)</p> <p>(b) $PN = 2000\cos(135^\circ - \alpha) = 200\sqrt{2}$ m or decimal equivalent</p> <p>(c) Time to closest approach = $\frac{\sqrt{20^2 - 12^2}}{\sqrt{20^2 - 12^2}}$</p> <p style="text-align: center;">$= \frac{2000\sin(135^\circ - \alpha)}{16}$</p> <p style="text-align: center;">Distance moved by Q = their $t \times 12$</p> <p style="text-align: center;">$= 1050\sqrt{2}$ m or decimal equivalent</p>	<p>M1</p> <p>M1 A1</p> <p>A1</p> <p style="text-align: right;">(4)</p> <p>M1A1ft A1</p> <p style="text-align: right;">(3)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">[12]</p>

Question Number	Scheme	Marks
Q4 (a)	$V = -mg2a \sin 2\theta - \frac{7}{20}mg(L - 4a \sin \theta)$ $= \frac{1}{5}mga(7 \sin \theta - 10 \sin 2\theta) - \frac{7}{20}mgL$	M1 B1 A1
		A1
(b)	$\frac{dV}{d\theta} = \frac{1}{5}mga(7 \cos \theta - 20 \cos 2\theta)$ $\frac{1}{5}mga(7 \cos \theta - 20 \cos 2\theta) = 0$ $7 \cos \theta - 20(2 \cos^2 \theta - 1) = 0$ $40 \cos^2 \theta - 7 \cos \theta - 20 = 0$ $(5 \cos \theta - 4)(8 \cos \theta + 5) = 0$ $\cos \theta = \frac{4}{5} \text{ or } (\cos \theta = -\frac{5}{8} \Rightarrow 2\theta > 180^\circ)$	(4)
		M1 A1
		DM1
		DM1
		A1
(c)	$\frac{d^2V}{d\theta^2} = \frac{1}{5}mga(-7 \sin \theta + 40 \sin 2\theta)$ $= \frac{1}{5}mga(-7 \sin \theta + 80 \sin \theta \cos \theta)$	DM1 A1 DM1
	When $\cos \theta = \frac{4}{5}$,	(8)
	$\frac{d^2V}{d\theta^2} = \frac{1}{5}mga\left(\frac{-21}{5} + 80 \times \frac{3}{5} \times \frac{4}{5}\right) = \frac{171}{25}mga$	M1 A1
	> 0 therefore stable	M1
		A1 cso
		(4) [16]

Question Number	Scheme	Marks
Q5 (a)	CLM: $2(\mathbf{i} + 2\mathbf{j}) + -2\mathbf{i} = 2\mathbf{j} + \mathbf{v}$ $\mathbf{v} = 2\mathbf{j} \text{ m s}^{-1}$	M1 A1 A1 (3)
(b)	$\mathbf{I} = 2(\mathbf{j} - (\mathbf{i} + 2\mathbf{j}))$ $= (-2\mathbf{i} - 2\mathbf{j}) \text{ Ns}$ Since \mathbf{I} acts along l.o.c.c. , l.o.c.c is parallel to $\mathbf{i} + \mathbf{j}$	M1 A1 A1 B1 (4)
(c)	Before $A:$ $(\mathbf{i} + 2\mathbf{j}) \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{3}{\sqrt{2}}$ $B:$ $-2\mathbf{j} \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{-2}{\sqrt{2}}$ After $A:$ $\mathbf{j} \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{1}{\sqrt{2}}$ $B:$ $2\mathbf{j} \cdot \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{2}{\sqrt{2}}$	M1 A3
	NIL: $e = \frac{\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - \frac{-2}{\sqrt{2}}} = \frac{1}{5}$	DM1 A1 (6) [13]

Question Number	Scheme	Marks
Q6 (a)	$(\rightarrow), T = m\ddot{y}$ <p>Hooke's Law:</p> $T = \frac{2mn^2ax}{2a} = mn^2x$ $x + y = \frac{1}{2}ft^2$ $\dot{x} + \dot{y} = ft$ $\ddot{x} + \ddot{y} = f$ <p>so, $(\rightarrow), mn^2x = m\ddot{y} = m(f - \ddot{x})$</p> $\ddot{x} + n^2x = f^{**}$	<p>M1</p> <p>B1</p> <p>B2</p> <p>DM1</p> <p>A1</p>
(b)	<p>C.F. : $x = A \cos nt + B \sin nt$</p> <p>P.I. : $x = \frac{f}{n^2}$</p> <p>Gen solution: $x = A \cos nt + B \sin nt + \frac{f}{n^2}$</p> $\dot{x} = -An \sin nt + Bn \cos nt$ $t = 0, x = 0 \Rightarrow A = -\frac{f}{n^2}$ $t = 0, \dot{x} = 0 \Rightarrow B = 0$ $x = \frac{f}{n^2}(1 - \cos nt)$	<p>(6)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1 A1ft</p> <p>M1 A1</p> <p>A1</p>
(c)	$\dot{x} = 0 \Rightarrow nt = \pi$ $x_{\max} = \frac{f}{n^2}(1 - (-1)) = \frac{2f}{n^2}$	<p>(8)</p> <p>M1</p> <p>M1 A1</p>
(d)	$\dot{y} = ft - \dot{x}$ $= f \frac{\pi}{n} - 0 = \frac{f\pi}{n}$	<p>M1</p> <p>A1</p> <p>(2)</p> <p>[19]</p>

June 2009
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
Q1	$\pm(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$ $((4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})) \cdot (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = \frac{1}{2}3v^2$ $12 = v$ $\mathbf{v} = \frac{12}{\sqrt{8^2 + (-4)^2 + 8^2}}(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$ $\mathbf{v} = (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) \text{ ms}^{-1}$	B1 M1 A1 f.t. A1 M1 DM1 A1 [7]
Q2	<p>C.F. is $\mathbf{r} = \mathbf{A} \cos 2t + \mathbf{B} \sin 2t$</p> <p>P.I. is $\mathbf{r} = \mathbf{p}e^{2t}$</p> $\dot{\mathbf{r}} = 2\mathbf{p}e^{2t}$ $\ddot{\mathbf{r}} = 4\mathbf{p}e^{2t}$ $4\mathbf{p}e^{2t} + 4\mathbf{p}e^{2t} = \mathbf{j}e^{2t}$ <p>so, (PI is) $\mathbf{r} = \frac{1}{8}\mathbf{j}e^{2t}$</p> <p>GS is $\mathbf{r} = \mathbf{A} \cos 2t + \mathbf{B} \sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$</p> $t = 0, \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{i} + \mathbf{j} = \mathbf{A} + \frac{1}{8}\mathbf{j} \Rightarrow \mathbf{i} + \frac{7}{8}\mathbf{j} = \mathbf{A}$ $\dot{\mathbf{r}} = -2\mathbf{A} \sin 2t + 2\mathbf{B} \cos 2t + \frac{1}{4}\mathbf{j}e^{2t}$ $t = 0, \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow 2\mathbf{i} = 2\mathbf{B} + \frac{1}{4}\mathbf{j} \Rightarrow \mathbf{i} - \frac{1}{8}\mathbf{j} = \mathbf{B}$ $\mathbf{r} = (\mathbf{i} + \frac{7}{8}\mathbf{j}) \cos 2t + (\mathbf{i} - \frac{1}{8}\mathbf{j}) \sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	B1 B1 B1 ft M1 A1 A1 ft DM1 A1 M1A1 A1 [11]

Question Number	Scheme	Marks
Q3 (a)	$mv = (m + \delta m)(v + \delta v) - (-\delta m)(c - v)$ $mv = mv + m\delta v + v\delta m + c\delta m - v\delta m$ $-m\delta v = c\delta m$ $\frac{dv}{dm} = -\frac{c}{m} *$	M1 A2 DM1 A1 (5)
(b)	$\frac{dm}{dt} = -m_0 k$ $\frac{dv}{dt} = \frac{dv}{dm} \times \frac{dm}{dt}$ $= -\frac{c}{m} \times -m_0 k$ $= \frac{cm_0 k}{m_0(1-kt)}$ $= \frac{ck}{(1-kt)}$	B1 M1 DM1 A1 (4) [9]

Question Number	Scheme	Marks
Q4 (a)	$\delta m = \frac{2Mx\delta x}{a^2}$ $\delta I = \frac{1}{3} \frac{2Mx\delta x}{a^2} (2x)^2$ $I = \int_0^a \frac{8Mx^3 dx}{3a^2}$ $= \frac{8M}{3a^2} \left[\frac{x^4}{4} \right]_0^a$ $= \frac{2}{3} Ma^2 *$	M1 A1 M1 A1 DM1 A1 (6)
(b)	$J.2a = \frac{2}{3} Ma^2 \omega$ $\frac{1}{2} \frac{2}{3} Ma^2 \omega^2 = Mg \frac{2a}{3} (1 + \cos 60^\circ)$ <p>solving for J</p> $J = M \sqrt{\frac{ag}{3}}$	M1 A1 M1 A2 DM1 A1 (7) [13]

Question Number	Scheme	Marks
Q5 (a)	$(2\mathbf{i} + \mathbf{j}) + (-2\mathbf{j} - \mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$ $\mathbf{F}_3 = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$ $ \mathbf{F}_3 = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6} \text{ N}$	M1 A1 M1 A1 (4)
(b)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) + (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \times (-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + ((y - z)\mathbf{i} + (-2z - x)\mathbf{j} + (x + 2y)\mathbf{k})$ $y - z = -1, -x - 2z = -3, x + 2y = 1$ $x = 1, y = 0, z = 1 \text{ is a solution}$ <p>so, $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ is a vector equn of line of action of \mathbf{F}_3</p>	M1 A3 DM1 DM1 M1 A1 (8)
(c)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \times (-2\mathbf{j} - \mathbf{k}) = \mathbf{G}$ $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = (\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \mathbf{G}$ $ \mathbf{G} = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11} \text{ N m}$	M1 A1 M1 A1 (4) [16]

Question Number	Scheme	Marks
Q6 (a)	$\frac{1}{3}2m(4a)^2 + \frac{1}{12}4ma^2 + 4m(4a)^2$ $= \frac{32}{3}ma^2 + \frac{1}{3}ma^2 + 64ma^2$ $= 75ma^2 \quad *$	B1 M1 A1 A1 (4)
(b)	$\frac{1}{2}75ma^2\omega^2 = 2mg2a(\cos\theta - \cos\alpha) + 4mg4a(\cos\theta - \cos\alpha)$ $a\omega^2 = \frac{8}{15}g(\cos\theta - \frac{24}{25}) = \frac{8}{375}g(25\cos\theta - 24)$ $X - 6mg\cos\theta = 2m2a\omega^2 + 4m4a\omega^2 = 20ma\omega^2$ $X = 6mg\cos\theta + 20m\frac{8}{375}g(25\cos\theta - 24)$ $= \frac{50mg\cos\theta}{3} - \frac{256mg}{25}$	M1 A2 A1 M1 A2 D M1 A1 (9)
(c)	$-2mg2a\sin\theta - 4mg4a\sin\theta = 75ma^2\ddot{\theta}$ $\ddot{\theta} = -\frac{4g}{15a}\sin\theta$ $\approx -\frac{4g}{15a}\theta, \text{ SHM}$ $\text{Time} = \frac{1}{4}2\pi\sqrt{\frac{15a}{4g}}$ $= \frac{\pi}{4}\sqrt{\frac{15a}{g}}$	M1 A1 A1 M1 M1 A1 (6) [19]

June 2009
6683 Statistics S1
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$(S_{pp}) = 38125 - \frac{445^2}{10}$ $= 18322.5$ <p style="text-align: right;">awrt 18300</p> $(S_{pt}) = 26830 - \frac{445 \times 240}{10}$ $= 16150$ <p style="text-align: right;">awrt 16200</p>	<p>M1 A1</p> <p>A1</p> <p style="text-align: right;">(3)</p>
(b)	$r = \frac{"16150"}{\sqrt{"18322.5" \times 21760}}$ $= 0.8088\dots$ <p style="text-align: right;">awrt 0.809</p>	<p>Using their values for method</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">(2)</p>
(c)	As the temperature increases the pressure increases.	<p>B1</p> <p style="text-align: right;">(1)</p> <p style="text-align: right;">[6]</p>
Notes	<p>1(a) M1 for seeing a correct expression $38125 - \frac{445^2}{10}$ or $26830 - \frac{445 \times 240}{10}$</p> <p>If no working seen, at least one answer must be exact to score M1 by implication.</p> <p>1(b) Square root and their values with 21760 all in the right places required for method. Anything which rounds to (awrt) 0.809 for A1.</p> <p>1(c) Require a correct statement in context using <u>temperature/heat</u> and <u>pressure</u> for B1.</p> <p>Don't allow "as <i>t</i> increases <i>p</i> increases".</p> <p>Don't allow proportionality.</p> <p>Positive correlation only is B0 since there is no interpretation.</p>	

