Question	Scheme	Marks	AOs
5(a)	Pond contains 1000 + 5*t* litres after *t* days	M1	3.3
	If *x* is the amount of pollutant in the pond after *t* days Rate of pollutant out = $20\times\dfrac{x}{1000+5t}$ g per day	M1	3.3
	Rate of pollutant in = 25 × 2 g = 50g per day	B1	2.2a
	$\dfrac{\mathrm{d}x}{\mathrm{d}t}=50-\dfrac{4x}{200+t}$ *	A1*	1.1b
		(4)	
(b)	$I=\mathrm{e}^{\int\frac{4}{200+t}\mathrm{d}t}=(200+t)^4\Rightarrow x(200+t)^4=\int 50(200+t)^4\,\mathrm{d}t$	M1	3.1b
	$x(200+t)^4=10(200+t)^5+c$	A1	1.1b
	$x=0,\ t=0\Rightarrow\ c=-3.2\times10^{12}$	M1	3.4
	$t=8\Rightarrow\ x=\ 10(200+8)-\dfrac{3.2\times10^{12}}{(200+8)^4}$	M1	1.1b
	= 370g	A1	2.2b
		(5)	
(c)	e.g. • The model should take into account the fact that the pollutant does not dissolve throughout the pond upon entry • The rate of leaking could be made to vary with the volume of water in the pond	B1	3.5c
		(1)	
		(10 marks)	

Notes:

(a)
M1: Forms an expression of the form 1000 + *kt* for the volume of water in the pond at time *t*
M1: Expresses the amount of pollutant out in terms of *x* and *t*
B1: Correct interpretation for pollutant entering the pond
A1*: Puts all the components together to form the correct differential equation

(b)
M1: Uses the model to find the integrating factor and attempts solution of their differential equation
A1: Correct solution
M1: Interprets the initial conditions to find the constant of integration
M1: Uses their solution to the problem to find the amount of pollutant after 8 days
A1: Correct number of grams

(c)
B1: Suggests a suitable refinement to the model

Notes
M1: A complete strategy to find *A*, *B* and *C* e.g. partial fractions. Allow slip when finding the constant but must be the correct form of partial fractions and correct identity. M1: Starts the process of differences to identify the relevant fractions at the start and end. Must have attempted a minimum of $r=0,\quad r=1,\quad \ldots\quad r=n-1$ and $r=n$ Follow through on their values of *A*, *B* and C. Look for $r=0\rightarrow \frac{A}{1}-\frac{B}{2}+\frac{C}{3}$ $\qquad r=1\rightarrow \frac{A}{2}-\frac{B}{3}+\frac{C}{4}$ $r=n-1\rightarrow \frac{A}{n}-\frac{B}{n+1}+\frac{C}{n+2}$ $\qquad r=n\rightarrow \frac{A}{n+1}-\frac{B}{n+2}+\frac{C}{n+3}$ A1: Correct fractions from the beginning and end that do not cancel stated. M1 Combines all 'their' fractions (at least two algebraic fractions) over their correct common denominator, does not need to be the lowest common denominator (allow a slip in the numerator). A1: Correct answer. **Note**: if they start with $r=1$ the maximum they can score is M1M0A0M1A0 **Note:** Proof by induction gains no marks

Question	Scheme	Marks	AOs
5(a)	The tank initially contains 100L. 3 L are entering every minute and 2 L are leaving every minute so overall 1 L increase in volume each minute so the tank contains 100 + *t* litres after *t* minutes	M1	3.3
	2 L leave the tank each minute and if there are *S*g of salt in the tank, the concentration will be $\frac{S}{100+t}\,g/L$ so salt leaves the tank at a rate of $2\times\frac{S}{100+t}\,g$ per minute	M1	3.3
	Salt enters the tank at a rate of $3\times 1g$ per minute	B1	2.2a
	$\therefore \frac{dS}{dt}=3-\frac{2S}{100+t}$ * cso	A1*	1.1b
		(4)	
(b)	$\frac{dS}{dt}+\frac{2S}{100+t}=3$		
	$I=e^{\int\frac{2}{100+t}dt}=(100+t)^2 \Rightarrow S(100+t)^2=\int 3(100+t)^2 dt$	M1	3.1b
	$S(100+t)^2=(100+t)^3(+c)$ OR $S(100+t)^2=30\,000t+300t^2+t^3(+c)$	A1	1.1b
	$t=0,\ S=0\Rightarrow c=-10^6$	M1	3.4
	$t=10\Rightarrow S=100+10-\frac{10^6}{(100+10)^2}$	dM1	1.1b

