Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, \ldots : u_{n+1}=\frac{1}{u_n},\ u_1=\frac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\ \sum_{r=1}^{16}\left(3+5r\right)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\frac{16}{2}(2(8)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=728+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\}\ \sum_{r=1}^{16}3\ +\ \sum_{r=1}^{16}\left(5r\right)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times16)+\frac{16}{2}(2(5)+15(5))\ +\ \frac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=48+680+131\,070\ =\ 131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum $=\ 10+17+26+39+60+97+166+299+560+1077+2106$	M1	3.1a
	$+4159+8260+16457+32846+65619=131\,798$ *	M1	1.1b
		M1	1.1b
		A1*	2.1
		(4)	
(ii)	$\left\{u_1=\frac{2}{3}\right\},\ u_2=\frac{3}{2},\ u_3=\frac{2}{3}, \ldots$ (***can be implied by later working***)	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\}\ 50\left(\frac{2}{3}\right)+50\left(\frac{3}{2}\right)$ or $50\left(\frac{2}{3}+\frac{3}{2}\right)$	M1	2.2a
	$=\frac{325}{3}$ $\left(\text{or } 108\frac{1}{3} \text{ or } 108.\dot{3} \text{ or } \frac{1300}{12} \text{ or } \frac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

Question	Scheme	Marks	AOs
8 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \ldots$		
	$= \dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \ldots\ldots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\ldots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}\ \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \ldots\ldots + \log_5 50) - (\log_5 2 + \log_5 3 + \ldots\ldots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1} = \dfrac{k(a_n + 2)}{a_n}$ once with $a_1 = 2$	M1	1.1b
	$(a_1 = 2), a_2 = 2k, a_3 = k+1, a_4 = \dfrac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\dfrac{k(k+3)}{k+1} = 2 \Rightarrow k^2 + 3k = 2k + 2 \Rightarrow k^2 + k - 2 = 0$ *	A1*	2.1
		(3)	
(b)	States that when $k = 1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4} = 2, a_{2/5} = -4, a_{3/6} = -1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26 \times (2 + -4 + -1) + 2 + -4$	M1	3.1a
	$= -80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + ... + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + ... + ar^n \Rightarrow S_n - rS_n = ...$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4 \times \dfrac{a(1-r^5)}{1-r}$ or $4 \times \dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1 - r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5 - 1)(r^5 - 3) = 0 \Rightarrow r^5 = ...$ or e.g. $1 - r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = ...$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Question Number	Scheme	Marks
5.		
(a)	$u_2 = 2 - \frac{4}{3} = \frac{2}{3},\ u_3 = 2 - \frac{4}{\frac{2}{3}} = -4,\ u_4 = 2 - \frac{4}{-4} = 3$	M1 A1 A1
		[3]
(b)	$u_{61} = 3$.	B1
		[1]
(c)	$\sum_{i=1}^{99} u_i = (3+\frac{2}{3}-4)+(3+\frac{2}{3}-4)+(3+\frac{2}{3}-4)+\ldots$	M1
	$\sum_{i=1}^{99} u_i = 33\times(\ldots+\ldots+\ldots)$, $=-11$	A1, A1
		[3]
(c)	**Alternative method** for part (c) Adds $n\times"3"+n\times"-4"+n\times"\frac{2}{3}"$	M1
	Uses n = 33	A1
	-11	A1
		[3]
		7 marks
	Notes	
(a)	**M1:** Attempt to use formula correctly (implied by first term correct, or given as 0.67, or third term following through from their second etc) **A1**: two correct answers **A1**: 3 correct answers (allow 0.6 recurring but not 0.667) Look for the values. Ignore the u_r label	
(b)	**B1**: cao (NB Use of AP is B0)	
(c)	**M1:** Uses sum of at least 3 terms found from part (a)) (may be implied by correct answer). Attempt to sum an AP here is M0. **A1**: obtains $33\times$(sum of three adjacent terms) or $11\times$(sum of nine adjacent terms) **A1**: - 11 cao (-11 implies both A marks) N.B. Use of n = 99 is M1A0A0	

Question Number	Scheme	Marks
11. (a)	Uses $(2p-6)-4p = 4p-60$ or $4p=\frac{60+(2p-6)}{2}$ or $60+2(4p-60)=2p-6$ or etc… or two correct equations with d	M1
	So $p = 9$ *	A1 * **[2]**
Alternative to (a)	Use p=9 to give 60, 36 and 12 and deduce d = -24 so conclude AP when p = 9	M1 A1 [2]
(b)	Uses $a + 19d$ with $a = 60$	M1
	Finds $d = 36 - 60 = -24$	B1
	So obtains -396	A1 **[3]**
(c)	Uses $\frac{n}{2}(2\times 60+(n-1)d)$	M1
	Uses $\frac{n}{2}(2\times 60-24(n-1))$	A1
	$= 12n\,(6-n)$ *	A1* **[3]**
		8 marks
	Notes	
(a)	M1**: Correct** equation to enable p to be found or two correct equations if d introduced and solving simultaneous equations to eliminate d and enable p to be found NB May add three terms and use sum formula giving e.g. $60+4p+2p-6=\frac{3}{2}(60+2p-6)$ A1: cso (Do not need intermediate step)	
(b)	M1: Correct formula with their value for d B1: d = -24 seen in (a) or (b) A1: -396 If all terms are found and added 60 + 36 + 12 + -12 + .. Need 20 terms for M1, need -24 implied by first 4 terms for B1 and correct answer for A1	
(c)	M1:Uses correct formula with their value for d A1: Correct value for d A1: given answer – must be no errors to award this mark Special case: Proves formula for sum of AP M1: Correct method of proof using their d A1: For d = -24 A1: given answer – must be no errors to award this mark	