Question Number	Scheme	Marks
<p>Q2 (a)</p> <div style="text-align: center;"> </div> <p>(b)(i) $\frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$ or equivalent</p> <p>(ii) $CNL + BNL + FNL = \frac{1}{2} \times \frac{4}{5} + \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{9}{10}$ $= \frac{4}{5}$ or equivalent</p> <p>(c) $P(F'/L) = \frac{P(F' \cap L)}{P(L)}$ $= \frac{\frac{1}{6} \times \frac{2}{5} + \frac{1}{2} \times \frac{1}{5}}{1 - (ii)}$ $= \frac{5}{30} = \frac{5}{6}$ or equivalent</p>	<p>Correct tree All labels Probabilities on correct branches</p> <p>(3)</p> <p>M1 A1 (2)</p> <p>M1 A1 (2)</p> <p>M1</p> <p>Attempt correct conditional probability but see notes</p> <p>numerator denominator</p> <p>A1 A1ft</p> <p>cao A1 (4) [11]</p>	
<p>Notes</p>	<p>Exact decimal equivalents required throughout if fractions not used e.g. 2(b)(i) 0.03 Correct path through their tree given in their probabilities award Ms 2(a) All branches required for first B1. Labels can be words rather than symbols for second B1. Probabilities from question enough for third B1 i.e. bracketed probabilities not required. Probabilities and labels swapped i.e. labels on branches and probabilities at end can be awarded the marks if correct. 2(b)(i) Correct answer only award both marks. 2(b)(ii) At least one correct path identified and attempt at adding all three multiplied pairs award M1 2(c) Require probability on numerator and division by probability for M1. Require numerator correct for their tree for M1. Correct formula seen and used, accept denominator as attempt and award M1 No formula, denominator must be correct for their tree or 1-(ii) for M1 1/30 on numerator only is M0, P(L/F') is M0.</p>	

Question Number	Scheme	Marks
Q3 (a) (b)	<p>1(cm) cao</p> <p>10 cm² represents 15 10/15 cm² represents 1</p> <p>Therefore frequency of 9 is $\frac{10}{15} \times 9$ or $\frac{9}{1.5}$</p> <p>height = 6(cm)</p> <p style="text-align: right;">or 1cm² represents 1.5</p> <p style="text-align: right;">Require $\times \frac{2}{3}$ or $\div 1.5$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;">[3]</p>
Notes	<p>If 3(a) and 3(b) incorrect, but their (a) x their (b)=6 then award B0M1A0</p> <p>3(b) Alternative method: f/cw=15/6=2.5 represented by 5 so factor x2 award M1 So f/cw=9/3=3 represented by 3x2=6. Award A1.</p>	

Question Number	Scheme	Marks
Q4 (a)	$Q_2 = 17 + \left(\frac{60 - 58}{29} \right) \times 2$ $= 17.1 \quad (17.2 \text{ if use } 60.5)$ <p style="text-align: right;">awrt 17.1 (or 17.2)</p>	M1 A1 (2)
(b)	$\sum fx = 2055.5 \quad \sum fx^2 = 36500.25$ <p style="text-align: center;">Exact answers can be seen below or implied by correct answers. Evidence of attempt to use midpoints with at least one correct</p> <p>Mean = 17.129... awrt 17.1</p> $\sigma = \sqrt{\frac{36500.25}{120} - \left(\frac{2055.5}{120} \right)^2}$ $= 3.28 \quad (s = 3.294)$ <p style="text-align: right;">awrt 3.3</p>	B1 B1 M1 B1 M1 A1 (6)
(c)	$\frac{3(17.129 - 17.1379...)}{3.28} = -0.00802$ <p style="text-align: right;">Accept 0 or awrt 0.0</p> <p>No skew/ slight skew</p>	M1 A1 B1 (3)
(d)	The skewness is very small. Possible.	B1 B1dep (2) [13]
Notes	<p>4(a) Statement of $17 + \frac{\text{freq into class}}{\text{class freq}} \times cw$ and attempt to sub or</p> $\frac{m - 17}{19 - 17} = \frac{60(.5) - 58}{87 - 58}$ <p>or equivalent award M1 cw=2 or 3 required for M1. 17.2 from cw=3 award A0.</p> <p>4(b) Correct $\sum fx$ and $\sum fx^2$ can be seen in working for both B1s Midpoints seen in table and used in calculation award M1 Require complete correct formula including use of square root and attempt to sub for M1. No formula stated then numbers as above or follow from (b) for M1 $(\sum fx)^2, \sum (fx)^2$ or $\sum f^2x$ used instead of $\sum fx^2$ in sd award M0 Correct answers only with no working award 2/2 and 6/6</p> <p>4(c) Sub in their values into given formula for M1</p> <p>4(d) No skew / slight skew / 'Distribution is almost symmetrical' / 'Mean approximately equal to median' or equivalent award first B1. Don't award second B1 if this is not the case. Second statement should imply 'Greg's suggestion that a normal distribution is suitable is possible' for second B1 dep. If B0 awarded for comment in (c) and (d) incorrect, allow follow through from the comment in (c).</p>	

Question Number	Scheme	Marks
Q5 (a)	$b = \frac{59.99}{33.381}$ $= 1.79713\dots$ <p style="text-align: right;">1.8 or awrt 1.80</p> $a = 32.7 - 1.79713\dots \times 51.83$ $= -60.44525\dots$ <p style="text-align: right;">awrt -60</p> $w = -60.445251\dots + 1.79713\dots l$ <p style="text-align: right;"><i>l</i> and <i>w</i> required and awrt 2sf</p>	M1 A1 M1 A1 A1ft (5)
(b)	$w = -60.445251\dots + 1.79713\dots \times 60$ $= 47.3825\dots$ <p style="text-align: right;">In range 47.3 – 47.6 inclusive</p>	M1 A1 (2)
(c)	It is extrapolating so (may be) unreliable.	B1, B1dep (2)
Notes	5(a) Special case $b = \frac{59.99}{120.1} = 0.4995 \text{ M0A0}$ $a = 32.7 - 0.4995 \times 51.83 \text{ M1A1}$ $w = 6.8 + 0.50l \text{ at least 2 sf required for A1}$ 5(b) Substitute into their answer for (a) for M1 5(c) ‘Outside the range on the table’ or equivalent award first B1	[9]

Question Number	Scheme	Marks								
Q6 (a)	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>3a</td> <td>2a</td> <td>a</td> <td>b</td> </tr> </table>	0	1	2	3	3a	2a	a	b	B1 (1)
0	1	2	3							
3a	2a	a	b							
(b)	$3a + 2a + a + b = 1$ $2a + 2a + 3b = 1.6$ $14a = 1.4$ $a = 0.1$ $b = 0.4$	or equivalent, using Sum of probabilities =1 or equivalent, using $E(X)=1.6$ Attempt to solve cao cao	M1 M1 M1dep B1 B1 (5)							
(c)	$P(0.5 < x < 3) = P(1) + P(2)$ $= 0.2 + 0.1$ $= 0.3$	$3a$ or their $2a$ +their a Require $0 < 3a < 1$ to award follow through	M1 A1 ft (2)							
(d)	$E(3X - 2) = 3E(X) - 2$ $= 3 \times 1.6 - 2$ $= 2.8$	cao	M1 A1 (2)							
(e)	$E(X^2) = 1 \times 0.2 + 4 \times 0.1 + 9 \times 0.4 (= 4.2)$ $\text{Var}(X) = "4.2" - 1.6^2$ $= 1.64$	**given answer** cso	M1 M1 A1 (3)							
(f)	$\text{Var}(3X - 2) = 9 \text{Var}(X)$ $= 14.76$	awrt 14.8	M1 A1 (2)							
			[15]							
Notes	<p>6(a) Condone a clearly stated in text but not put in table.</p> <p>6(b) Must be attempting to solve 2 different equations so third M dependent upon first two Ms being awarded. Correct answers seen with no working B1B1 only, 2/5 Correctly verified values can be awarded M1 for correctly verifying sum of probabilities =1, M1 for using $E(X)=1.6$ M0 as no attempt to solve and B1B1 if answers correct.</p> <p>6(d) 2.8 only award M1A1</p> <p>6(e) Award first M for at least two non-zero terms correct. Allow first M for correct expression with a and b e.g. $E(X^2) = 6a+9b$ Given answer so award final A1 for correct solution.</p> <p>6(f) 14.76 only award M1A1</p>									

Question Number	Scheme	Marks
Q7(a) (i) (ii) (b) (c) (d)	$P(A \cup B) = a + b$ $P(A \cup B) = a + b - ab$ $P(R \cup Q) = 0.15 + 0.35 = 0.5$ $P(R \cap Q) = P(R Q) \times P(Q) = 0.1 \times 0.35 = 0.035$ $P(R \cup Q) = P(R) + P(Q) - P(R \cap Q) \quad \text{OR} \quad P(R) = P(R \cap Q') + P(R \cap Q)$ $0.5 = P(R) + 0.35 - 0.035$ $P(R) = 0.185$ $= 0.15 + \text{their (c)}$ $= 0.15 + 0.035$ $= 0.185$	cao B1 or equivalent B1 (2) 0.5 B1 (1) M1 A1 0.035 (2) M1 A1 0.185 (2) [7]
Notes	7(a) (i) Accept $a + b - 0$ for B1 Special Case If answers to (i) and (ii) are (i) $P(A)+P(B)$ and (ii) $P(A)+P(B)-P(A)P(B)$ award B0B1 7(a)(i) and (ii) answers must be clearly labelled or in correct order for marks to be awarded.	

Question Number	Scheme	Marks
Q8 (a)	<p>Let the random variable X be the lifetime in hours of bulb</p> $P(X < 830) = P\left(Z < \frac{\pm(830 - 850)}{50}\right)$ <p style="text-align: right;">Standardising with 850 and 50</p> $= P(Z < -0.4)$ $= 1 - P(Z < 0.4)$ <p style="text-align: right;">Using 1-(probability>0.5)</p> $= 1 - 0.6554$ $= 0.3446 \text{ or } 0.344578 \text{ by calculator}$ <p style="text-align: right;">awrt 0.345</p>	M1 M1 A1 (3)
(b)	0.3446×500 $= 172.3$ <p style="text-align: right;">Their (a) x 500 Accept 172.3 or 172 or 173</p>	M1 A1 (2)
(c)	<p>Standardise with 860 and σ and equate to z value $\frac{\pm(818 - 860)}{\sigma} = z$ value</p> $\frac{818 - 860}{\sigma} = -0.84(16) \text{ or } \frac{860 - 818}{\sigma} = 0.84(16) \text{ or } \frac{902 - 860}{\sigma} = 0.84(16) \text{ or equiv.}$ <p style="text-align: right;">$\pm 0.8416(2)$ 50 or awrt 49.9</p> $\sigma = 49.9$	M1 A1 B1 A1 (4)
(d)	<p>Company Y as the <u>mean</u> is greater for Y.</p> <p>They have (approximately) the same <u>standard deviation</u> or <u>sd</u></p> <p style="text-align: right;">both</p>	B1 B1 (2) [11]
Notes	<p>8(a) If 1-z used e.g. 1-0.4=0.6 then award second M0 8(c) M1 can be implied by correct line 2 A1 for completely correct statement or equivalent. Award B1 if 0.8416(2) seen Do not award final A1 if any errors in solution e.g. negative sign lost. 8(d) Must use statistical terms as underlined.</p>	

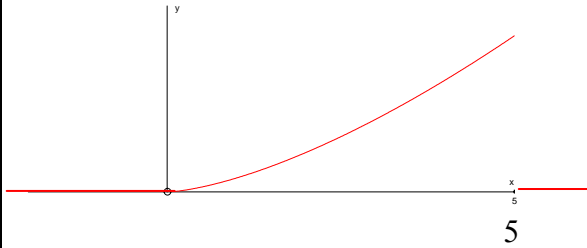
June 2009
6684 Statistics S2
Mark Scheme

Question Number	Scheme	Marks
Q1 (a)	$[X \sim B(30, 0.15)]$ $P(X \leq 6) = 0.8474$	awrt 0.847 M1, A1 (2)
(b)	$Y \sim B(60, 0.15) \approx \text{Po}(9)$ $P(Y \leq 12) = 0.8758$	for using Po(9) B1 M1, A1 (3)
[N.B. normal approximation gives 0.897, exact binomial gives 0.894]		[5]
(a)	M1 for a correct probability statement $P(X \leq 6)$ or $P(X < 7)$ or $P(X=0) + P(X=1) + P(X=2) + P(X=4) + P(X=5) + P(X=6)$. (may be implied by long calculation) Correct answer gets M1 A1. allow 84.74%	
(b)	B1 may be implied by using Po(9). Common incorrect answer which implies this is 0.9261 M1 for a correct probability statement $P(X \leq 12)$ or $P(X < 13)$ or $P(X=0) + P(X=1) + \dots + P(X=12)$ (may be implied by long calculation) and attempt to evaluate this probability using their Poisson distribution. Condone $P(X \leq 13) = 0.8758$ for B1 M1 A1 Correct answer gets B1 M1 A1 Use of normal or exact binomial get B0 M0 A0	