	OR $S(100+10)^2=(100+10)^3(+c)\Rightarrow S=\ldots$		
	$=\text{awrt } 27\,(\text{g})$ or $\dfrac{3310}{121}\,(\text{g})$	A1	2.2b
		(5)	
(c)	Concentration is $\left(100+t-\dfrac{10^6}{(100+t)^2}\right)\div(100+t)=0.9$ OR $S=0.9(100+t)\Rightarrow 0.9(100+t)=(100+t)-\dfrac{10^6}{(100+10)^2}$ OR $S=0.9(100+t)\Rightarrow 0.9(100+t)^3=(100+t)^3-10^6$	M1	3.4
	$(100+t)^3=10^7\Rightarrow t=\ldots$ OR $t^3+300t^2+30\,000t-9\,000\,000=0\Rightarrow t=\ldots$	dM1	1.1b
	$t=\text{awrt } 115\,(\text{minutes})$	A1	2.2b
		(3)	
(d)	E.g. • It is unlikely that mixing is instantaneous • The model will only be valid when the tank is not full • When the valve is closed, the model is not valid • It is unlikely that the concentration of salt water entering the tank remains exactly the same	B1	3.5a
		(1)	
		(13 marks)	

Notes

(a)

M1: A suitable explanation for the "100 + t" e.g. as a minimum (v) = 100 + 3t – 2t = 100 + t

M1: A suitable explanation for the $\dfrac{2S}{100+t}$

There need to be some explanation (words) for this part of the formula.

e.g. the concentration of (salt) $=\dfrac{S}{100+t}$ therefore (salt) out $=2\times\dfrac{S}{100+t}=\dfrac{2S}{100+t}$

e.g. salt out = $\dfrac{2S}{\text{volume of water}}=\dfrac{2S}{100+t}$

Note: M0 for $2\times\dfrac{S}{100+t}=\dfrac{2S}{100+t}$ only with no explanation

B1: Correct interpretation for the "3" e.g. salt in = 3 or $\dfrac{\mathrm{d}S}{\mathrm{d}t}$ in = 3

<u>Note:</u> Salt **water** in = 3 is **B0**

Question	Scheme	Marks	AOs
7(a)	$(1+t)\frac{dP}{dt}+P=t^{\frac{1}{2}}(1+t)\Rightarrow\frac{dP}{dt}+\frac{P}{1+t}=t^{\frac{1}{2}}$	B1	1.1b
	$I=e^{\int\frac{1}{1+t}dt}=1+t\Rightarrow P(1+t)=\int t^{\frac{1}{2}}(1+t)dt=...$	M1	3.1b
	$P(1+t)=\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+c$	A1	1.1b
	$t=0,\ P=5\Rightarrow c=5$	M1	3.4
	$P=\frac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}=\frac{\frac{2}{3}8^{\frac{3}{2}}+\frac{2}{5}8^{\frac{5}{2}}+5}{9}=...$	M1	1.1b
	= 10 277 bacteria (allow awrt 10 300)	A1	2.2b
		(6)	
(b)	$P=\frac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}\Rightarrow\frac{dP}{dt}=\frac{(1+t)\left(t^{\frac{1}{2}}+t^{\frac{3}{2}}\right)-\left(\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5\right)}{(1+t)^2}$ Alt: $P+(1+t)\frac{dP}{dt}=t^{\frac{1}{2}}+t^{\frac{3}{2}}\Rightarrow\frac{dP}{dt}=\frac{t^{\frac{1}{2}}+t^{\frac{3}{2}}-\frac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}}{(1+t)}$	M1 A1ft	3.4 1.1b
	$\left(\frac{dP}{dt}\right)_{t=1}=\frac{dP}{dt}=\frac{5\times10-\left(\frac{16}{3}+\frac{64}{5}+5\right)}{(5)^2}=\frac{403}{375}$	dM1	3.1a
	$\frac{403}{375}\times1000=\frac{3224}{3}(=awrt\,1070)$ bacteria per hour	A1	3.2a
		(4)	
	(b) Alternative:		
	$P=\frac{\frac{2}{3}t^{\frac{3}{2}}+\frac{2}{5}t^{\frac{5}{2}}+5}{(1+t)}=\frac{\frac{16}{3}+\frac{64}{5}+5}{(1+4)}$	M1	3.4
	$=\frac{347}{75}$	A1ft	1.1b
	$(1+t)\frac{dP}{dt}+P=t^{\frac{1}{2}}(1+t)\Rightarrow5\frac{dP}{dt}+\frac{347}{75}=2\times5\Rightarrow\frac{dP}{dt}=\frac{403}{375}$	dM1	3.1a
	$\frac{403}{375}\times1000=\frac{3224}{3}(=1075)$ bacteria per hour	A1	3.2a
		(4)	
(c)	E.g. • The number of bacteria increases indefinitely which is not realistic	B1	3.5b
		(1)	
			(11 marks)