Question Number	Scheme	Marks
9(i)	$\sum_{r=1}^{20}(3+5r)=8+13+18+..........+103$	M1
	Use of $S_n=\frac{n}{2}\left(2a+(n-1)d\right)$ or $S_n=\frac{n}{2}\left(a+l\right)$ with a=3or 8, n=19 or 20, d=5 and l=103	M1
	$S_{20}=\frac{20}{2}\left(8+103\right)=1110$	A1
		(3)
(ii)	$\sum_{r=0}^{\infty}\frac{a}{4^r}=16\Rightarrow\frac{a}{1}+\frac{a}{4}+\frac{a}{16}....=16$ $\quad r=\frac{1}{4}$ oe	B1
	Use of $S_\infty=\frac{a}{1-r}$ with $0<\lvert r\rvert<1$ and $S_\infty=16$	M1
	$16=\frac{a}{1-'r'}\Rightarrow a=..$	dM1
	$a=12$	A1
		(4)
		(7 marks)

Question Number	Scheme	Marks
5. (a)	$S_n = a + (a+d) + (a+2d)+ ... + (a+(n-1)d)$	M1
	$S_n = (a+(n-1)d) + (a+(n-2)d) + ... + (a+d) + a$	M1
	$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + ... + (2a+(n-1)d)$	M1
	$S_n = \frac{n}{2}[2a+(n-1)d]$* See notes below for those who use triangle numbers in their proof	A1*
		[4]
(b)	**Uses** either $\frac{n}{2}(2\times a+(n-1)7)$ or $\frac{n}{2}(a+497)$ or $7\times\sum_{i=1}^{71} i$	M1
	i.e $\frac{71}{2}(2\times 7+70\times 7)$ or $\frac{72}{2}(2\times 0+71\times 7)$ or $\frac{71}{2}(7+497)$ or $7\times\frac{71}{2}(72)$	A1
	$= 17892$	A1
		[3]
		7 marks
	Notes	

(a) **M1**: List terms including **at least first two** and **a last term which may be *a* + *nd* or *a* + (*n* – 1)*d* or *L***

M1: List terms in reverse including **at least their last term (or correct last term) and finally their first term**

M1: The LHS should be $2S$. The RHS must follow from at least two terms correctly matching in the addition and should include at least two terms which are each **correctly** $\{2a+(n-1)d\}$ or $(a + L)$ **or should** be $n\{2a+(n-1)d\}$ or $n(a + L)$

A1: Need some indication of at least three terms being added (i.e at least three terms and their pairs listed with terms correctly matching or three additions seen) and also need to achieve final answer with no errors and if L was used need to state that $L = a + (n - 1)d$

NB: Some candidates use a variation of

$$\sum_{r=1}^{n}(a+(r-1)d = \sum_{r=1}^{n} a + d\sum_{r=1}^{n}(r-1) = na + d\frac{n}{2}(n+1) - dn \text{ or } na + d\frac{(n-1)}{2}(n)$$

And conclude that $S_n = \frac{n}{2}[2a+(n-1)d]$. This gains the full 4 marks M1M1M1A1, but must be completely correct.

(b) **M1:Uses** correct formula (with their a and n) with d =7 or with last term correct

A1: Uses consistent and correct a and n

A1: Correct answer

Question Number	Scheme	Marks
8.		
(a)	$u_2 = 3k - 12,\ u_3 = 3(u_2) - 12$	M1
	$u_2 = 3k - 12,\ u_3 = 9k - 48$	A1
	$u_4 = 3(9k - 48) - 12 = 27k - 156$ (ft their u_3) .	M1 A1ft
		[4]
(b)	$27k - 156 = 15$ so $k =$	M1
	$k = 6\frac{1}{3}$ or $\frac{19}{3}$ or 6.33 (3sf)	A1
		[2]
(c)	$\sum_{i=1}^{4} u_i = 6\frac{1}{3} + 7 + 9 + 15$ or $\sum_{i=1}^{4} u_i = k + 3k - 12 + 9k - 48 + 27k - 156$	M1
	$= 40k - 216$, $= 37\frac{1}{3}$ or $\frac{112}{3}$	A1ft, A1cao
		[3]
		9 marks
	Notes	

(a) **M1:** Attempt to use formula twice to find u_2 and u_3

A1: two correct simplified answers

M1: Attempt again to find u_4

A1ft: 4th term correct and simplified - follow through their u_3

(b) **M1**: Put their 4th term (not 5th) equal to 15 and attempt to find $k =$

A1: accept any correct fraction or decimal answer (allow 6.33 or better here)

(c) **M1:** Uses 1st term and their following 3 terms **with plus signs** (either numerical or in terms of *k*). Must be using terms from iteration and not formula for an AP or GP. May make a copying slip.

A1ft: for $40k - 216$ or follow through on their k so check $40k$ -216 for their k

A1: obtains $37\frac{1}{3}$ (must be exact) if exact answer given, then isw

Those who use 6.3 will obtain 36 They should have M1A1ftA0 – should have used exact *k* to give exact answer here.

Those who use 6.33 will obtain 37.2 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

Those who use 6.333 will obtain 37.32 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

6.3333 will obtain 37.332 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

6.33333 will obtain 37.3332 etc All these answers should have M1A1ftA0 – should have used exact *k* to give exact answer here. Etc

Special case: Those who use $k = 6$ will obtain $6 + 6 + 6 + 6 = 24$ This is M1 A0 A0 in part (c) – as over simplified