Question Number	Scheme	Marks
Q3 (a)	<i>A statistic</i> is a function of X_1, X_2, \dots, X_n that does not contain any unknown parameters	B1 B1 (2)
(b)	The <u>probability</u> distribution of Y or the distribution of all possible values of Y (o.e.)	B1 (1)
(c)	Identify (ii) as not a statistic Since <u>it contains</u> unknown parameters <u>μ and σ</u> .	B1 dB1 (2)
		[5]
(a)	Examples of other acceptable wording: B1 e.g. is a function of the sample or the data / is a quantity calculated from the sample or the data / is a random variable calculated from the sample or the data B1 e.g. does not contain any unknown parameters/quantities contains only known parameters/quantities <u>only</u> contains values of the sample Y is a function of X_1, X_2, \dots, X_n that does not contain any unknown parameters is a function of the values of a sample with no unknowns is a function of the sample values is a function of all the data values A random variable calculated from the sample A random variable consisting of any function A function of a value of the sample A function of the sample which contains no other values/ parameters	B1B1 B1B1 B1B0 B1B0 B1B0 B0B0 B1B0 B1B0
(b)	Examples of other acceptable wording All possible values of the statistic together with their associated probabilities	
(c)	1 st B1 for selecting only (ii) 2 nd B1 for a reason. This is dependent upon the first B1. Need to mention at least one of μ (mean) or σ (standard deviation or variance) or unknown parameters. Examples since it contains μ B1 since it contains σ B1 since it contains unknown parameters/quantities B1 since it contains unknowns B0	

Question Number	Scheme	Marks
Q4 (a)	$X \sim B(20, 0.3)$ $P(X \leq 9) = 0.9520$ so $P(X \leq 2) = 0.0355$ $P(X \geq 10) = 0.0480$ Therefore the critical region is $\{X \leq 2\} \cup \{X \geq 10\}$	M1 A1 A1 A1A1 (5)
(b)	$0.0355 + 0.0480 = 0.0835$ awrt (0.083 or 0.084)	B1 (1)
(c)	11 is in the critical region there is evidence of a <u>change/ increase</u> in the <u>proportion/number</u> of <u>customers buying single tins</u>	B1ft B1ft (2)
(a)	M1 for B(20,0.3) seen or used 1 st A1 for 0.0355 2 nd A1 for 0.048 3 rd A1 for $(X) \leq 2$ or $(X) < 3$ or $[0,2]$ They get A0 if they write $P(X \leq 2/ X < 3)$ 4 th A1 $(X) \geq 10$ or $(X) > 9$ or $[10,20]$ They get A0 if they write $P(X \geq 10/ X > 9)$ $10 \leq X \leq 2$ etc is accepted To describe the critical regions they can use any letter or no letter at all. It does not have to be X .	
(b)	B1 correct answer only	
(c)	1 st B1 for a correct statement about 11 and their critical region. 2 nd B1 for a correct comment in context consistent with their CR and the value 11	
	Alternative solution 1 st B0 $P(X \geq 11) = 1 - 0.9829 = 0.0171$ since no comment about the critical region 2 nd B1 a correct contextual statement.	

Question Number	Scheme	Marks
Q5 (a)	$X = \text{the number of errors in 2000 words}$ so $X \sim \text{Po}(6)$ $P(X \geq 4) = 1 - P(X \leq 3)$ $= 1 - 0.1512 = 0.8488$ awrt 0.849	B1 M1 A1 (3)
(b)	$Y = \text{the number of errors in 8000 words. } Y \sim \text{Po}(24)$ so use a <u>Normal</u> approx $Y \approx N(24, \sqrt{24}^2)$ Require $P(Y \leq 20) = P\left(Z < \frac{20.5 - 24}{\sqrt{24}}\right)$ $= P(Z < -0.714\dots)$ $= 1 - 0.7611$ $= 0.2389$ awrt (0.237~0.239)	M1 A1 M1 M1 A1 M1 A1 (7)
	[N.B. Exact Po gives 0.242 and no ± 0.5 gives 0.207]	[10]
(a)	B1 for seeing or using Po(6) M1 for $1 - P(X \leq 3)$ or $1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$ A1 awrt 0.849 SC If B(2000, 0.003) is used and leads to awrt 0.849 allow B0 M1 A1 If no distribution indicated awrt 0.8488 scores B1M1A1 but any other awrt 0.849 scores B0M1A1	
(b)	1 st M1 for identifying the normal approximation 1 st A1 for [mean = 24] and [sd = $\sqrt{24}$ or var = 24] These first two marks may be given if the following are seen in the standardisation formula : 24 $\sqrt{24}$ or awrt 4.90 2 nd M1 for attempting a continuity correction (20/ 28 \pm 0.5 is acceptable) 3 rd M1 for standardising using their mean and their standard deviation. 2 nd A1 correct z value awrt ± 0.71 or this may be awarded if see $\frac{20.5 - 24}{\sqrt{24}}$ or $\frac{27.5 - 24}{\sqrt{24}}$ 4 th M1 for 1 - a probability from tables (must have an answer of < 0.5) 3 rd A1 answer awrt 3 sig fig in range 0.237 – 0.239	

Question Number	Scheme	Marks
Q6 (a) (b) (c) (d) (e) (f) (g)	<p>$P(A > 3) = \frac{2}{5} = 0.4$</p> <p>$(0.4)^3 = 0.064$ or $\frac{8}{125}$</p> $f(y) = \frac{d}{dy}(F(y)) = \begin{cases} \frac{3y^2}{125} & 0 \leq y \leq 5 \\ 0 & \text{otherwise} \end{cases}$  <p>Shape of curve and start at (0,0)</p> <p>Point (5, 0) labelled and curve between 0 and 5 and pdf ≥ 0</p> <p>Mode = 5</p> $E(Y) = \int_0^5 \left(\frac{3y^3}{125} \right) dy = \left[\frac{3y^4}{500} \right]_0^5 = \frac{15}{4} \text{ or } 3.75$ $P(Y > 3) = \begin{cases} \int_3^5 \frac{3y^2}{125} dy = 1 - \frac{27}{125} = \frac{98}{125} = 0.784 \\ \text{or } 1 - F(3) \end{cases}$	<p>B1 (1)</p> <p>M1, A1 (2)</p> <p>M1A1 (2)</p> <p>B1</p> <p>B1 (2)</p> <p>B1 (1)</p> <p>M1M1A1 (3)</p> <p>M1A1 (2) [13]</p>
(a) (b) (c) (d) (e) (f) (g)	<p>B1 correct answer only (cao). Do not ignore subsequent working</p> <p>M1 for cubing their answer to part (a) A1 cao</p> <p>M1 for attempt to differentiate the cdf. They must decrease the power by 1 A1 fully correct answer including 0 otherwise. Condone < signs</p> <p>B1 for shape. Must curve the correct way and start at (0,0). No need for y = 0 (patios) lines B1 for point (5,0) labelled and pdf only existing between 0 and 5, may have y=0 (patios) for other values</p> <p>B1 cao</p> <p>1st M1 for attempt to integrate their $yf(y) y^n \rightarrow y^{n+1}$. 2nd M1 for attempt to use correct limits A1 cao</p> <p>M1 for attempt to find $P(Y > 3)$. e.g. writing \int_3^5 their $f(y)$ must have correct limits or writing $1 - F(3)$</p>	

Question Number	Scheme	Marks
Q7 (a)	$E(X) = 2$ (by symmetry)	B1 (1)
(b)	$0 \leq x < 2$, gradient = $\frac{1}{2} = \frac{1}{4}$ and equation is $y = \frac{1}{4}x$ so $a = \frac{1}{4}$ $b - \frac{1}{4}x$ passes through $(4, 0)$ so $b = 1$	B1 B1 (2)
(c)	$E(X^2) = \int_0^2 \left(\frac{1}{4}x^3\right) dx + \int_2^4 \left(x^2 - \frac{1}{4}x^3\right) dx$ $= \left[\frac{x^4}{16}\right]_0^2 + \left[\frac{x^3}{3} - \frac{x^4}{16}\right]_2^4$ $= 1 + \frac{64-8}{3} - \frac{256-16}{16} = 4\frac{2}{3}$ or $\frac{14}{3}$	M1M1 A1 M1A1
(d)	$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{14}{3} - 2^2 = \frac{2}{3}$ (so $\sigma = \sqrt{\frac{2}{3}} = 0.816$) (*)	M1 A1cso (7)
(e)	$P(X \leq q) = \int_0^q \frac{1}{4}x dx = \frac{1}{4}q$, $\frac{q^2}{2} = 1$ so $q = \sqrt{2} = 1.414$ awrt 1.41	M1A1, A1 (3)
	$2 - \sigma = 1.184$ so $2 - \sigma, 2 + \sigma$ is wider than IQR, therefore greater than 0.5	M1, A1 (2) [15]
(a)	B1 cao	
(b)	B1 for value of a . B1 for value of b	
(c)	1 st M1 for attempt at $\int ax^3$ using their a . For attempt they need x^4 . Ignore limits. 2 nd M1 for attempt at $\int bx^2 - ax^3$ use their a and b . For attempt need to have either x^3 or x^4 . Ignore limits 1 st A1 correct integration for both parts 3 rd M1 for use of the correct limits on each part 2 nd A1 for either getting 1 and $3\frac{2}{3}$ or awrt 3.67 somewhere or $4\frac{2}{3}$ or awrt 4.67 4 th M1 for use of $E(X^2) - [E(X)]^2$ must add both parts for $E(X^2)$ and only have subtracted the mean ² once. You must see this working 3 rd A1 $\sigma = \sqrt{\frac{2}{3}}$ or $\sqrt{0.66667}$ or better with no incorrect working seen.	
(d)	M1 for attempting to find LQ, integral of either part of $f(x)$ with their 'a' and 'b' = 0.25 Or their $F(x) = 0.25$ i.e. $\frac{ax^2}{2} = 0.25$ or $bx - \frac{ax^2}{2} + 4a - 2b = 0.25$ with their a and b If they add both parts of their $F(x)$, then they will get M0.	
(e)	1 st A1 for a correct equation/expression using their 'a' 2 nd A1 for $\sqrt{2}$ or awrt 1.41 M1 for a reason based on their quartiles • Possible reasons are $P(2 - \sigma < X < 2 + \sigma) = 0.6498$ allow awrt 0.65 • $1.184 < LQ(1.414)$ A1 for correct answer > 0.5 NB you must check the reason and award the method mark. A correct answer without a correct reason gets M0 A0	

Question Number	Scheme	Marks
Q8 (a)	$X \sim \text{Po}(2)$ $P(X=4) = \frac{e^{-2} \times 2^4}{4!} = 0.0902$	awrt 0.09 M1 A1 (2)
(b)	$Y \sim \text{Po}(8)$ $P(Y>10) = 1 - P(Y \leq 10) = 1 - 0.8159 = 0.18411\dots$	awrt 0.184 B1 M1A1 (3)
(c)	$F = \text{no. of faults in a piece of cloth of length } x$ $F \sim \text{Po}(x \times \frac{2}{15})$ $e^{-\frac{2x}{15}} = 0.80$ $e^{-\frac{2}{15} \times 1.65} = 0.8025\dots, \quad e^{-\frac{2}{15} \times 1.75} = 0.791\dots$ These values are either side of 0.80 therefore $x = 1.7$ to 2 sf	M1A1 M1 A1 (4)
(d)	Expected number with no faults = $1200 \times 0.8 = 960$ Expected number with some faults = $1200 \times 0.2 = 240$ So expected profit = $960 \times 0.60 - 240 \times 1.50, \quad = \text{£}216$	M1 A1 M1, A1 (4) [13]
(a)	M1 for use of Po(2) may be implied A1 awrt 0.09	
(b)	B1 for Po(8) seen or used M1 for $1 - P(Y \leq 10)$ oe A1 awrt 0.184	
(c)	1 st M1 for forming a suitable Poisson distribution of the form $e^{-\lambda} = 0.8$ 1 st A1 for use of lambda as $\frac{2x}{15}$ (this may appear after taking logs) 2 nd M1 for attempt to consider a range of values that will prove 1.7 is correct OR for use of logs to show lambda = ... 2 nd A1 correct solution only. Either get 1.7 from using logs or stating values either side	
S.C	for $e^{-\frac{2}{15} \times 1.7} = 0.797\dots \approx 0.80 \quad \therefore x = 1.7$ to 2 sf allow 2 nd M1A0	
(d)	1 st M1 for one of the following $1200p$ or $1200(1-p)$ where $p = 0.8$ or $2/15$. 1 st A1 for both expected values being correct or two correct expressions. 2 nd M1 for an attempt to find expected profit, must consider with and without faults 2 nd A1 correct answer only.	