Question	Scheme		Marks	AOs
8(a)	Volume of paint = 30 litres therefore Rate of paint out $= 3 \times \frac{r}{30}$ litres per second		M1	3.3
	$\frac{\mathrm{d}r}{\mathrm{d}t} = 2 - \frac{r}{10}$		A1	1.1b
			(2)	
(b)	Rearranges $\frac{\mathrm{d}r}{\mathrm{d}t} + \frac{r}{10} = 2$ and attempts integrating factor IF = $\mathrm{e}^{\int \frac{1}{10}\mathrm{d}t} = \ldots$	Separates the variables $\int \frac{1}{20-r}\,\mathrm{d}r = \frac{1}{10}\mathrm{d}t$ $\Rightarrow \ldots$	M1	3.1a
	$r\mathrm{e}^{\frac{t}{10}} = \int 2\mathrm{e}^{\frac{t}{10}}\,\mathrm{d}t \Rightarrow r\mathrm{e}^{\frac{t}{10}} = \lambda \mathrm{e}^{\frac{t}{10}}(+c)$	Integrates to the form $\lambda \ln(20-r) = \frac{1}{10}t(+c)$	M1	1.1b
	$r\mathrm{e}^{\frac{t}{10}} = 20\mathrm{e}^{\frac{t}{10}} + c$	$-\ln(20-r) = \frac{1}{10}t + c$	A1ft	1.1b
	$t = 0, r = 10 \Rightarrow c = \ldots$		M1	3.4
	$r = \frac{20\mathrm{e}^{\frac{t}{10}} - 10}{\mathrm{e}^{\frac{t}{10}}} = 15$ rearranges to achieve $\mathrm{e}^{\frac{t}{10}} = \alpha$ and solves to find a value for t or $r = 20 - 10\mathrm{e}^{-\frac{t}{10}} = 15$ rearranges to achieve $\mathrm{e}^{-\frac{t}{10}} = \beta$ and solves to find a value for t	$-\ln(20-15) = \frac{1}{10}t - \ln 10$ Leading to a value for t	M1	3.4
	$t =$ awrt 7 seconds		A1	2.2b
			(6)	
(c)	The model predicts 7 seconds but it actually takes 9 seconds so (over) 2 seconds out (over 20%), therefore it is not a good model		B1ft	3.5a
			(1)	
				(9 marks)

Notes:

(a)

M1: Clearly identifies that Rate of paint out $= 3 \times \frac{r}{\text{their volume}}$. It is a "show that" question so there must be clearly reasoning. Just answer with no reasoning scores M0.

A1: Puts all the components together to form the correct differential equation.

Question	Scheme	Marks	AOs
3(a)	$\frac{dy}{dx}+y\tan x=e^{2x}\cos x$ IF $=e^{\int \tan x\,dx}=e^{\ln\sec x}=\sec x\Rightarrow \sec x\frac{dy}{dx}+y\sec x\tan x$ $=e^{2x}$ $\Rightarrow y\sec x=\int e^{2x}\ dx$	M1	3.1a
	$y\sec x=\frac{1}{2}e^{2x}(+c)$	A1	1.1b
	$y=\left(\frac{1}{2}e^{2x}+c\right)\cos x$	A1	1.1b
		(3)	
(b)	$x=0,\ y=3\Rightarrow c=\ldots\{2.5\}$	M1	3.1a
	$y=\left(\frac{1}{2}e^{2x}+\frac{5}{2}\right)\cos x=0\Rightarrow \cos x=0\Rightarrow x=\ldots$	M1	1.1b
	$x=\frac{\pi}{2}$	A1	1.1b
		(3)	
		(6 marks)	

Notes:

(a)

M1: Finds the integrating factor and attempts the solution of the differential equation.