Question Number	Scheme	Marks
10 (a)	$u_2=\frac{8}{3}$ or $2\frac{2}{3}$, $u_3=\frac{16}{9}$ or $1\frac{7}{9}$, $u_4=\frac{32}{27}$ or $1\frac{5}{27}$	M1, A1
		[2]
(b)	$u_{20}=4\times\left(\frac{2}{3}\right)^{19}$;$=0.00180$ or 0.0018 or exact equivalent	M1; cao A1
		[2]
(c)	Use $\sum_{i=1}^{16}u_i=\frac{4(1-\left(\frac{2}{3}\right)^{16})}{1-\frac{2}{3}}$	M1
	Find 12 - *their* $\sum_{i=1}^{16}u_i$	dM1
		A1
	$= 12 - 11.9817$ = awrt 0.0183	**[3]**
(d)	12 is the **sum to infinity** (and all terms are **positive)** so sum is less than 12	B1
	Or $\sum_{i=1}^{n}u_i=\frac{4(1-\left(\frac{2}{3}\right)^{n})}{1-\frac{2}{3}}=12-12\left(\frac{2}{3}\right)^n$ *and* $\left(\frac{2}{3}\right)^n>0$ so is less than 12	**[1]**
		[8 marks]

(a)

M1 Any one term is 2/3 the previous term. Accept for example $u_2=\text{awrt }2.67$

A1 All 3 terms correct. Accept exact equivalents $u_2=\frac{8}{3}$ or $2\frac{2}{3}$, $u_3=\frac{16}{9}$ or $1\frac{7}{9}$, $u_4=\frac{32}{27}$ or $1\frac{5}{27}$

(b)

M1 Uses correct nth term formula ar^{n-1} with $a=4, n=20$ and $r=\frac{2}{3},\frac{3}{2}$ or awrt 0.7

Condone for the M mark use of ar^{n-1} with $a=\frac{8}{3}$ (awrt 2.67), $n=20$ and $r=\frac{2}{3},\frac{3}{2}$ or awrt 0.7

Expressions such as $4\times\left(\frac{2}{3}\right)^{19}$, $\frac{8}{3}\times\left(\frac{2}{3}\right)^{18}$ and $\frac{2^{n+1}}{3^{n-1}}\rightarrow\frac{2^{21}}{3^{19}}$ are correct and sufficient for M1

A1 Accept any of $0.0018, 0.00180$, 1.80×10^{-3} or 1.8×10^{-3}

(c)

M1 Uses the correct sum formula $S=\frac{a\left(r^n-1\right)}{(r-1)}$ or $S=\frac{a\left(1-r^n\right)}{(1-r)}$ with $a=4, r=\frac{2}{3},\frac{3}{2}$ or awrt 0.7, $n=16$

Condone the sum formula $S=\frac{a\left(r^n-1\right)}{(r-1)}$ or $S=\frac{a\left(1-r^n\right)}{(1-r)}$ with $a=\frac{8}{3}$ (awrt 2.67), $r=\frac{2}{3},\frac{3}{2}$ or awrt 0.7, $n=16$

dM1 Dependent upon the previous M mark. Score for an attempt at finding $12-\sum_{i=1}^{16}u_i$

A1 awrt 0.0183

Note: Some candidates may list all 16 terms which is acceptable provided the answer is accurate

(d)

B1 Need a reason + a minimal conclusion. Eg The sum to infinity =12 **and** sum is less than 12
Allow sum to infinity is 12, hence true.

Question Number	Scheme	Marks
5(i)		
(a)	$(U_2)=\dfrac{4}{4-3}=4$	B1
		(1)
(b)	$\sum_{n=1}^{100} U_n = 100\times 4 = 400$	M1A1
		(2)
5(ii)	$\sum_{r=1}^{n}(100-3r)<0 \Rightarrow 97+94+91+...(100-3r)<0$	
	$\sum$AP with $a=97, d=-3, n=n, S<0 \Rightarrow 0=\dfrac{n}{2}(2\times 97+(n-1)\times -3)<0$	M1
	$\Rightarrow \dfrac{n}{2}(197-3n)<0 \Rightarrow n>65.\dot{6}$	dM1
	$\Rightarrow n=66$	A1
		(3) **(6 marks)**
(ii) ALT I	$\sum_{r=1}^{n}(100-3r)<0 \Rightarrow \sum_{r=1}^{n}3r > \sum_{r=1}^{n}100$	
	$\Rightarrow 3\dfrac{n(n+1)}{2}>100n$	M1 M1A1
	$\Rightarrow n>65.\dot{6} \Rightarrow n=66$	

(i)(a)

B1 States that U_2 is 4. Accept $\dfrac{4}{1}$ but not $\dfrac{4}{4-3}$ and remember to isw.

Note that $U_1=4$ so be sure that you don't award this B1

(i)(b)

M1 Uses the method that $\sum_{n=1}^{100} U_n = k\times 4$ where $k=100$ or 99

You may see the AP formula being used which is fine as long as $a=4,\ d=0$ and $n=99/100$

Look for expression of the form $\dfrac{100}{2}\{2\times 4+99\times 0\}$ OR $\dfrac{100}{2}\{4+4\}$

A1 400

Question	Scheme	Marks
6.		
(a)	$u_2 = 24,\ u_3 = 16$ and $u_4 = \dfrac{32}{3}$	M1, A1 **[2]**
(b)	$r = \dfrac{2}{3}$	B1 **[1]**
(c)	$u_{11} = ar^{10} = 36 \times (r)^{10}$.	M1
	$u_{11} = ar^{10} = 36 \times \left(\dfrac{2}{3}\right)^{10} = \left(\dfrac{4096}{6561}\right)$	
	$= 0.6243$	A1 **[2]**
(d)	$\sum_{i=1}^{6} u_i = \dfrac{36(1-(\frac{2}{3})^6)}{1-\frac{2}{3}}$ or $\sum_{i=1}^{6} u_i = 36+24+16+\dfrac{32}{3}+u_5+u_6$	M1
	$= 98\frac{14}{27}$	A1cao **[2]**
(e)	$\sum_{i=1}^{\infty} u_i = \dfrac{36}{1-\frac{2}{3}} = 108$	M1 A1 **[2]**
		9 marks
	Notes	

(a)

M1: Attempt to use formula correctly at least twice. It may be seen for example in u_3 and u_4