June 2009
6691 Statistics S3
Mark Scheme

Question Number	Scheme	Marks
Q1	<p>(a) Randomly select a number between 00 and 499 (001 and 500) select every 500th person</p> <p>(bi) <u>Quota</u> Advantage: <u>Representative</u> sample can be achieved (with small sample size) <u>Cheap</u> (costs kept to a minimum) not “quick” Administration relatively <u>easy</u> Disadvantage Not possible to estimate sampling errors (due to lack of randomness) Not a random process Judgment of interviewer can affect choice of sample – <u>bias</u> Non-response not recorded Difficulties of defining controls e.g. social class</p> <p>(bii) <u>Systematic</u> Advantage: <u>Simple</u> or <u>easy</u> to use not “quick” or “cheap” or “efficient” It is suitable for large <u>samples</u> (not populations) Disadvantage Only random if the ordered list is (truly) random Requires a list of the population <u>or</u> must assign a number to each member of the pop.</p>	<p>B1 B1 (2)</p> <p>B1</p> <p>B1</p> <p>(2)</p> <p>B1</p> <p>B1 (2)</p> <p>[6]</p>
(a)	<p>1st B1 for idea of using random numbers to select the first from 1 - 500 (o.e.) 2nd B1 for selecting every 500th (name on the list)</p> <p style="text-align: center;">If they are clearly trying to carry out <u>stratified</u> sample then score B0B0</p>	
(b)	<p>Score B1 for any one line</p>	
(i)	<p>1st B1 for Quota advantage 2nd B1 for Quota disadvantage</p>	
(ii)	<p>3rd B1 for Systematic Advantage 4th B1 for Systematic Disadvantage</p>	

Question Number	Scheme	Marks
Q2	<p>(a) Limits are $20.1 \pm 1.96 \times 0.5$</p> <p style="text-align: center;"><u>(19.1, 21.1)</u></p> <p>(b) 98 % confidence limits are</p> $20.1 \pm 2.3263 \times \frac{0.5}{\sqrt{10}}$ <p style="text-align: center;"><u>(19.7, 20.5)</u></p> <p>(c) The growers claim is not correct Since 19.5 does not lie in the interval (19.7, 20.5)</p>	<p>M1 B1 A1cso (3)</p> <p>M1 B1 A1A1 (4)</p> <p>B1 dB1 (2) [9]</p>
	<p>(a) M1 for $20.1 \pm z \times 0.5$. Need 20.1 and 0.5 in correct places with no $\sqrt{10}$ B1 for $z = 1.96$ (or better) A1 for awrt 19.1 <u>and</u> awrt 21.1 but must have scored both M1 and B1 [Correct answer only scores 3/3]</p> <p>(b) M1 for $20.1 \pm z \times \frac{0.5}{\sqrt{10}}$, need to see 20.1, 0.5 and $\sqrt{10}$ in correct places B1 for $z = 2.3263$ (or better) 1st A1 for awrt 19.7 2nd A1 for awrt 20.5 [Correct answer only scores M1B0A1A1]</p> <p>(c) 1st B1 for rejection of the claim. Accept “unlikely” or “not correct” 2nd dB1 Dependent on scoring 1st B1 in this part for rejecting grower’s claim for an argument that supports this. Allow comment on <u>their</u> 98% CI from (b)</p>	

Question Number	Scheme	Marks																																																							
Q3 (a)	<table border="1"> <thead> <tr> <th></th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> <th>F</th> <th>G</th> <th>H</th> <th>I</th> <th>J</th> </tr> </thead> <tbody> <tr> <td>BMI</td> <td>1</td> <td>6</td> <td>3</td> <td>8</td> <td>4</td> <td>5</td> <td>7</td> <td>2</td> <td>9</td> <td>10</td> </tr> <tr> <td>or</td> <td>10</td> <td>5</td> <td>8</td> <td>3</td> <td>7</td> <td>6</td> <td>4</td> <td>9</td> <td>2</td> <td>1</td> </tr> <tr> <td>Finishing position</td> <td>3</td> <td>5</td> <td>1</td> <td>9</td> <td>6</td> <td>4</td> <td>10</td> <td>2</td> <td>7</td> <td>8</td> </tr> <tr> <td>d^2</td> <td>4</td> <td>1</td> <td>4</td> <td>1</td> <td>4</td> <td>1</td> <td>9</td> <td>0</td> <td>4</td> <td>4</td> </tr> </tbody> </table> <p> $\sum d^2 = 32$ (298) $r_s = 1 - \frac{6 \times 32}{10 \times 99}$ $= 0.80606\dots$ (-0.80606) accept $\pm \frac{133}{165}$ <u>awrt ± 0.806</u> </p>		A	B	C	D	E	F	G	H	I	J	BMI	1	6	3	8	4	5	7	2	9	10	or	10	5	8	3	7	6	4	9	2	1	Finishing position	3	5	1	9	6	4	10	2	7	8	d^2	4	1	4	1	4	1	9	0	4	4	<p>M1</p> <p>M1</p> <p>M1 A1ft</p> <p>A1 (5)</p>
	A	B	C	D	E	F	G	H	I	J																																															
BMI	1	6	3	8	4	5	7	2	9	10																																															
or	10	5	8	3	7	6	4	9	2	1																																															
Finishing position	3	5	1	9	6	4	10	2	7	8																																															
d^2	4	1	4	1	4	1	9	0	4	4																																															
(b)	<p>$H_0 : \rho = 0, H_1 : \rho > 0,$</p> <p>Critical value is $(\pm)0.5636$</p> <p>(0.806 > 0.5636 therefore) in critical region/ reject H_0</p>	<p>B1 B1</p> <p>B1</p> <p>M1 A1ft</p>																																																							
(c)	<p>The lower the BMI the higher the position in the race./ support for doctors belief</p> <p>The position is already ranked OR Position is not Normally distributed</p>	<p>(5)</p> <p>B1 (1)</p> <p>[11]</p>																																																							
(a)	<p>1st M1 for attempt to rank BMI scores</p> <p>2nd M1 for attempt at $\sum d^2$ (<u>must</u> be using ranks)</p> <p>3rd M1 for use of the correct formula with their $\sum d^2$. If answer is not correct an expression is required.</p> <p>1st A1ft for a correct expression. ft their $\sum d^2$ but only if all 3 Ms are scored</p> <p>2nd A1 awrt ± 0.806 (but sign must be compatible with their $\sum d^2$)</p>	<p>No ranking can score 3rd M1 only</p>																																																							
(b)	<p>2nd B1 for $\rho > 0$ (or < 0 but must be one tail and consistent with their ranking)</p> <p>3rd B1 for critical value that is compatible with their H_1. If one-tail must be ± 0.5636 if two-tail must be ± 0.6485 [Condone wrong sign]</p> <p>M1 for a correct statement relating their r_s with their cv. e.g. “reject H_0”, “in critical region”, “significant result” May be implied by a correct comment</p> <p>A1ft for correct comment in context. Must mention low/high BMI and race/fitness <u>or</u> doctor’s belief. Comment should be <u>one-tailed</u>. Allow positive <u>correlation</u> between... but <u>NOT</u> ...positive <u>relationship</u>...</p>	<p>No H_1 assume one-tail for 3rd B1</p>																																																							
(c)	<p>B1 for a correct and relevant comment either based on the fact that the data was originally partially ordered <u>or</u> on the underlying normal assumption “Quicker” or “easier” score B0</p>																																																								

Question Number	Scheme	Marks
Q4	$X \sim N(55, 3^2) \text{ therefore } \bar{X} \sim N\left(55, \frac{9}{8}\right)$ $P(\bar{X} > 57) = P\left(Z > \frac{57-55}{\sqrt{\frac{9}{8}}}\right) = P(Z > 1.8856\dots)$ $= 1 - 0.9706$ $= 0.0294$ <p style="text-align: right;"><u>0.0294~0.0297</u></p>	B1 B1 M1 M1 A1 [5]
ALT	<p>1st B1 for $\bar{X} \sim$ normal and $\mu = 55$, may be implied but must be \bar{X}</p> <p>2nd B1 for $\text{Var}(\bar{X})$ or st. dev of \bar{X} e.g. $\bar{X} \sim N(55, \frac{9}{8})$ or $\bar{X} \sim N\left(55, \left(\frac{3}{\sqrt{8}}\right)^2\right)$ for B1B1 Condone use of X if they clearly mean \bar{X} so $X \sim N(55, \frac{9}{8})$ is OK for B1B1</p> <p>1st M1 for an attempt to standardize with 57 and mean of 55 and their st. dev. $\neq 3$</p> <p>2nd M1 for 1 - tables value. Must be trying to find a probability < 0.5</p> <p>A1 for answers in the range 0.0294~0.0297</p> $\sum_1^8 X_i \sim N(8 \times 55, 8 \times 3^2)$ <p>1st B1 for $\sum X \sim$ normal and mean = 8×55</p> <p>2nd B1 for variance = 8×3^2</p> <p>1st M1 for attempt to standardise with 57×8, mean of 55×8 and their st dev $\neq 3$</p>	

Question Number	Scheme	Marks																		
Q5 (a)	$\lambda = \frac{0 \times 40 + 1 \times 33 + 2 \times 14 + 3 \times 8 + 4 \times 5}{100} = 1.05$	M1 A1 (2)																		
(b)	Using Expected frequency = $100 \times P(X=x) = 100 \times \frac{e^{-1.05} 1.05^x}{x!}$ gives $r = 36.743$ $s = 19.290$	M1 A1 A1 (3)																		
(c)	H_0 : Poisson distribution is a suitable model H_1 : Poisson distribution is not a suitable model	B1																		
	<table border="1"> <thead> <tr> <th>Number of goals</th> <th>Frequency</th> <th>Expected frequency</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>40</td> <td>34.994</td> </tr> <tr> <td>1</td> <td>33</td> <td>36.743</td> </tr> <tr> <td>2</td> <td>14</td> <td>19.290</td> </tr> <tr> <td>3</td> <td>8</td> <td>6.752</td> </tr> <tr> <td>≥ 4</td> <td>5</td> <td>2.221</td> </tr> </tbody> </table>	Number of goals	Frequency	Expected frequency	0	40	34.994	1	33	36.743	2	14	19.290	3	8	6.752	≥ 4	5	2.221	M1
Number of goals	Frequency	Expected frequency																		
0	40	34.994																		
1	33	36.743																		
2	14	19.290																		
3	8	6.752																		
≥ 4	5	2.221																		
	$\nu = 4 - 1 - 1 = 2$ CR : $\chi^2_2(0.05) > 5.991$	B1ft B1																		
	$\sum \frac{(O-E)^2}{E} = \frac{(40-34.9937)^2}{34.9937} + \dots + \frac{(13-8.972443)^2}{8.972443}$ $[=0.7161\dots+0.3813\dots+1.4508\dots+1.80789\dots]$ $= 4.356$. (ans in range 4.2 – 4.4)	M1 A1																		
	Not in critical region Number of goals scored can follow a Poisson distribution / managers claim is justified	A1 ft (7)																		
		[12]																		
(a)	M1 for an attempt to find the mean- at least 2 terms on numerator seen Correct answer only will score both marks																			
(b)	M1 for use of correct formula (ft their mean). 1 st A1 for r , 2 nd A1 for s (19.29 OK)																			
(c)	1 st B1 Must have both hypotheses and mention Poisson at least once inclusion of their value for mean in hypotheses is B0 but condone in conclusion 1 st M1 for an attempt to pool ≥ 4 2 nd B1ft for $n - 1 - 1 = 2$ i.e realising that they must subtract 2 from their n 3 rd B1 for 5.991 only 2 nd M1 for an attempt at the test statistic, at least 2 correct expressions/values (to 3sf) 1 st A1 for answers in the range 4.2~4.4 2 nd A1 for correct comment in context based on their test statistic and their cv that mentions goals or manager. Dependent on 2 nd M1 Condone mention of Po(1.05) in conclusion Score A0 for inconsistencies e.g. “significant” followed by “manager’s claim is justified”																			

Question Number	Scheme	Marks
Q6 (a)	<p>$\mu_U \sim$ mean length of upper shore limpets, $\mu_L \sim$ mean length of lower shore limpets</p> <p>$H_0 : \mu_u = \mu_L$</p> <p>$H_1 : \mu_u < \mu_L$</p> <p style="text-align: right;">both</p> $\text{s.e.} = \sqrt{\frac{0.42^2}{120} + \frac{0.67^2}{150}}$ $= 0.0668$ $z = \frac{5.05 - 4.97}{0.0668} = (\pm)1.1975$ <p style="text-align: right;">awrt \pm 1.20</p> <p>Critical region is $z \geq 1.6449$, or probability = awrt (0.115 or 0.116) $z = \pm 1.6449$</p> <p>(1.1975 < 1.6449) therefore not in critical region / accept H_0/not significant (or $P(Z \geq 1.1975) = 0.1151$, $0.1151 > 0.05$ or z not in critical region)</p> <p>There is no evidence that the limpets on the upper shore are shorter than the limpets on the lower shore.</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>dM1 A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(8)</p> <p>B1</p> <p>B1</p> <p>(2)</p> <p>[10]</p>
(a)	<p>1st B1 If μ_1, μ_2 used then it must be clear which refers to upper shore. Accept sensible choice of letters such as u and l.</p> <p>1st M1 Condone minor slips e.g. $\frac{0.67^2}{120}$ or $\frac{0.67}{150} + \frac{0.42^2}{120}$ etc i.e. swapped n or one sd and one variance but M0 for $\sqrt{\frac{0.67}{150} + \frac{0.42}{120}}$</p> <p>1st A1 can be scored for a fully correct expression. May be implied by awrt 1.20</p> <p>2nd dM1 is dependent upon the 1st M1 but can fit their se value if this mark is scored.</p> <p>2nd A1 for awrt (\pm) 1.20</p> <p>3rd M1 for a correct statement based on their z value and their cv. No cv is M0A0 If using probability they must compare their p (<0.5) with 0.05 (o.e) so can allow $0.884 < 0.95$ to score this 3rd M1 mark. May be implied by their contextual statement and M1A0 is possible.</p>	
(b)	<p>3rd A1 for a correct comment to accept null hypothesis that mentions <u>length of limpets</u> on the two <u>shores</u>.</p> <p>1st B1 for one correct statement. Accept "samples are independent"</p> <p>2nd B1 for both statements</p>	

