

## Functions

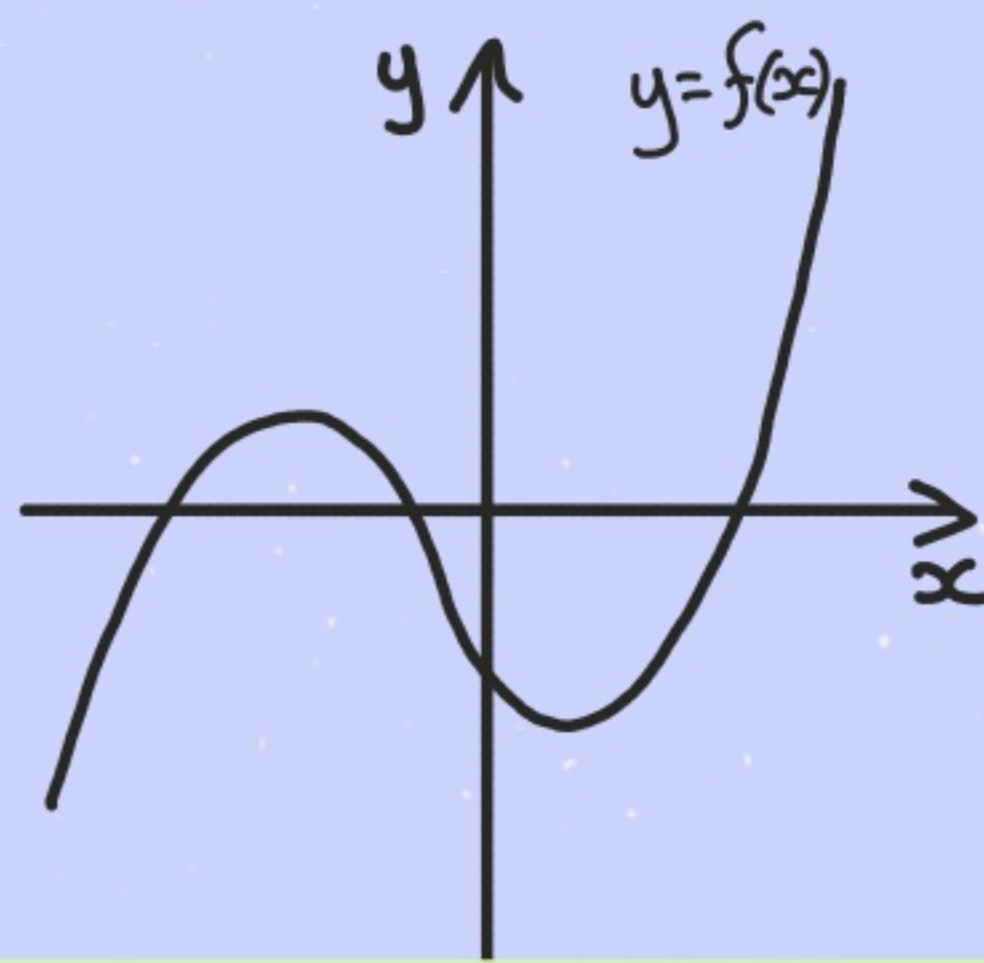
### Key Formulae

#### The Modulus Function

$$\bullet |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} \quad |f(x)| = \begin{cases} f(x) & f(x) \geq 0 \\ -f(x) & f(x) < 0 \end{cases}$$