A1: All three correct exact simplified answers. Allow $10.\dot{6}$

(b)

B1: Accept $\dfrac{2}{3}$ or equivalent such as $\dfrac{24}{36}$ Allow awrt 0.667

(c)

M1: Uses $u_{11} = ar^{10} = 36 \times (r)^{10}$ with their r

A1: Accept awrt 0.6243 or $\dfrac{4096}{6561}$

(d)

M1: Uses correct sum formula with a = 36 and their r **or** alternatively for adding their first six terms.
FYI Sight of 36, 24, 16, 10.7, 7.1, 4.7 followed by 98.5 implies this mark. (You may only see the first 4 terms in part a)

A1: Obtains $= 98\frac{14}{27}$ (must be exact). For information $\dfrac{2660}{27} = 98\frac{14}{27}$ Allow $98.\dot{5}1\dot{8}$

(e)

M1: Uses correct sum to infinity formula with a = 36 and either $r = \dfrac{2}{3}$ or their r as long as $|r| < 1$

A1: Obtains 108 (must be exact)

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="7">4 (a)</td><td>$S_9 = 54$
$\Rightarrow 54 = \frac{9}{2}(2a+8d)$
or
$\Rightarrow 54 = \frac{9}{2}(a+a+8d)$</td><td>Uses a correct sum formula with $n = 9$ and $S_9 = 54$</td><td>M1</td></tr>
<tr><td>$\Rightarrow a+4d = 6$*</td><td>cso</td><td>A1*</td></tr>
<tr><td colspan="2">Listing:</td><td></td></tr>
<tr><td colspan="2">$a+a+d+a+2d+....+a+8d = 54$
$\Rightarrow 9a+36d = 54$
Scores M1 for attempting to sum 9 terms (both lines needed)
or
$a+a+d+a+2d+a+3d+a+4d+a+5d+a+6d+a+7d+a+8d = 54$
Scores M1 on its own and then A1 if they complete correctly.</td><td></td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td colspan="2"></td><td>(2)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td rowspan="6">(b)</td><td>$a+7d = \frac{1}{2}(a+6d)$
or
$\frac{1}{2}(a+7d) = a+6d$</td><td>Uses $t_8 = \frac{1}{2}t_7$ or $\frac{1}{2}t_8 = t_7$ to produce one of these equations.</td><td>M1</td></tr>
<tr><td>$\Rightarrow 6-4d+7d = \frac{1}{2}(6-4d+6d)$
$\Rightarrow d = ...$</td><td>Uses the given equation from (a) and their second linear <u>equation</u> in a and d and proceeds to find a value for either a or d.</td><td>M1</td></tr>
<tr><td rowspan="2">$\Rightarrow d = -1.5, a = 12$</td><td>A1: Either $d = -1.5$ (oe) or $a = 12$</td><td rowspan="2">A1A1</td></tr>
<tr><td>A1: Both $d = -1.5$(oe) and $a = 12$</td></tr>
<tr><td colspan="2">Note that use of $\frac{1}{2}t_8 = t_7$ in (b) gives a = 30 and d = -6</td><td></td></tr>
<tr><td colspan="2"></td><td>(4)</td></tr>
<tr><td></td><td colspan="2"></td><td>(6 marks)</td></tr>
</table>

Question Number	Scheme	Marks
9.(a)	$a=7k-5,\ ar=5k-7$ and $ar^2=2k+10$	B1
	(So $r=$) $\dfrac{5k-7}{7k-5}=\dfrac{2k+10}{5k-7}$ or $(7k-5)(2k+10)=(5k-7)^2$ or equivalent	M1
	See $(5k-7)^2=25k^2-70k+49$	M1
	$14k^2+60k-50=25k^2-70k+49 \rightarrow\ 11k^2-130k+99=0$ *	A1cso * **(4)**
(b)	$(k-11)(11k-9)$ so $k=$	M1
	k = 9/11 only* (after rejecting 11)	A1*
	N.B. Special case k = 9/11 can be verified in (b) (1 mark only)	
	$11\times\left(\dfrac{9}{11}\right)^2-130\times\left(\dfrac{9}{11}\right)+99=\dfrac{81}{11}-\dfrac{1170}{11}+\dfrac{1089}{11}=0$ M1A0	**(2)**
(c)	$a=\dfrac{8}{11}$	B1
	$\dfrac{5\times\frac{9}{11}-7}{7\times\frac{9}{11}-5}$ *or* $\dfrac{2\times\frac{9}{11}+10}{5\times\frac{9}{11}-7}$ so $r=-4$	B1
	(i) Fourth term = $ar^3=-\dfrac{512}{11}$	M1A1
	(ii) $S_{10}=\dfrac{a(1-r^{10})}{(1-r)}=\dfrac{\frac{8}{11}(1-(-4)^{10})}{(1-(-4))}=-152520$	M1A1
		(6) **[12]**

Notes

(a) Mark parts (a) and (b) together

B1: Correct statement (needs all three terms)**– this may be omitted and implied** by correct statement in k only, as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately)

M1: Valid Attempt to eliminate a and r and to obtain equation in k only

M1: Correct expansion of $(5k-7)^2=25k^2-70k+49$ - may have four terms $(5k-7)^2=25k^2-35k-35k+49$

A1cso: No incorrect work seen. The printed answer is obtained including "=0".

(b) M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula – see notes at start of mark scheme) **or** see 9/11 substituted and given as "=0" for M1A0

A1*: 9/11 **only and** 11 should be seen and rejected. Accept 9/11 underlined or k= 9/11 written on following line. Alternatively (k − 11) may be seen in the factorisation and a statement 'k not integer' given with k=9/11 stated.