Question Number	Scheme	Marks
Q7 (a)	<p>Estimate of Mean = $\frac{600.9}{5} = 120.18$</p> <p>Estimate of Variance = $\frac{1}{4} \left\{ 72216.31 - \frac{600.9^2}{5} \right\}$ or $\frac{0.148}{4} = 0.037$</p> <p>(b) $P(-0.05 < \mu - \hat{\mu} < 0.05) = 0.90$ or $P(-0.05 < \bar{X} - \mu < 0.05) = 0.90$ [\leq is OK]</p> $\frac{0.05}{\frac{0.2}{\sqrt{n}}} = 1.6449$ $n = \frac{1.6449^2 \times 0.2^2}{0.05^2}$ $n = 43.29\dots$ $n = 44$	<p>M1A1</p> <p>M1 A1ft A1 (5)</p> <p>B1</p> <p>M1 A1</p> <p>dM1</p> <p>A1</p> <p>A1 (6) [11]</p>
(a)	<p>1st M1 for an attempt at $\sum x$ (accept 600 to 1sf)</p> <p>1st A1 for $\frac{600.9}{5} = \text{awrt } 120$ or awrt 120.2. No working give M1A1 for awrt 120.2</p> <p>2nd M1 for the use of a correct formula including a reasonable attempt at $\sum x^2$ (Accept 70 000 to 1sf) or $\sum (x - \bar{x})^2 = 0.15$ (to 2 dp)</p> <p>2nd A1ft for a correct expression with correct $\sum x^2$ but can fit their <u>mean</u> (for expression - no need to check values if it is incorrect)</p> <p>3rd A1 for 0.037 Correct answer with no working scores 3/3 for variance</p> <p>(b) B1 for a correct probability statement <u>or</u> “width of 90% CI = $0.05 \times 2 = 0.1$”</p> <p>1st M1 for $\frac{0.05}{\frac{0.2}{\sqrt{n}}} = z$ value <u>or</u> $2 \times \frac{0.2}{\sqrt{n}} \times z = 0.1$</p> <p>Condone 0.5 instead of 0.05 <u>or</u> missing 2 <u>or</u> 0.05 for 0.1 for M1</p> <p>1st A1 for a correct equation including 1.6449</p> <p>2nd dM1 Dependent upon 1st M1 for rearranging to get $n = \dots$ Must see “squaring”</p> <p>2nd A1 for $n = \text{awrt } 43.3$</p> <p>3rd A1 for rounding up to get $n = 44$</p> <p>Using e.g. 1.645 instead of 1.6449 can score all the marks except the 1st A1</p>	<p>1st B1 may be implied by 1st A1 scored or correct equation.</p>

Question Number	Scheme	Marks
Q8 (a)	$E(4X-3Y) = 4E(X) - 3E(Y)$ $= 4 \times 30 - 3 \times 20$ $= 60$	M1 A1 (2)
(b)	$\text{Var}(4X-3Y) = 16 \text{Var}(X) + 9 \text{Var}(Y)$ $= 16 \times 9 + 9 \times 4$ $= 180$	16 or 9; adding M1; M1 A1 (3)
(c)	$E(B) = 80$ $\text{Var}(B) = 16$ $E(B - A) = 20$ $\text{Var}(B - A) = 196$ $P(B - A > 0) = P\left(Z > \frac{-20}{\sqrt{196}}\right) = [P(Z > -1.428\dots)]$ $= 0.923 \dots$	$E(B) - E(A)$ ft on 180 and 16 B1 B1 M1 A1ft stand. using their mean and var dM1 awrt 0.923 – 0.924 A1 (6) [11]
(a)	M1 for correct use of $E(aX + bY)$ formula	
(b)	1 st M1 for $16\text{Var}(X)$ or $9\text{Var}(Y)$ 2 nd M1 for <u>adding</u> variances Key points are the 16, 9 and +. Allow slip e.g using $\text{Var}(X)=4$ etc to score Ms	
(c)	1 st M1 for attempting $B - A$ and $E(B - A)$ or $A - B$ and $E(A - B)$ This mark may be implied by an attempt at a correct probability e.g. $P\left(Z > \frac{0 - (80 - 60)}{\sqrt{180 + 16}}\right)$. To be implied we must see the “0” 1 st A1ft for $\text{Var}(B - A)$ can ft their $\text{Var}(A) = 180$ and their $\text{Var}(B) = 16$ 2 nd dM1 Dependent upon the 1 st M1 in part (c). for attempting a correct probability i.e. $P(B-A > 0)$ or $P(A-B < 0)$ and standardising with their mean and variance. They must standardise properly with the 0 to score this mark 2 nd A1 for awrt 0.923 ~ 0.924	

June 2009
6686 Statistics S4
Mark Scheme

Question Number	Scheme	Marks
Q1	<p>$H_0: \mu = 5; H_1: \mu < 5$</p> <p>CR: $t_9(0.01) > 2.821$</p> <p>$\bar{x} = 4.91$</p> $s^2 = \frac{1}{9} \left(241.2 - \frac{49.1^2}{10} \right) = 0.0132222$ $t = \frac{ 4.91 - 5 }{\frac{\sqrt{0.013222}}{\sqrt{10}}} = \pm 2.475$ <p>Since 2.475 is not in the critical region there is insufficient evidence to reject H_0 and conclude that the mean diameter of the bolts is not less than (not equal to) 5mm.</p>	<p style="text-align: center;">both</p> <p>B1 B1 B1</p> <p>s= awrt 0.115 M1 A1</p> <p>2.47 – 2.48 M1 A1</p> <p>A1ft</p> <p style="text-align: right;">[8]</p>

Question Number	Scheme	Marks
Q2 (a)	The differences are normally distributed	B1 (1)
(b)	The data is collected in pairs or small sample size and variance unknown or samples not independent	B1 (1)
(c)	<p>d: 2.5, 1.6, 1.6, -1.9, -0.6, 4.5 at least 2 correct</p> <p>$(\Sigma d = 7.7, \Sigma d^2 = 35.59) \bar{d} = \pm 1.2833, sd = 2.2675. (\text{Var} = 5.141)$</p> <p>$H_0: \mu_d = 0, H_1: \mu_d > 0$ ($H_1: \mu_d < 0$ if $d = -2.5, -1.6, -1.6$ etc) both depend on their d's</p> <p>$t = \frac{\pm 1.2833\sqrt{6}}{2.2675} = \pm 1.386\dots$ formula and substitution, 1.38 – 1.39</p> <p>Critical value $t_5(5\%) = 2.015$ (1 tail)</p> <p>Not significant. Insufficient evidence to support that the device reduces CO₂ emissions.</p>	M1 A1, A1 B1 M1, A1 B1 A1 ft (8)
(d)	<p>The idea that the device reduces CO₂ emissions has been rejected when in fact it does reduce emissions.</p> <p>OR</p> <p>Concluding that the device does not reduce emissions when in fact it does (if not in context can get B1 only)</p>	B1 B1 (2)
	<p>(b) Allow because the same car has been used</p> <p>(c) awrt $\pm 1.28, 2.27$</p>	[12]

Question Number	Scheme	Marks
3	<p>(a) Size is the probability of H_0 being rejected when it is in fact true. or $P(\text{reject } H_0 / H_0 \text{ is true})$ oe</p> <p>(b) The power of the test is the probability of rejecting H_0 when H_1 is true. or $P(\text{rejecting } H_0 / H_1 \text{ is true}) / P(\text{rejecting } H_0 / H_0 \text{ is false})$ oe</p> <p>(c) $X \sim B(12, 0.5)$ $P(X \leq 2) = 0.0193$ $P(X \geq 10) = 0.0193$ \therefore critical region is $\{X \leq 2 \cup X \geq 10\}$</p> <p>(d)(i) $P(\text{Type II error}) = P(3 \leq X \leq 9 \mid p = 0.4)$ $= P(X \leq 9) - P(X \leq 2)$ $= 0.9972 - 0.0834$ $= 0.9138$</p> <p>(ii) Power = $1 - 0.9138$ $= 0.0862$</p> <p>(e) Increase the sample size Increase the significance level/larger critical region</p>	<p>B1 (1)</p> <p>B1 (1)</p> <p>B1 M1 A1A1 (4)</p> <p>M1 M1dep A1</p> <p>B1 ft (4)</p> <p>B1 B1 (2)</p> <p>[12]</p>
Notes	<p>(d) (i) first M1 for either correct area or follow through from their critical region 2nd M1 dependent on them having the first M1. for finding their area correctly A1 cao</p> <p>(ii) B1 follow through from their (i)</p>	

Question Number	Scheme	Marks
Q4 (a)	$H_0 : \sigma_A^2 = \sigma_B^2, H_1 : \sigma_A^2 \neq \sigma_B^2$ <p>critical values $F_{12,8}=3.28$ and $\frac{1}{F_{8,12}} = 0.35$</p> $\frac{s_B^2}{s_A^2} = 2.40 \left(\frac{s_A^2}{s_B^2} = 0.416 \right)$ <p>Since 2.40 (0.416) is not in the critical region we accept H_0 and conclude there is no evidence that the two variances are different.</p>	B1 B1 M1A1 A1ft (5)
(b)	$S_p^2 = \frac{8 \times 1.02 + 12 \times 2.45}{20}$ $= 1.878$ $(27.94 - 25.54) \pm 2.086 \times \sqrt{1.878} \times \sqrt{\frac{1}{9} + \frac{1}{13}}$ <p>(1.16, 3.64)</p>	M1 A1 B1M1 A1ft A1 A1 (7)
(c)	<p>To calculate the confidence interval the variances need to be equal. In part (a) the test showed they are equal.</p>	B1 B1 (2) [14]

Question Number	Scheme	Marks
Q5 (a)	95% confidence interval for μ is 2.145 $560 \pm t_{14}(2.5\%) \sqrt{\frac{25.2}{15}} = 560 \pm 2.145 \sqrt{\frac{25.2}{15}} = (557.2, 562.8)$	B1 M1 A1 A1 (4)
(b)	95% confidence interval for σ^2 is $5.629 < \frac{14 \times 25.2}{\sigma^2} < 26.119$ $\sigma^2 < 62.675 \quad \sigma^2 > 13.507$ $13.507 < \sigma^2 < 62.675$ awrt 13.5, 62.7	B1, M1, B1 A1, A1 (5)
(c)	Require $P(X > 565) = P\left(Z > \frac{565 - \mu}{\sigma}\right)$ to be as large as possible OR $\frac{565 - \mu}{\sigma}$ to be as small as possible; both imply highest σ and μ . $\frac{565 - 562.8}{\sqrt{62.675}} = 0.28$ $P(Z > 0.28) = 1 - 0.6103 = 0.3897$ awrt 0.39 – 0.40	M1 M1A1 M1 A1 (5) [14]
(c)	M1 for using their largest σ and μ M1 for using $\frac{x - \mu}{\sigma}$ M1 1 – their prob	

Question Number	Scheme	Marks
Q6 (a)	$E\left(\frac{2}{3}X_1 + \frac{1}{2}X_2 + \frac{5}{6}X_3\right) = \frac{2}{3} \times \frac{k}{2} + \frac{1}{2} \times \frac{k}{2} + \frac{5}{6} \times \frac{k}{2} = k$ $E(X_1 + X_2 + X_3) = k \Rightarrow \text{unbiased}$	M1 A1 B1 (3)
(b)	$E(aX_1 + bX_2) = a\frac{k}{2} + b\frac{k}{2} = k$ $a + b = 2$ $\text{Var}(aX_1 + bX_2) = a^2\frac{k^2}{12} + b^2\frac{k^2}{12}$ $= a^2\frac{k^2}{12} + (2-a)^2\frac{k^2}{12}$ $= (2a^2 - 4a + 4)\frac{k^2}{12}$ $= (a^2 - 2a + 2)\frac{k^2}{6} \quad (*) \text{ since answer given}$	M1 A1 M1A1 M1
(c)	<p>Min value when $(2a-2)\frac{k^2}{6} = 0$ $\frac{d}{da}(\text{Var}) = 0$, all correct, condone missing $\frac{k^2}{6}$</p> $\Rightarrow 2a - 2 = 0$ $a = 1, b = 1.$ $\frac{d^2(\text{Var})}{da^2} = \frac{2k^2}{6} > 0 \quad \text{since } k^2 > 0 \text{ therefore it is a minimum}$ $\text{min variance} = (1 - 2 + 2)\frac{k^2}{6}$ $= \frac{k^2}{6}$ <p>Alternative</p> $\frac{k^2}{6}(a-1)^2 - \frac{k^2}{6} + \frac{2k^2}{6}$ $\frac{k^2}{6}(a-1)^2 + \frac{k^2}{6}$ $\text{Min when } \frac{k^2}{6}(a-1)^2 = 0$ $a = 1 \quad b = 1$ $\text{min var} = k^2/6$	A1 cso (6) M1A1 A1A1 M1 B1 (6) M1 A1 M1 A1A1 B1