Look for I.F. $=e^{\int \tan x dx}\Rightarrow y\times$ 'their I.F.' $=\int e^{2x}\cos x\times$ 'their I.F.' dx

A1: Correct solution condone missing + *c*

A1: Correct general solution, Accept equivalents of the form *y* = f(*x*), such as $y=\frac{e^{2x}}{2\sec x}+\frac{c}{\sec x}$

(b)

M1: Uses $x=0\quad y=3$ to find the constant of integration. Allow if done as part of part (a) and allow for their answer to (a) as long as it has a constant of integration to find.

M1: Sets $y=0$ in an equation of the form $y=(Ae^{2x}+c)\cos x$ (oe) where *A* is 1, 2 or $\frac{1}{2}$, with their *c* or constant *c* and makes a valid attempt to solve the equation to find a value for *x*. (Allow even if the constant of integration has not been found).

A1: Depends on both M's. Awrt 1.57 or $\frac{\pi}{2}$ only. There must have been an attempt to find the constant of integration, but allow from a correct answer to (a) as long as a positive value for *c* has been found (can be scored from implicit form).

Question	Scheme	Marks	AOs
6(a)	$\frac{dV}{dt}=3-\frac{4}{1+e^{0.8t}}\pm kV$ (where k is constant)	M1	3.3
	$t=0, V=10, \frac{dV}{dt}=-3 \Rightarrow -3=3-\frac{4}{1+1}-10k \Rightarrow k=...$	dM1	3.4
	$\Rightarrow 10k=4 \Rightarrow k=\frac{2}{5} \Rightarrow \frac{dV}{dt}=3-\frac{4}{1+e^{0.8t}}-0.4V$ *	A1*	2.1
		(3)	
(b)	$\frac{d}{dt}\left(\arctan e^{0.4t}\right)=\frac{1}{1+\left(e^{0.4t}\right)^2}\times ke^{0.4t}$	M1	1.1b
	$\frac{d}{dt}\left(\arctan e^{0.4t}\right)=\frac{2e^{0.4t}}{5\left(1+e^{0.8t}\right)}$ oe	A1	1.1b
		(2)	
	Alternative to part (b):		
	$y=\arctan e^{0.4t} \Rightarrow \tan y=e^{0.4t} \Rightarrow \sec^2 y\frac{dy}{dx}=0.4e^{0.4t}$	M1	1.1b
	$\frac{dy}{dx}=\frac{0.4e^{0.4t}}{\sec^2 y}=\frac{0.4e^{0.4t}}{1+\tan^2 y}=\frac{0.4e^{0.4t}}{1+\left(e^{0.4t}\right)^2}$	A1	1.1b
		(2)	
(c)	$\frac{dV}{dt}+0.4V=3-\frac{4}{1+e^{0.8t}} \Rightarrow I.F.\left(=e^{\int 0.4\,dt}\right)=e^{0.4t}$	B1	2.2a
	$e^{0.4t}V=\int 3e^{0.4t}-\frac{4e^{0.4t}}{1+e^{0.8t}}\,dt$	M1	1.1b
	$=Ae^{0.4t}-B\arctan\left(e^{0.4t}\right)(+c)$	M1	1.1b
	$e^{0.4t}V=7.5e^{0.4t}-10\arctan\left(e^{0.4t}\right)\ (+c)$	A1	1.1b
	$V=10, t=0 \Rightarrow 10=7.5-10\arctan 1+c \Rightarrow c=...$	M1	3.4
	$V=7.5-10e^{-0.4t}\arctan\left(e^{0.4t}\right)+2.5(\pi+1)e^{-0.4t}$	A1	2.1
		(6)	
(d)	E.g. $V(10)\approx 7.4$ litres so the model is not very accurate as it predicts approximately 7.5% below the actual level.	B1ft	3.5a
		(1)	
		(12 marks)	