(c) Mark parts (i) and (ii) together

B1: $a=\dfrac{8}{11}$ or any equivalent (If not stated explicitly or used in formula may be implied by correct answer to (ii))

B1:Substitutes k = 9/11 completely and obtain $r=-4$ (If not stated explicitly, may be implied by correct answer to (i) or (ii))

(i) M1: Use of correct formula with n = 4 a and/or r may still be in terms of k or uses $(2k+10)\times r$. May assume $r=k$.

A1: Correct exact answer

(ii) M1: Use of correct formula with n = 10 a and/or r may still be in terms of k May assume $r=k$ A1 : -152520 cao

NB Correct formula **with negative sign** in numerator followed by the incorrect $(8/11)(1+4^{10})/(1-(-4))$ usually found equal to 152520.2909 with no negative sign can be allowed M1A0 but if the incorrect numerical expression appears on its own with no formula then M0A0

Listing terms can get: B1 (first term) B1 M1A1 (implied by correct 4th term) M1A1 (implied by −152520)

Question Number	Scheme	Marks
5.(i)	**Mark (a) and (b) together**	
(a)	$a+ar=34$ or $\frac{a(1-r^2)}{(1-r)}=34$ or $\frac{a(r^2-1)}{(r-1)}=34$; $\frac{a}{1-r}=162$	B1; B1
(Way 1)	Eliminate a to give $(1+r)(1-r)=\frac{17}{81}$ or $1-r^2=\frac{34}{162}$ .. (not a cubic)	aM1
	(and so $r^2=\frac{64}{81}$ and) $r=\frac{8}{9}$ only	aA1 **(4)**
(b)	Substitute their $r=\frac{8}{9}$ ($0<r<1$) to give $a=$ $a=18$	bM1 bA1 **(2)**
(Way 2) Part (b) first	Eliminate r to give $\frac{34-a}{a}=1-\frac{a}{162}$	bM1
	gives $a=18$ or 306 and rejects 306 to give $a=18$	bA1
Then part (a) again	Substitute a = 18 to give $r=$	aM1
	$r=\frac{8}{9}$	aA1
(ii)	$\frac{42(1-\frac{6}{7}^n)}{1-\frac{6}{7}}>290$ (For trial and improvement approach see notes below)	M1
	to obtain So $(\frac{6}{7})^n<(\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n>(\frac{294}{4})$ or $(\frac{6}{7})^n<(\frac{2}{147})$	A1
	So $n>\frac{\log''(\frac{4}{294})''}{\log(\frac{6}{7})}$ or $\log_{\frac{6}{7}}''(\frac{4}{294})''$ or equivalent but must be log of positive quantity	M1
	(i.e. $n>$ 27.9) so $n=28$	A1 **(4)**

Notes

(i) (a) **B1**: Writes **a correct** equation connecting a and r and 34 (allow equivalent equations – may be implied)
B1: Writes a **correct** equation connecting a and r and 162 (allow equivalent equation – may be implied)

Way 1: a**M1**: Eliminates a correctly for these two equations to give $(1+r)(1-r)=\frac{17}{81}$ or $(1+r)(1-r)=\frac{34}{162}$ or equivalent – **not a cubic** – should have factorized (1 - r) to give a correct quadratic

a**A1:** Correct value for r. Accept 0.8 recurring or 8/9 (not 0.889) Must only have positive value.

b**M1**: Substitutes their r ($0<r<1$) into a correct formula to give value for a. Can be implied by $a=18$

b**A1**: must be 18 (not answers which round to 18)

Way 2: Finds *a* first - B1, B1: As before then award the (b) M and A marks before the (a) M and A marks

b**M1**: Eliminates r correctly to give $\frac{34-a}{a}=1-\frac{a}{162}$ or $a^2-324a+5508=0$ or equivalent

b**A1:** Correct value for a so $a=18$ only. (Only award after 306 has been rejected)

a**M1**: Substitutes their 18 to give $r=$

a**A1**: $r=\frac{8}{9}$ only

(ii) **M1:** Allow n **or** $n-1$ and any symbols from ">", "<", or "=" etc **A1 :** Must be power n (**not** $n-1$) with any symbol

M1: Uses logs correctly on $(\frac{6}{7})^n$ or $(\frac{7}{6})^n$ **not on** $(36)^n$ to get as far as n Allow any symbol

A1: $n=28$ cso (any errors with inequalities earlier e.g. failure to reverse the inequality when dividing by the negative $\log(\frac{6}{7})$ or any contradictory statements must be penalised here) Those with equals throughout may gain this mark if they follow 27.9 by n=28. Just $n=28$ without mention of 27.9 is only allowed following correct inequality work.

Special case: Trial and improvement: Gives $n=28$ as $S=$ awrt 290.1 (M1A1)and when $n=27$ $S=$ (awrt) 289 so $n=28$ (M1A1) – $n=28$ with no working is M1A0M0A0 and insufficient accuracy is M1A0M1A0

Uses nth term instead of sum of n terms – over simplified – do not treat as misread – award 0/4

Question Number	Scheme		Marks
6(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $=160$	M1: Use of a correct S_∞ formula	M1A1
		A1: 160	
	Accept correct answer only (160)		
			[2]
(b)	$S_{12} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}}$; $=127.77324...$	M1: Use of a correct S_n formula with $n = 12$ (condone missing brackets around 7/8)	M1A1
		A1: **awrt** 127.8	
	T & I in (b) requires all 12 terms to be calculated correctly for M1 and A1 for awrt 127.8		
			[2]
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working. (condone missing brackets around 7/8)(Allow $=, <, >, \geq, \leq$) but see note below.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $+\left(\frac{7}{8}\right)^N$ oe (Allow $=, <, >, \geq, \leq$) but see note below. **Dependent on the previous M1**	**d**M1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the power law of logarithms or takes logs base 0.875 correctly to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ (Allow $=, <, >, \geq, \leq$) but see note below.	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but not $N > 44$	A1 **cso**
	An incorrect **<u>inequality</u>** statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			[4]
			Total 8
	<u>Trial & Improvement Method in (c):</u>		
	1st M1: Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		
	2nd M1: Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		
	3rd M1: For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		
	A1: $N = 44$ **cso**		
	Answer of $N = 44$ only with no working scores no marks		