June 2009
6689 Decision Mathematics D1
Mark Scheme

Question Number	Scheme	Marks																		
Q1	<p>(a) AD, AE, DB; DC, CF</p> <p>(b)</p> <div style="text-align: center;"> </div> <p>(c)</p> <p>Weight 595 (km)</p> <p>Notes: (a) 1M1: Using Prim – first 2 arcs probably but condone starting from another vertex. 1A1: first three arcs correct 2A1: all correct. (b) 1B1: CAO (c) 1B1: CAO condone lack of km.</p> <p><u>Apply the misread rule, if not listing arcs or not starting at A.</u> So for M1 (only) Accept numbers across the top (condoning absence of 6) Accept full vertex listing Accept full arc listing starting from vertex other than A</p> <table style="width: 100%; border: none;"> <tr> <td>[AD AE DB DC CF]</td> <td>{1 4 5 2 3 6}</td> <td>ADEBCF</td> </tr> <tr> <td>BD AD AE CD CF</td> <td>{3 1 5 2 4 6}</td> <td>BDAECF</td> </tr> <tr> <td>CD AD AE BD CF</td> <td>{3 5 1 2 4 6}</td> <td>CDAEBF</td> </tr> <tr> <td>DA AE DB CD CF</td> <td>{2 4 5 1 3 6}</td> <td>DAEBCF</td> </tr> <tr> <td>EA AD DB DC CF</td> <td>{2 4 5 3 1 6}</td> <td>EADBCF</td> </tr> <tr> <td>FC CD AD AE BD</td> <td>{4 6 2 3 5 1}</td> <td>FCDAEB</td> </tr> </table>	[AD AE DB DC CF]	{1 4 5 2 3 6}	ADEBCF	BD AD AE CD CF	{3 1 5 2 4 6}	BDAECF	CD AD AE BD CF	{3 5 1 2 4 6}	CDAEBF	DA AE DB CD CF	{2 4 5 1 3 6}	DAEBCF	EA AD DB DC CF	{2 4 5 3 1 6}	EADBCF	FC CD AD AE BD	{4 6 2 3 5 1}	FCDAEB	<p>M1 A1; A1 (3)</p> <p>B1 (1)</p> <p>B1 (1)</p> <p>[5]</p>
[AD AE DB DC CF]	{1 4 5 2 3 6}	ADEBCF																		
BD AD AE CD CF	{3 1 5 2 4 6}	BDAECF																		
CD AD AE BD CF	{3 5 1 2 4 6}	CDAEBF																		
DA AE DB CD CF	{2 4 5 1 3 6}	DAEBCF																		
EA AD DB DC CF	{2 4 5 3 1 6}	EADBCF																		
FC CD AD AE BD	{4 6 2 3 5 1}	FCDAEB																		

Question Number	Scheme	Marks
Q2		
(a)	$\frac{230}{60} = 3.8\dot{3}$ so 4 needed	M1 A1 (2)
(b)	Bin 1: 32 17 9 Bin 2: 45 12 Bin 3: 23 28 Bin 4: 38 16 Bin 5: 10	M1 A1 A1 A1 (4)
(c)	e.g. Bin 1: 32 28 Bin 2: 38 12 10 Bin 3: 45 9 Bin 4: 23 17 16	M1 A1 A1 (3)
	Notes:	
(a)	1M1: Their 230 divided by 60, some evidence of correct method 3.8 enough. 1A1: cso 4.	
(b)	1M1: Use of first fit. Probably 32, 45 and 17 correctly placed. 1A1: 32, 45, 17, 23, 38 and 28 placed correctly 2A1: 32, 45, 17, 23, 38, 28, 16, 9 placed correctly. 3A1: cao	
(c)	1M1: Use of full bin – at least one full bin found and 5 numbers placed. 1A1: 2 full bins found	
	Eg [32+28 and 38+12+10] [23+28+9 and 16+12+32] [32+28 and 23+16+12+9] [38+12+10 and 23+28+9]	
	2A1: A 4 bin solution found.	
	Special case for (b) misread using first fit decreasing.	
	Give M1A1 (max)	
	Bin 1: 45 12	
	Bin 2: 38 17	
	Bin 3: 32 28	
	Bin 4: 23 16 10 9	
	M1 for placing 45, 38, 32, 28 and 23 correctly	
	A1 for cao.	[9]

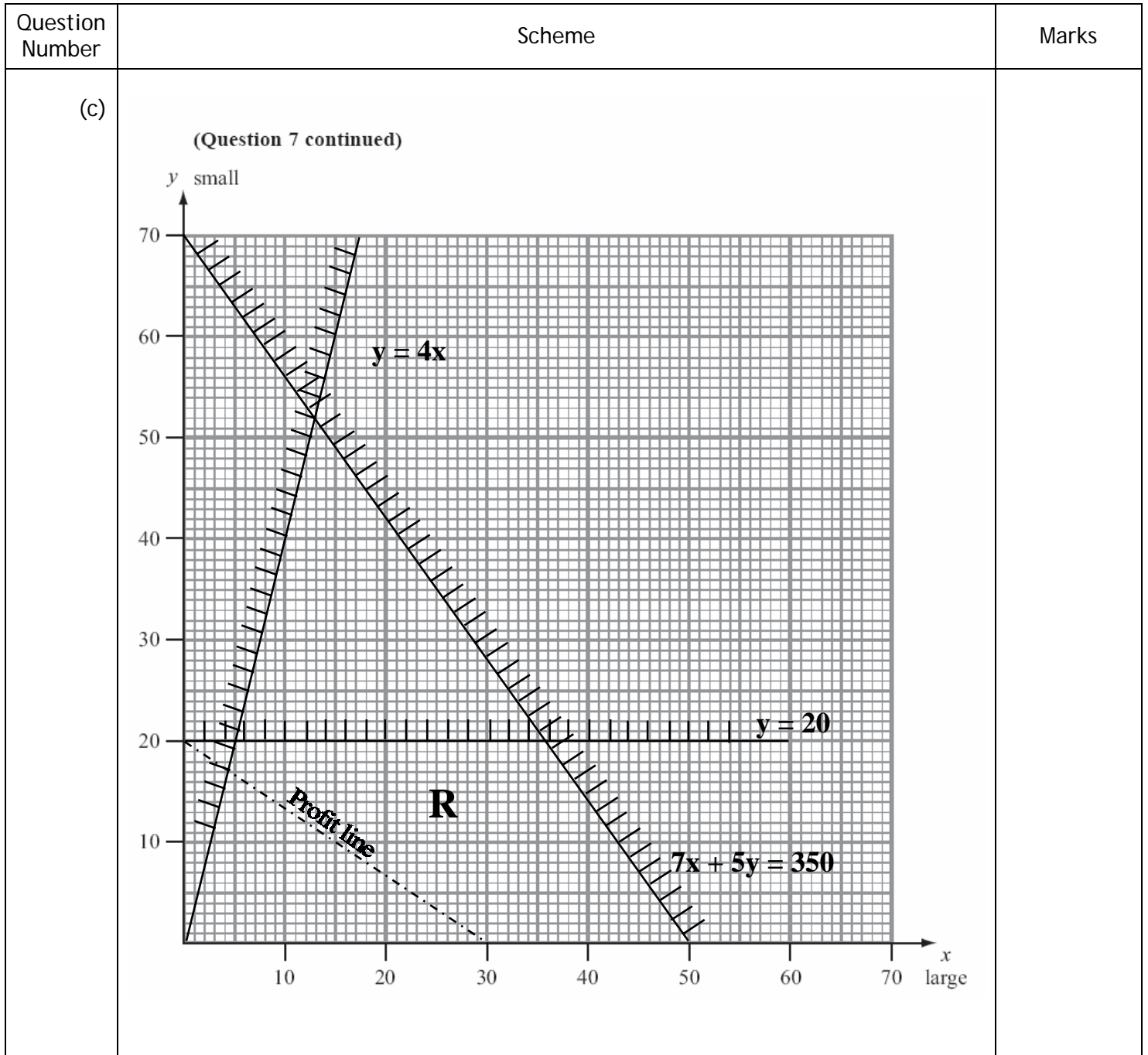
Question Number	Scheme	Marks
Q3	<p>(a) $H - 2 = M - 5 = R - 4$ change status to give</p> <p>(b) $C = 3$ (E unmatched) $H = 2$ $M = 5$ $R = 4$ $S = 1$</p> <p>(c) e.g. C is the only person who can do 3 and the only person who can do 6</p> <p>e.g. $E - 5 = M - 2 = H - 1 = S - 3 = C - 6$ change status to give</p> <p>$C = 6$ $E = 5$ $H = 1$ $M = 2$ $R = 4$ $S = 3$</p> <p>Notes:</p> <p>(a) 1M1: Path from H to 4 1A1: correct path and change status 2A1: CAO must follow from correct path.</p> <p>(b) 1B1: CAO or e.g reference to E 5 M 2 H 1 S</p> <p>(c) 1M1: Path from E to 6 1A1: CAO do not penalise lack of change status a second time. 2A1: CAO must follow from a correct path</p>	<p>M1 A1</p> <p>A1 (3)</p> <p>B1 (1)</p> <p>M1 A1</p> <p>A1 (3)</p> <p>[7]</p>

Question Number	Scheme	Marks																																																																		
Q4	<table border="1" data-bbox="400 320 1155 604"> <tr><td>M</td><td>J</td><td>E</td><td>K</td><td>H</td><td>B</td><td>L</td><td>P</td><td>N</td><td>D</td><td>B</td></tr> <tr><td>B</td><td>M</td><td>J</td><td>E</td><td>K</td><td>H</td><td>L</td><td>P</td><td>N</td><td>D</td><td>H</td></tr> <tr><td>B</td><td>E</td><td>D</td><td>H</td><td>M</td><td>J</td><td>K</td><td>L</td><td>P</td><td>N</td><td>D L</td></tr> <tr><td>B</td><td>D</td><td>E</td><td>H</td><td>J</td><td>K</td><td>L</td><td>M</td><td>P</td><td>N</td><td>(E) K P</td></tr> <tr><td>B</td><td>D</td><td>E</td><td>H</td><td>J</td><td>K</td><td>L</td><td>M</td><td>N</td><td>P</td><td>(J) N</td></tr> <tr><td>B</td><td>D</td><td>E</td><td>H</td><td>J</td><td>K</td><td>L</td><td>M</td><td>N</td><td>P</td><td>(M)</td></tr> </table> <p data-bbox="663 651 895 689" style="text-align: center;">Sort completed</p> <p data-bbox="225 741 671 824">$\left[\frac{1+10}{2} \right] = 6$ Katie reject left</p> <p data-bbox="225 880 730 963">$\left[\frac{7+10}{2} \right] = 9$ Natsuko reject right</p> <p data-bbox="225 1019 687 1102">$\left[\frac{7+8}{2} \right] = 8$ Miri reject right</p> <p data-bbox="288 1111 659 1149">7 = Louis name found</p> <p data-bbox="220 1240 325 1274">Notes:</p> <p data-bbox="240 1283 1273 1321">(a) 1M1: quick sort, pivots, p, identified, two sublists one <p one >p.</p> <p data-bbox="288 1328 1070 1364">If choosing one pivot only per iteration, M1 only.</p> <p data-bbox="300 1370 1150 1406">1A1: first pass correct, next pivot(s) chosen consistently.</p> <p data-bbox="277 1413 1190 1449">2A1ft: second pass correct, next pivot(s) chosen consistently</p> <p data-bbox="277 1456 1158 1491">3A1ft: third pass correct, next pivot(s) chosen consistently</p> <p data-bbox="304 1498 1265 1576">4A1: cso List re-written or end statement made or each element been chosen as a pivot.</p> <p data-bbox="225 1583 1107 1619">(b) 1M1: binary search, choosing pivot rejecting half list.</p> <p data-bbox="304 1626 818 1662">If using unordered list then M0.</p> <p data-bbox="312 1668 654 1704">If choosing J M1 only</p> <p data-bbox="304 1711 1257 1747">1A1: first two passes correct, condone 'sticky' pivots here, bod.</p> <p data-bbox="288 1753 900 1789">2A1ft: third pass correct, pivots rejected.</p> <p data-bbox="312 1796 895 1832">3A1: cso, including success statement.</p> <p data-bbox="220 1839 1318 1917">Special case for (b) – If just one letter out of order, award maximum of M1A1A0A0</p>	M	J	E	K	H	B	L	P	N	D	B	B	M	J	E	K	H	L	P	N	D	H	B	E	D	H	M	J	K	L	P	N	D L	B	D	E	H	J	K	L	M	P	N	(E) K P	B	D	E	H	J	K	L	M	N	P	(J) N	B	D	E	H	J	K	L	M	N	P	(M)	<p data-bbox="1362 327 1465 356">M1 1A1</p> <p data-bbox="1362 423 1437 452">2A1ft</p> <p data-bbox="1362 508 1437 537">3A1ft</p> <p data-bbox="1362 640 1517 674">4A1 (5)</p> <p data-bbox="1362 770 1401 799">M1</p> <p data-bbox="1362 904 1410 934">1A1</p> <p data-bbox="1362 1039 1437 1068">2A1ft</p> <p data-bbox="1362 1111 1517 1144">3A1 (4)</p> <p data-bbox="1485 1173 1525 1207">[9]</p>
M	J	E	K	H	B	L	P	N	D	B																																																										
B	M	J	E	K	H	L	P	N	D	H																																																										
B	E	D	H	M	J	K	L	P	N	D L																																																										
B	D	E	H	J	K	L	M	P	N	(E) K P																																																										
B	D	E	H	J	K	L	M	N	P	(J) N																																																										
B	D	E	H	J	K	L	M	N	P	(M)																																																										