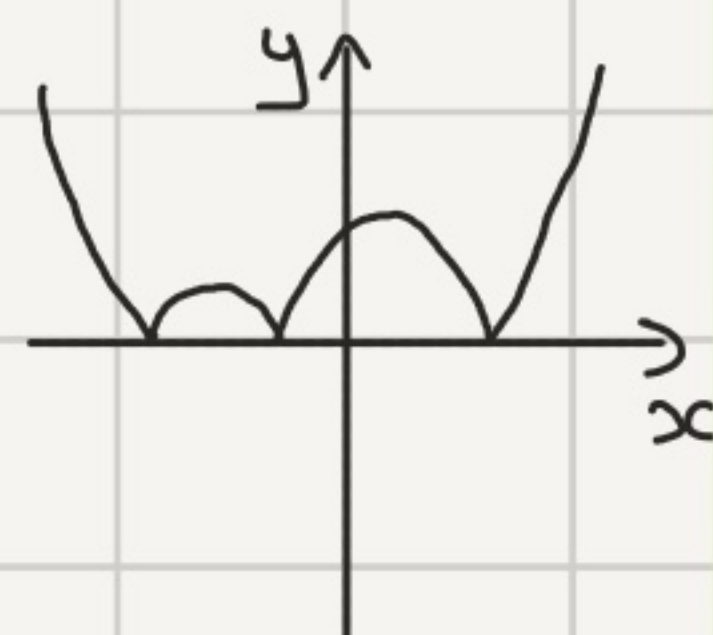
### Key Graphs

Let the graph of  $y=f(x)$  be

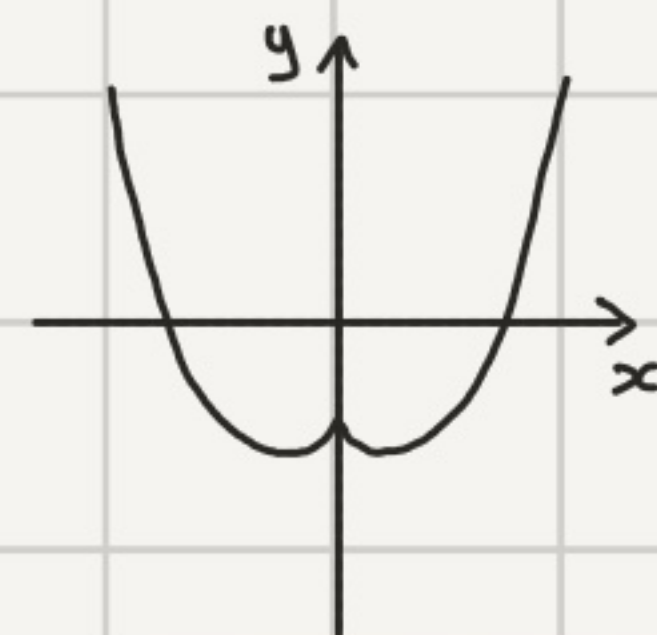


#### Sketching the Modulus Function

- $y = |f(x)|$ 
  - 1) Sketch the same as  $y=f(x)$  above the  $x$ -axis.
  - 2) Reflect where  $y=f(x)$  is below the  $x$ -axis in the  $x$ -axis.



- $y = f(|x|)$ 
  - 1) Sketch  $y=f(x)$  for  $x \geq 0$ .
  - 2) Reflect what you've drawn in the  $y$ -axis.



#### Sketching Transformations

- $y = f(x) + a$  translation by vector  $\begin{pmatrix} 0 \\ a \end{pmatrix}$
- $y = f(x+a)$  translation by vector  $\begin{pmatrix} -a \\ 0 \end{pmatrix}$
- $y = -f(x)$  reflection in the  $x$ -axis.
- $y = f(-x)$  reflection in the  $y$ -axis.
- $y = af(x)$  stretch by factor  $a$  along the  $y$ -axis.
- $y = f(ax)$  stretch by factor  $\frac{1}{a}$  along the  $x$ -axis.