Question Number	Scheme	Marks
5.(a)	$a = 4p,\ ar = (3p+15)$ and $ar^2 = 5p+20$	B1
	(So $r =$) $\dfrac{5p+20}{3p+15} = \dfrac{3p+15}{4p}$ or $4p(5p+20) = (3p+15)^2$ or equivalent	M1
	See $(3p+15)^2 = 9p^2 + 90p + 225$	M1
	$20p^2 + 80p = 9p^2 + 90p + 225 \to\ 11p^2 - 10p - 225 = 0$ *	A1 *
		(4)
(b)	$(p-5)(11p+45)$ so $p =$	M1
	$p = 5$ only (after rejecting - 45/11)	A1
	<u>N.B. Special case p = 5 can be verified in (b) (1 mark only)</u>	
	$11\times 5^2 - 10\times 5 - 225 \quad = 275 - 50 - 225 = 0$ M1A0	
		(2)
(c)	$\dfrac{3\times 5+15}{4\times 5}$ or $\dfrac{5\times 5+20}{3\times 5+15}$	M1
	$r = \dfrac{3}{2}$	A1
		(2)
(d)	$S_{10} = \dfrac{20\left(1-\left(\text{"}\frac{3}{2}\text{"}\right)^{10}\right)}{\left(1-\text{"}\frac{3}{2}\text{"}\right)}$	M1A1ft
	(= 2266.601568…) = 2267	A1
		(3) **Total 11**

	Notes for Question 5
(a)	B1: Correct statement (needs all three terms)– **this may be omitted and implied** by correct statement in p only as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately)
	M1: Valid Attempt to eliminate a and r and to obtain equation in p only
	M1: Correct expansion of $(3p+15)^2 = 9p^2 + 90p + 225$
	A1cso: No incorrect work seen. The printed answer is obtained.
	NB Those who show $p = 5$ in part (a) obtain no credit for this
(b)	M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula) Must appear in part (b) – not part (a)
	A1: 5 **only and** -45/11 should be seen and rejected or ($11p$ + 45) seen and statement $p > 0$
(c)	M1: Substitutes $p = 5$ completely and attempt ratio (correct way up)
	A1: 1.5 or any equivalent
(d)	M1: Use of correct formula with $n = 10$ a and/or r may still be in terms of p
	A1ft: Correct expression ft on their r only – must have $a = 20$ and power = 10 here
	A1 2267 (accept awrt 2267)

Question	Scheme	Marks	AOs
12 (a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $$p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $$A = \frac{32000}{'1.0658'^4} \text{ or } A = \frac{50000}{'1.0658'^{11}}$$	M1	1.1b
	$A = 24795 \to 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p /1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
			(10 marks)

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p =$ awrt 1.0658

Both marks can be awarded from incorrect but consistent interpretations of t. Eg.

$32000 = Ap^5, 50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or ' ≈ 24800 '. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

Question	Scheme	Marks	AOs
7 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t = 0, V = 20\,000 \Rightarrow A = 20000$	M1	1.1b
	Eg. Substitutes $t = 1, V = 16000 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$	dM1	3.1b
	$V = 20000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t = 10$ in their $V = 20000e^{-0.223t} \Rightarrow V = (£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V = 18000e^{-0.223t} + 2000$	B1ft	3.3
		(1)	
			(7 marks)

(a) Option 1

M1: For $V = Ae^{\pm kt}$ Do not allow if k is fixed, eg $k = -0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Substitutes $t = 0 \Rightarrow A = 20000$ into their exponential model

Candidates may start by simply writing $V = 20000e^{kt}$ which would be M1 M1

dM1: Substitutes $t = 1 \Rightarrow 16000 = 20000e^{-1k} \Rightarrow k = ..$via the correct use of logs.

It is dependent upon both previous M's.

A1: $V = 20000e^{-0.223t}$ (with accuracy to at least 3sf) or $V = 20000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares V = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".

Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''

Do not allow ''it is not a good model because it is not the same''

In the alternative it is for comparing their value of t with 10 and making a suitable comment as to the reliability of their model with a reason.

$V = 20000e^{-0.223t} \Rightarrow 2000 = 20000e^{-0.223t} \Rightarrow t = 10.3$ years.

Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)

B1ft: For a correct statement. Eg states that the value of their '-0.223' should become less negative.

Alt states that the value of their '0.223' should become smaller. If they refer to k then refer to the model and apply the same principles.

Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	= 36.915 minutes = 36 minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** th km is $6\times1.05=6\times1.05^1$ **6**th km is $6\times1.05\times1.05=6\times1.05^2$ **7**th km is $6\times1.05\times1.05\times1.05=6\times1.05^3$ Hence the time for the r th km is $6\times1.05^{r-4}$	B1	3.4
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20}6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20th km $=\frac{6.3\left(1.05^{16}-1\right)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\frac{6.3\left(1.05^{16}-1\right)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

(a)

M1: For using model to calculate the total time.

Sight of 24 minutes + $6\times1.05+6\times1.05^2$ or equivalent is required. Eg 24 + 6.3 + 6.615

Alternatively in seconds 24 minutes + 378 sec (6min 18 s) +396.9 (6 min 37 s)

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$ or equivalent working in seconds

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the r th km and the index of 1.05
Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\frac{a\left(r^n-1\right)}{r-1}$, if seen, must be correct in part (c) for all relevant marks

M1: For the overall strategy of finding the total time.

Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum6\times1.05^n$ or $24+\frac{6\left(1.05^{20}-1\right)}{1.05-1}$

The geometric sequence formula, must be used with $r=1.05$ oe but condone slips on a and n

Question	Scheme	Marks	AOs
5 (a)	Uses $115 = 28 + 5d \Rightarrow d = (17.4)$	M1	3.1b
	Uses $28 + 2 \times "17.4" = ...$	M1	3.4
	$= 62.8$ (km h $^{-1}$)	A1	1.1b
		(3)	
(b)	Uses $115 = 28r^5 \Rightarrow r = (1.3265)$	M1	3.1b
	Uses $28 \times "1.3265^4" = ...$ or $\frac{115}{"1.3265"}$	M1	3.4
	$= 86.7$ (km h $^{-1}$)	A1	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

M1: Translates the problem into maths using n^{th} term $= a + (n-1)d$ and attempts to find d

Look for either $115 = 28 + 5d \Rightarrow d = ...$ or an attempt at $\frac{115-28}{5}$ condoning slips

It is implied by use of $d = 17.4$ Note that $115 = 28 + 6d \Rightarrow d = ...$ is M0

M1: Uses the model to find the fastest speed the car can go in 3rd gear using $28 + 2"d"$ or equivalent. This can be awarded following an incorrect method of finding "d"

A1: 62.8 km/ h Lack of units are condoned. Allow exact alternatives such as $\frac{314}{5}$

(b)

M1: Translates the problem into maths using n^{th} term $= ar^{n-1}$ and attempts to find r

It must use the 1st and 6th gear and not the 3rd gear found in part (a)

Look for either $115 = 28r^5 \Rightarrow r = ...$ o.e. or $\sqrt[5]{\frac{115}{28}}$ condoning slips.

It is implied by stating or using $r = \text{awrt } 1.33$

M1: Uses the model to find the fastest speed the car can go in 5th gear using $28 \times "r^4"$ or $\frac{115}{"r"}$ o.e.

This can be awarded following an incorrect method of finding "r"

A common misread seems to be finding the fastest speed the car can go in 3rd gear as in (a).

Providing it is clear what has been done, e.g. $u_3 = 28 \times "r^2"$ it can be awarded this mark.

A1: awrt 86.7 km/h Lack of units are condoned. Expressions must be evaluated.

Question Number	Scheme	Marks
9.(a)	Uses $300 \times (1.05)^{23}$	M1
	Obtains 921 or 922 or 920	A1
		[2]
(b)	Uses $S = \dfrac{300(1.05^{24} - 1)}{1.05 - 1}$ Must have correct r and n but can use their a (e.g. 315)	M1
	13351 (accept awrt 13400)	A1
		[2]
(c)	Uses $300(1.05)^{n-1} > 3000$ Or $300(1.05)^{n-1} = 3000$	M1
	$(n-1)\log 1.05 > \log 10$ Or $(n-1)\log 1.05 = \log 10$ Or $(n-1) = \log_{1.05} 10$ Or correct equivalent log work ft	M1
	$n > 48.19$ $N = 49$	A1
		[3]
		7 marks
	Notes	
(a)	M1: for correct statement of formula with correct *a, r* and *n* A1: cao (This answer implies the M1)	
(b)	M1: Correct formula with r = 1.05 and n = 24 ft their a (If they list all the terms – correct answer implies method mark) A1: answers which round to 13400 are acceptable	
(c)	M1: Correct inequality or uses equality and interprets correctly later (ft their a) M1: Correct algebra then correct use of logs on their previous line (may follow use of =, or use of n instead of n -1) Can get M0M1A0 A1: need to see 49 or 49th month **Special case**: Uses sum formula: If they reach $(1.05)^n > 1\frac{1}{2}$ and then use logs correctly to give $n\log(1.05) > \log 1\frac{1}{2}$ then give M0M1A0 If trial and error is used then the correct answer implies the method. So 49 is M1M1A1 and 48 scores M1M0A0. Similar marks follow answer only with no working.	

Question Number	Scheme	Marks
12.(a)	Uses $275000\times(1.1)^5$ or finds £442890.25 or uses $275000\times(1.1)^4$ or finds £402627.50	M1
	Finds both of the above and subtracts to give £40 262.75 and concludes approx. £40300*	M1 A1*
	Or	[3]
	Uses $275000\times(1.1)^5$ -$275000\times(1.1)^4$, $=awrt\,40260 = 40300\ (3sf)$ *	M1 M1,A1* [3]
(b)	Puts $275000\times(1.1)^{n-1} > 1000000$ or $275000\times(1.1)^{n-1} = 1000000$	M1
	$(1.1)^{n-1} > \dfrac{1000000}{275000}$ (or $\frac{40}{11}$ or 3.63 or 3.64). Or	M1
	$(1.1)^{n-1} = \dfrac{1000000}{275000}$ (or $\frac{40}{11}$ or 3.63 or 3.64)	
	$n-1 > \dfrac{\log\left(\frac{40}{11}\right)}{\log 1.1}$ or $n-1 = \dfrac{\log\left(\frac{40}{11}\right)}{\log 1.1}$	M1
	($n > 14.5$ or $n > 14.6$ or $n = 15$) so the year is 2030	A1
		[4]
(c)	Uses $S = \dfrac{275000(1.1^n - 1)}{1.1-1}$ or uses $S = \dfrac{275000(1-1.1^n)}{1-1.1}$	M1
	Uses n = 11 in formula	A1
	Awrt £5 096 100	A1
	Or: adds 11 terms £275000 + 302500 + 332750 + 366025 + 402627.5 + 442890.25 + 487179.275 + 535897.2025 + 589486.9228 + 648435.615 + 713279.1765 = awrt 5096100 (see notes below)	[3]
		10 marks
	Notes	