Question Number	Scheme	Marks
Q5 (a)	<p> $CD + EG = 45 + 38 = 83$ $CE + DG = 39 + 43 = 82 \leftarrow$ $CG + DE = 65 + 35 = 100$ Repeat CE and DG Length $625 + 82 = 707$ (m) </p> <p> DE (or 35) is the smallest So finish at C. New route $625 + 35 = 660$ (m) </p> <p>Notes:</p> <p>(a) 1M1: Three pairings of their four odd nodes 1A1: one row correct 2A1: two rows correct 3A1: three rows correct 4A1ft: ft their least, but must be the correct shortest route arcs on network. (condone DG) 5A1ft: $625 +$ their least = a number. Condone lack of m</p> <p>(b) 1M1: Identifies their shortest from a choice of at least 2 rows. 1A1ft: ft from their least or indicates C. 2A1ft = 1Bft: correct for their least. (Indept of M mark)</p>	<p> M1 1A1 2A1 3A1 4A1ft 5A1ft (6) </p> <p> M1 A1ft A1ft=1B1 (3) </p> <p>[9]</p>

Question Number	Scheme	Marks
<p>Q6</p> <p>(a)</p> <p>Route: A E H I</p> <p>(b)</p> <p>Shortest distance from A to G is 28 km</p> <p>Notes:</p> <p>(a) 1M1: Small replacing big in the working values at C or F or G or I 1A1: Everything correct in boxes at A, B, D and F 2A1ft: ft boxes at E and C handled correctly but penalise order of labelling only once 3A1ft: ft boxes at G and H handled correctly but penalise order of labelling only once 4A1ft: ft boxes at I handled correctly but penalise order of labelling only once 5A1: route cao A E H I</p> <p>(b) 1B1ft: ft their final label at G condone lack of km</p>		<p>M1</p> <p>1A1</p> <p>2A1ft</p> <p>3A1ft</p> <p>4A1ft</p> <p>5A1</p> <p>B1ft</p> <p>[7]</p>

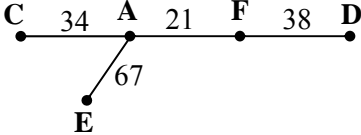
Question Number	Scheme	Marks
Q7 (a)	$7x + 5y \leq 350$	M1 A1 (2)
(b)	$y \leq 20$ e.g. make at most 20 small baskets $y \leq 4x$ e.g. the number of small (y) baskets is at most 4 times the number of large baskets (x). {E.g if $y = 40$, $x = 10, 11, 12$ etc. or if $x = 10$, $y = 40, 39, 38$ }	B1 B1 (2)
(c)	(see graph next page) Draw three lines correctly Label R	B3,2,1,0 B1 (4)
(d)	(P=) $2x + 3y$	B1 (1)
(e)	Profit line or point testing. $x = 35.7$ $y = 20$ precise point found. Need integers so optimal point in R is (35, 20); Profit (£)130	M1 A1 B1 B1;B1 (5)
[14]		
Notes:		
(a) 1M1: Coefficients correct (condone swapped x and y coefficients) need 350 and any inequality		
1A1: cso.		
(b) 1B1: cao		
2B1: cao, test their statement, need both = and < aspects.		
(c) 1B1: One line drawn correctly		
2B1: Two lines drawn correctly		
3B1: Three lines drawn correctly. Check (10, 40) (0, 0) and axes		
4B1: R correct, but allow if one line is slightly out (1 small square).		
(d) 1B1: cao accept an expression.		
(e) 1M1: Attempt at profit line or attempt to test at least two vertices in their feasible region.		
1A1: Correct profit line or correct testing of at least three vertices.		
Point testing: (0,0) P = 0; (5,20) P = 70; (50,0) P = 100		
$\left(35\frac{5}{7}, 20\right) = \left(\frac{250}{7}, 20\right) P = 131\frac{3}{7} = \frac{920}{7}$		
also (35, 20) P = 130. Accept (36,20) P = 132 for M but not A.		
Objective line: Accept gradient of 1/m for M mark or line close to correct gradient.		
1B1: cao – accept x co-ordinates which round to 35.7		
2B1: cao		
3B1: cao		

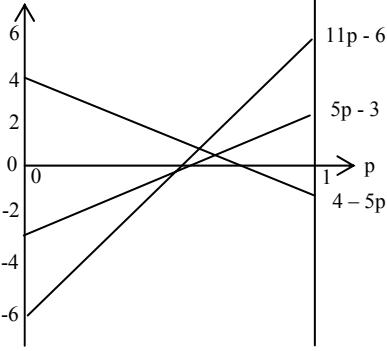


Question Number	Scheme	Marks
<p>Q8</p> <p>(a)</p> <p>(b) A C J L</p> <p>(c) Total float for M = $56(ft) - 46 - 9 = 1$ Total float for H = $47 - 12 - 21 = 14$</p> <p>(d)</p> <p>(e)</p> <p>1pm day 16: C 1pm day 31: C F G H</p>	<p>(a)</p> <p>(b) A C J L</p> <p>(c)</p> <p>Total float for M = $56(ft) - 46 - 9 = 1$ Total float for H = $47 - 12 - 21 = 14$</p> <p>(d)</p> <p>(e)</p> <p>1pm day 16: C 1pm day 31: C F G H</p>	<p>M1 A1 M1 A1 (4)</p> <p>B1 (1)</p> <p>M1 A1ft B1 (3)</p> <p>M1 A1 M1,A1 (4)</p> <p>B1ft B2ft,1ft,0 (3)</p> <p>[15]</p>

June 2009
6690 Decision Mathematics D2
Mark Scheme

Question Number	Scheme	Marks
Q1		
(a)	There are more tasks than people.	B1 (1)
(b)	Adds a row of zeros	B1 (1)
(c)	$\begin{bmatrix} 15 & 11 & 14 & 12 \\ 13 & 8 & 17 & 13 \\ 14 & 9 & 13 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0 & 3 & 1 \\ 5 & 0 & 9 & 5 \\ 5 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \rightarrow \begin{bmatrix} 3 & 0 & 2 & 0 \\ 4 & 0 & 8 & 4 \\ 4 & 0 & 3 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ Either $\begin{bmatrix} 3 & 3 & 2 & 0 \\ 1 & 0 & 5 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix}$ Or $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 4 \\ 2 & 0 & 1 & 5 \\ 0 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 0 & 4 \\ 0 & 4 & 0 & 2 \end{bmatrix}$	B1;M1A1
(d)	J – 4, M – 2, R – 3, (D – 1)	A1 (6)
	Minimum cost is (£)33.	B1 (1)
		[9]

Question Number	Scheme	Marks
Q2	<p>(a) In the classical problem each vertex must be visited only once. In the practical problem each vertex must be visited at least once.</p> <p>(b) A F D B E C A {1 4 6 3 5 2 } $21 + 38 + 58 + 36 + 70 + 34 = 257$</p> <p>(c) 257 is the better upper bound, it is lower.</p> <p>(d) R.M.S.T.</p> <div style="text-align: center;">  </div> <p>Lower bound is $160 + 36 + 58 = 254$</p> <p>(e) Better lower bound is 254, it is higher</p> <p>(f) $254 < \text{optimal} \leq 257$</p> <p>Notes:</p> <p>(a) 1B1: Generous, on the right lines bod gets B1 2B1: cao, clear answer.</p> <p>(b) 1M1: Nearest Neighbour each vertex visited once (condone lack of return to start) 1A1: Correct route cao – must return to start. 2A1: 257 cao</p> <p>(c) 1B1ft: ft their lowest.</p> <p>(d) 1M1: Finding correct RMST (maybe implicit) 160 sufficient 1A1: cao tree or 160. 2M1: Adding 2 least arcs to B, 36 and 58 only 2A1: 254</p> <p>(e) 1B1ft: ft their highest</p> <p>(f) 1B1: cao</p>	<p>B2, 1, 0 (2)</p> <p>M1 A1 A1 (3)</p> <p>B1ft (1)</p> <p>M1 A1 M1A1 (4)</p> <p>B1ft</p> <p>B1 (2)</p> <p>[12]</p>

Question Number	Scheme	Marks												
Q3														
(a)	Row minima $\{-5, -4, -2\}$ row maximin $= -2$ Column maxima $\{1, 6, 13\}$ col minimax $= 1$ $-2 \neq 1$ therefore not stable.	M1 A1 A1 (3)												
(b)	Column 1 dominates column 3, so column 3 can be deleted.	B1 (1)												
(c)	<table border="1" data-bbox="432 618 1123 757"> <thead> <tr> <th></th> <th>A plays 1</th> <th>A plays 2</th> <th>A plays 3</th> </tr> </thead> <tbody> <tr> <th>B plays 1</th> <td>5</td> <td>-1</td> <td>2</td> </tr> <tr> <th>B plays 2</th> <td>-6</td> <td>4</td> <td>-3</td> </tr> </tbody> </table>		A plays 1	A plays 2	A plays 3	B plays 1	5	-1	2	B plays 2	-6	4	-3	B1 B1 (2)
	A plays 1	A plays 2	A plays 3											
B plays 1	5	-1	2											
B plays 2	-6	4	-3											
(d)	Let B play row 1 with probability p and row 2 with probability $(1-p)$ If A plays 1, B's expected winnings are $11p - 6$ If A plays 2, B's expected winnings are $4 - 5p$ If A plays 3, B's expected winnings are $5p - 3$	M1 A1 M1 A1 M1 A1 A1 (7)												
														
	$5p - 3 = 4 - 5p$ $10p = 7$ $p = \frac{7}{10}$													
	B should play 1 with a probability of 0.7 2 with a probability of 0.3 and never play 3													
	The value of the game is 0.5 to B													
		[13]												

Question Number	Scheme	Marks
Q4	<p>(a) Value of cut $C_1 = 34$; Value of cut $C_2 = 45$</p> <p>(b) S B F G T or S B F E T – value 2 Maximum flow = 28</p> <p>Notes: (a) 1B1: cao 2B1: cao (b) 1M1: feasible flow-augmenting route and a value stated 1A1: a correct flow-augmenting route and value 1A1= B1: cao</p>	<p>B1; B1 (2)</p> <p>M1 A1 A1=B1 (3)</p> <p>[5]</p>
Q5	<p>(a) $x = 0, y = 0, z = 2$</p> <p>(b) $P - 2x - 4y + \frac{5}{4}r = 10$</p> <p>Notes: (a) 1B1: Any 2 out of 3 values correct 2B1: All 3 values correct. (b) 1M1: One equal sign, modulus of coefficients correct. All the right ingredients. 1A1: cao – condone terms of zero coefficient</p>	<p>B2,1,0 (2)</p> <p>M1 A1 (2)</p> <p>[4]</p>