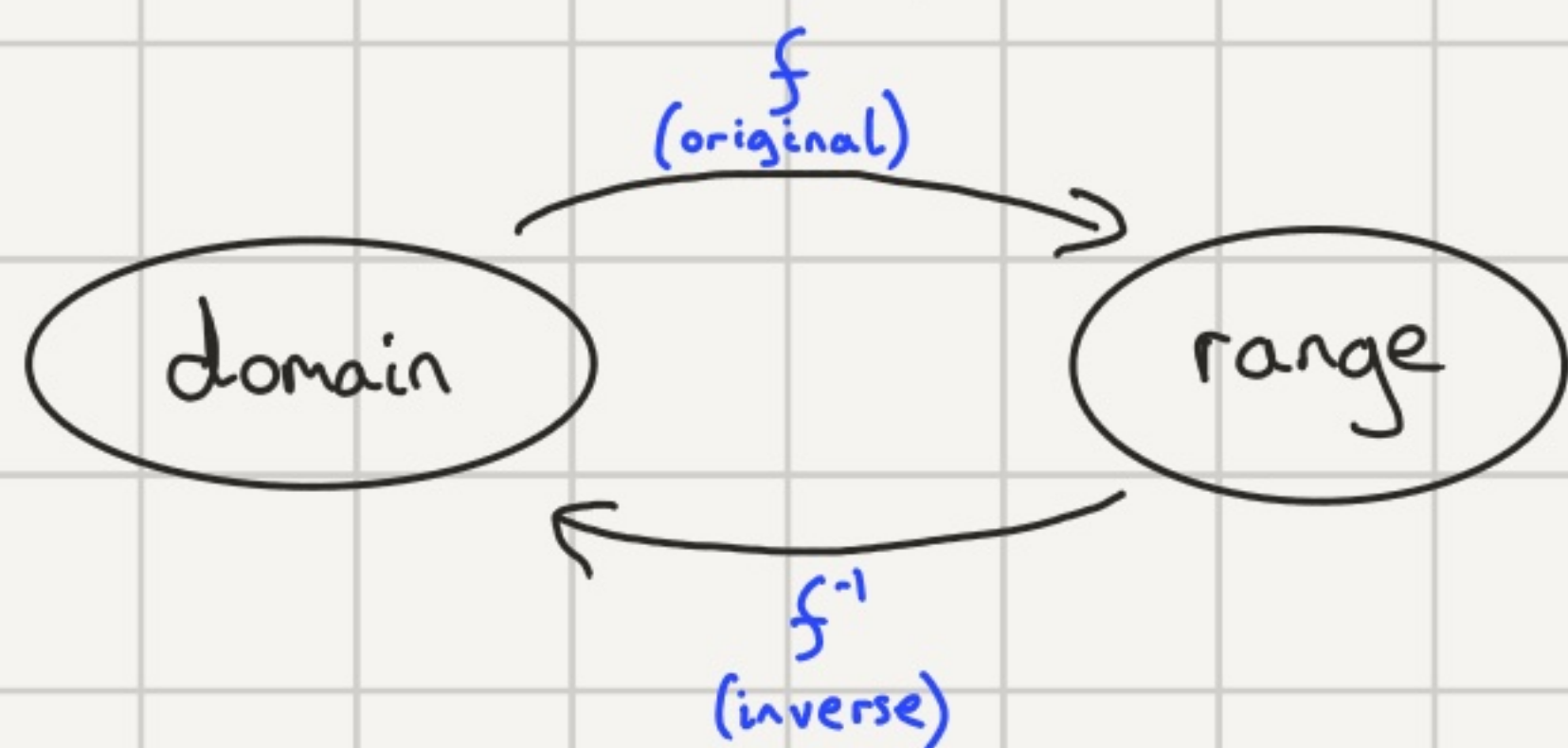
## Key Concepts

### Domain & Range

- The domain of a function is the set of input values.
- The range of a function is the set of output values.

### Inverse Functions

- What is an inverse function?  
An inverse function maps the range of the original function back to the domain of the original function.



- Inverse functions only exist for one-to-one functions.
- The graph of an inverse function can be found by reflecting the original function in the line  $y=x$ .

### Types of Mapping

- **one-to-one** Each  $x$  value maps to one  $y$  value. Each  $y$  value has only one  $x$  value that maps to it.
- **many-to-one** Each  $x$  value maps to one  $y$  value. Some  $y$  values have more than one  $x$  value that map to them.
- **one-to-many** Some  $x$  values map to more than one  $y$  value.
- What's a function? A one-to-one or many-to-one mapping is called a function.

### Composite Functions

- $f(g(x))$  means substitute  $x$  into  $g$ , and substitute the result into  $f$ .
- $f \circ g(x) = f(g(x))$ .