Question Number	Scheme	Marks																									
Q6																											
(a)	The supply is equal to the demand	B1 (1)																									
(b)	<table border="1" style="margin-left: 20px;"> <tr><td></td><td>A</td><td>B</td><td>C</td></tr> <tr><td>X</td><td>16</td><td>6</td><td></td></tr> <tr><td>Y</td><td></td><td>9</td><td>8</td></tr> <tr><td>Z</td><td></td><td></td><td>15</td></tr> </table>		A	B	C	X	16	6		Y		9	8	Z			15	B1 (1)									
	A	B	C																								
X	16	6																									
Y		9	8																								
Z			15																								
(c)	<table border="1" style="margin-left: 20px;"> <tr><td></td><td>A</td><td>B</td><td>C</td></tr> <tr><td>X</td><td>16-θ</td><td>6+θ</td><td></td></tr> <tr><td>Y</td><td></td><td>9-θ</td><td>8+θ</td></tr> <tr><td>Z</td><td>θ</td><td></td><td>15-θ</td></tr> </table>		A	B	C	X	16- θ	6+ θ		Y		9- θ	8+ θ	Z	θ		15- θ	M1 A1									
	A	B	C																								
X	16- θ	6+ θ																									
Y		9- θ	8+ θ																								
Z	θ		15- θ																								
	Value of $\theta = 9$, exiting cell is YB	A1 (3)																									
(d)	<table style="margin-left: 20px;"> <tr><td></td><td></td><td>17</td><td>8</td><td>20</td></tr> <tr><td></td><td></td><td>A</td><td>B</td><td>C</td></tr> <tr><td>0</td><td>X</td><td>7</td><td>15</td><td></td></tr> <tr><td>-5</td><td>Y</td><td></td><td></td><td>17</td></tr> <tr><td>-11</td><td>Z</td><td>9</td><td></td><td>6</td></tr> </table>			17	8	20			A	B	C	0	X	7	15		-5	Y			17	-11	Z	9		6	M1 A1
		17	8	20																							
		A	B	C																							
0	X	7	15																								
-5	Y			17																							
-11	Z	9		6																							
	$XC = 7 - 0 - 20 = -13$																										
	$YA = 16 + 5 - 17 = 4$																										
	$YB = 12 + 5 - 8 = 9$	A1 (3)																									
	$ZB = 10 + 11 - 8 = 13$																										
	<table border="1" style="margin-left: 20px;"> <tr><td></td><td>A</td><td>B</td><td>C</td></tr> <tr><td>X</td><td>7-θ</td><td>15</td><td>θ</td></tr> <tr><td>Y</td><td></td><td></td><td>17</td></tr> <tr><td>Z</td><td>9+θ</td><td></td><td>6-θ</td></tr> </table>		A	B	C	X	7- θ	15	θ	Y			17	Z	9+ θ		6- θ	M1 A1									
	A	B	C																								
X	7- θ	15	θ																								
Y			17																								
Z	9+ θ		6- θ																								
	Value of $\theta = 6$, entering cell XC, exiting cell ZC	M1 A1																									
	<table border="1" style="margin-left: 20px;"> <tr><td></td><td>A</td><td>B</td><td>C</td></tr> <tr><td>X</td><td>1</td><td>15</td><td>6</td></tr> <tr><td>Y</td><td></td><td></td><td>17</td></tr> <tr><td>Z</td><td>15</td><td></td><td></td></tr> </table>		A	B	C	X	1	15	6	Y			17	Z	15			A1 (3)									
	A	B	C																								
X	1	15	6																								
Y			17																								
Z	15																										
	Cost (£) 524	B1 (1)																									
		[12]																									

Question Number	Scheme	Marks																																																																																																																																																																																		
Q7																																																																																																																																																																																				
(a)	<table border="1" style="width:100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Stage</th> <th>State (in £1000s)</th> <th>Action (in £1000s)</th> <th>Dest. (in £1000s)</th> <th>Value (in £1000s)</th> </tr> </thead> <tbody> <tr><td></td><td>250</td><td>250</td><td>0</td><td>300*</td></tr> <tr><td>1</td><td>200</td><td>200</td><td>0</td><td>240*</td></tr> <tr><td></td><td>150</td><td>150</td><td>0</td><td>180*</td></tr> <tr><td></td><td>100</td><td>100</td><td>0</td><td>120*</td></tr> <tr><td></td><td>50</td><td>50</td><td>0</td><td>60*</td></tr> <tr><td></td><td>0</td><td>0</td><td>0</td><td>0*</td></tr> <tr><td></td><td>250</td><td>280</td><td>0</td><td>$200 + 0 = 280$</td></tr> <tr><td></td><td></td><td>200</td><td>50</td><td>$235 + 60 = 295$</td></tr> <tr><td></td><td></td><td>150</td><td>100</td><td>$190 + 120 = 310^*$</td></tr> <tr><td></td><td></td><td>100</td><td>150</td><td>$125 + 180 = 305$</td></tr> <tr><td></td><td></td><td>50</td><td>200</td><td>$65 + 240 = 305$</td></tr> <tr><td></td><td></td><td>0</td><td>250</td><td>$0 + 300 = 300$</td></tr> <tr><td>2</td><td>200</td><td>200</td><td>0</td><td>$235 + 0 = 235$</td></tr> <tr><td></td><td></td><td>150</td><td>50</td><td>$190 + 60 = 250^*$</td></tr> <tr><td></td><td></td><td>100</td><td>100</td><td>$125 + 120 = 245$</td></tr> <tr><td></td><td></td><td>50</td><td>150</td><td>$65 + 180 = 245$</td></tr> <tr><td></td><td></td><td>0</td><td>200</td><td>$0 + 240 = 240$</td></tr> <tr><td></td><td>150</td><td>150</td><td>0</td><td>$190 + 0 = 190^*$</td></tr> <tr><td></td><td></td><td>100</td><td>50</td><td>$125 + 60 = 185$</td></tr> <tr><td></td><td></td><td>50</td><td>100</td><td>$65 + 120 = 185$</td></tr> <tr><td></td><td></td><td>0</td><td>150</td><td>$0 + 180 = 180$</td></tr> <tr><td></td><td>100</td><td>100</td><td>0</td><td>$125 + 0 = 125^*$</td></tr> <tr><td></td><td></td><td>50</td><td>50</td><td>$65 + 60 = 125^*$</td></tr> <tr><td></td><td></td><td>0</td><td>100</td><td>$0 + 120 = 120$</td></tr> <tr><td></td><td>50</td><td>50</td><td>0</td><td>$65 + 0 = 65^*$</td></tr> <tr><td></td><td></td><td>0</td><td>50</td><td>$0 + 60 = 60$</td></tr> <tr><td></td><td>0</td><td>0</td><td>0</td><td>$0 + 0 = 0^*$</td></tr> <tr><td>3</td><td>250</td><td>250</td><td>0</td><td>$300 + 0 = 300$</td></tr> <tr><td></td><td></td><td>200</td><td>50</td><td>$230 + 65 = 295$</td></tr> <tr><td></td><td></td><td>150</td><td>100</td><td>$170 + 125 = 295$</td></tr> <tr><td></td><td></td><td>100</td><td>150</td><td>$110 + 190 = 300$</td></tr> <tr><td></td><td></td><td>50</td><td>200</td><td>$55 + 250 = 305$</td></tr> <tr><td></td><td></td><td>0</td><td>250</td><td>$0 + 310 = 310^*$</td></tr> </tbody> </table> <p>Maximum income £310 000</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <tr> <td>Scheme</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>Invest (in £1000s)</td> <td>100</td> <td>150</td> <td>0</td> </tr> </table>	Stage	State (in £1000s)	Action (in £1000s)	Dest. (in £1000s)	Value (in £1000s)		250	250	0	300*	1	200	200	0	240*		150	150	0	180*		100	100	0	120*		50	50	0	60*		0	0	0	0*		250	280	0	$200 + 0 = 280$			200	50	$235 + 60 = 295$			150	100	$190 + 120 = 310^*$			100	150	$125 + 180 = 305$			50	200	$65 + 240 = 305$			0	250	$0 + 300 = 300$	2	200	200	0	$235 + 0 = 235$			150	50	$190 + 60 = 250^*$			100	100	$125 + 120 = 245$			50	150	$65 + 180 = 245$			0	200	$0 + 240 = 240$		150	150	0	$190 + 0 = 190^*$			100	50	$125 + 60 = 185$			50	100	$65 + 120 = 185$			0	150	$0 + 180 = 180$		100	100	0	$125 + 0 = 125^*$			50	50	$65 + 60 = 125^*$			0	100	$0 + 120 = 120$		50	50	0	$65 + 0 = 65^*$			0	50	$0 + 60 = 60$		0	0	0	$0 + 0 = 0^*$	3	250	250	0	$300 + 0 = 300$			200	50	$230 + 65 = 295$			150	100	$170 + 125 = 295$			100	150	$110 + 190 = 300$			50	200	$55 + 250 = 305$			0	250	$0 + 310 = 310^*$	Scheme	1	2	3	Invest (in £1000s)	100	150	0	<p>1M1 A1</p> <p>A1</p> <p>2M1</p> <p>A1</p> <p>A1</p> <p>3M1 A1ft</p> <p>B1 B1 (10)</p> <p>B1 B1 B1 (3)</p> <p>[13]</p>
Stage	State (in £1000s)	Action (in £1000s)	Dest. (in £1000s)	Value (in £1000s)																																																																																																																																																																																
	250	250	0	300*																																																																																																																																																																																
1	200	200	0	240*																																																																																																																																																																																
	150	150	0	180*																																																																																																																																																																																
	100	100	0	120*																																																																																																																																																																																
	50	50	0	60*																																																																																																																																																																																
	0	0	0	0*																																																																																																																																																																																
	250	280	0	$200 + 0 = 280$																																																																																																																																																																																
		200	50	$235 + 60 = 295$																																																																																																																																																																																
		150	100	$190 + 120 = 310^*$																																																																																																																																																																																
		100	150	$125 + 180 = 305$																																																																																																																																																																																
		50	200	$65 + 240 = 305$																																																																																																																																																																																
		0	250	$0 + 300 = 300$																																																																																																																																																																																
2	200	200	0	$235 + 0 = 235$																																																																																																																																																																																
		150	50	$190 + 60 = 250^*$																																																																																																																																																																																
		100	100	$125 + 120 = 245$																																																																																																																																																																																
		50	150	$65 + 180 = 245$																																																																																																																																																																																
		0	200	$0 + 240 = 240$																																																																																																																																																																																
	150	150	0	$190 + 0 = 190^*$																																																																																																																																																																																
		100	50	$125 + 60 = 185$																																																																																																																																																																																
		50	100	$65 + 120 = 185$																																																																																																																																																																																
		0	150	$0 + 180 = 180$																																																																																																																																																																																
	100	100	0	$125 + 0 = 125^*$																																																																																																																																																																																
		50	50	$65 + 60 = 125^*$																																																																																																																																																																																
		0	100	$0 + 120 = 120$																																																																																																																																																																																
	50	50	0	$65 + 0 = 65^*$																																																																																																																																																																																
		0	50	$0 + 60 = 60$																																																																																																																																																																																
	0	0	0	$0 + 0 = 0^*$																																																																																																																																																																																
3	250	250	0	$300 + 0 = 300$																																																																																																																																																																																
		200	50	$230 + 65 = 295$																																																																																																																																																																																
		150	100	$170 + 125 = 295$																																																																																																																																																																																
		100	150	$110 + 190 = 300$																																																																																																																																																																																
		50	200	$55 + 250 = 305$																																																																																																																																																																																
		0	250	$0 + 310 = 310^*$																																																																																																																																																																																
Scheme	1	2	3																																																																																																																																																																																	
Invest (in £1000s)	100	150	0																																																																																																																																																																																	
(b)	<p>Stage: Scheme being considered</p> <p>State: Money available to invest</p> <p>Action: Amount chosen to invest</p>																																																																																																																																																																																			

Question Number	Scheme	Marks
Q8	<p>E.g. Add 6 to make all elements positive</p> $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$ <p>Let Laura play 1, 2 and 3 with probabilities p_1, p_2 and p_3 respectively Let V = value of game + 6</p> <p>e.g. Maximise $P = V$ Subject to:</p> $V - 4p_1 - 13p_2 - 7p_3 \leq 0$ $V - 14p_1 - 10p_2 - p_3 \leq 0$ $V - 5p_1 - 3p_2 - 10p_3 \leq 0$ $p_1 + p_2 + p_3 \leq 1$ $p_1, p_2, p_3 \geq 0$ <p>Notes: 1B1: Making all elements positive 2B1: Defining variables 3B1: Objective, cao word and function 1M1: At least one constraint in terms of their variables, must be going down columns. Accept = here. 1A1ft: ft their table. One constraint in V correct. 2A1ft: ft their table. Two constraints in V correct. 3A1: CAO all correct .</p> <p>Alt using x_i method</p> <p>Now additionally need: let $x_i = \frac{p_i}{v}$ for 2B1</p> $\text{minimise } (P) = x_1 + x_2 + x_3 = \frac{1}{v}$ <p>subject to:</p> $4x_1 + 13x_2 + 7x_3 \geq 1$ $14x_1 + 10x_2 + x_3 \geq 1$ $5x_1 + 3x_2 + 10x_3 \geq 1$ $x_i \geq 0$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1 A3,2ft,1ft ,0</p> <p>(7)</p> <p>[7]</p>

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467
Fax 01623 450481

Email publications@linneydirect.com

Order Code UA021531 Summer 2009

For more information on Edexcel qualifications, please visit www.edexcel.com/quals

Edexcel Limited. Registered in England and Wales no.4496750
Registered Office: One90 High Holborn, London, WC1V 7BH