Question Number	Scheme	Marks
8. (a)	$238 = a + k\,d$ **or** $108 = a + k\,d$ with any values for k	M1
	$238 = a + (14)d$ or $108 = a + (24)d$ or $"d" = \pm 13$	A1
	$238 = a + (14)d$ **and** $108 = a + (24)d$	A1
	Solves their simultaneous equations to obtain $a =$; (so a) $= 420$	M1; A1 [5]
(b)	Uses $\frac{25}{2}(2 \times a + (25-1) \times "-13")$ **or** Uses $\frac{25}{2}(a + 108)$, to obtain $= 6600$	M1 , A1 [2]
		7 marks

(a)

M1 Score for $238 = a + k\,d$ **or** $108 = a + k\,d$ with any non- zero integer value for k

A1 One of $238 = a + (14)d$, or $108 = a + (24)d$ or $"d" = \pm 13$
The $"d" = \pm 13$ can be achieved from equations such as $238 = a + (13)d$, or $108 = a + (23)d$

A1 Both $238 = a + (14)d$, **and** $108 = a + (24)d$

M1 Finds the value of a by solving a pair of simultaneous equations in a and d

A1 Achieves ($a =$) 420

In an alternative, working by a method of differences, you may see very few formulae: The scheme can be easily applied,

1st M1 Seeing $\frac{238-108}{10}$ or $\frac{108-238}{10}$

1st A1 For ± 13

2nd A1 For $238 + 13 \times 14$ or $108 + 13 \times 24$.
Note that this is achieved after the award of the next mark. It is scored as the third mark of (a) on e –pen.

2nd M1 Sight of $238 + 'k' \times d$ or $108 + 'k' \times d$ with any non- zero integer values for k and their d

3rd A1 Achieves 420

(b)

M1 Uses a correct sum formula $S = \frac{n}{2}\left(2a + (n-1)d\right)$ with $n = 25$ and their values of a and d

Alternatively uses $S = \frac{n}{2}\left(a + l\right)$ with $n = 25, l = 108$ and their value of a.

A1 cao 6600

Question Number	Scheme	Marks
9.(a)	$130000 \times (1.02) = 132600*$ or $2\% = 2600$ and $130000 + 2600 = 132600*$	B1
		[1]
(b)	$(r =)\ 1.02$	B1
		[1]
(c)	Uses $130000 \times (1.02)^{N-1} > 260000$ **or** $130000 \times (1.02)^{N-1} = 260000$	M1
	So $(1.02)^{N-1} > 2$	A1
	$(N-1)\log_{10}(1.02) > \log_{10} 2$ or $(N-1)\log_{10}(1.02) = \log_{10} 2$ or $(N-1) > \log_{1.02} 2$ or $(N-1) = \log_{1.02} 2$	M1
	$N > \dfrac{\log_{10} 2}{\log_{10}(1.02)} + 1*$	A1cso
		[4]
(d)	$(N =)\ 37$	B1
		[1]
		7 marks
	Notes	
(a)	**B1:** A reason must be provided for this mark as the answer is printed. **Allow both** $130000 \times (1 + 2\%)$ **and** $130000 \times (102\%)$ **as both give the correct answer when entered this way on a calculator. But not** $130000 \times 1 + 2\%$	
(b)	**B1:** For 1.02 oe e.g. allow $\dfrac{51}{50}$	
(c)	**M1: Correct** inequality or equality – may use *r* or their *r* or 1.02 and may use *N* or *n*. **A1:** $(1.02)^{N-1} > 2$ cao. Allow $(1.02)^{n-1} > 2$ **M1:** Correct use of logs power rule on their previous line which must have come from using the n^{th} term of a GP. Condone missing brackets for this mark e.g. $N - 1\log_{10}(1.02) > \log_{10} 2$. (May follow use of $=$ instead of $>$ or use of *r* instead of 1.02 or use of *N* instead of *N* - 1). These cases can get M0A0M1. Allow the base to be absent or just 'ln' for this mark. If the inequality sign is reversed at this point, still allow the M1. **A1*:** Answer is **exactly** as printed (**including the bases**) and **all** inequality work should be correct and all previous marks scored and **no missing brackets earlier.** Allow this mark to score from a correct previous line provided the power rule is used. So fully correct work leading to $(N-1)\log_{10}(1.02) > \log_{10} 2 \Rightarrow N > \dfrac{\log_{10} 2}{\log_{10}(1.02)} + 1$ **scores the final M1A1 but** $(1.02)^{N-1} > 2 \Rightarrow N > \dfrac{\log_{10} 2}{\log_{10}(1.02)} + 1$ **scores M0A0 (no explicit use of power rule)**	
(d)	**B1:** Only need $N = 37$ – may follow trial and error or uses logs to a different base. Do not allow $N \geq 37$ or $N > 37$ or $N = 37.0$	

!

Question Number	Scheme		Marks
14 (a)	Allow the use of S or S_n throughout without penalty. $S = a + ar + ar^2 + ...ar^{n-1}$ **and** $rS = ar + ar^2 + ar^3 + ...ar^n$ There must be a minimum of '3' terms and must include the first and the nth term. Condone for this mark only $S = a + ar + ar^2 + ...ar^n$ **and** $rS = ar + ar^2 + ar^3 + ...ar^{n+1}$ and allow commas instead of +'s but see note below.		M1
	$S - rS = a - ar^n$	Subtracts either way around. **As a special case allow** $S - rS = a + ar^n$. For this mark, their S and their rS must be different but it must be S and rS they are considering with possible missing terms or slips.	M1
	$\Rightarrow S(1-r) = a(1-r^n) \Rightarrow S = \dfrac{a(1-r^n)}{(1-r)}$	**d**M1: **Dependent upon both previous M's**. It is for taking out a common factor of S and achieving $S = \ldots$ A1*: Fully correct proof with **no errors or omissions. The use of commas instead of +'s is an error.** $S = \dfrac{a(r^n - 1)}{(r-1)}$ without reaching the printed answer is A0	**d**M1A1*
			(4)
(a) Way 2	$S = \dfrac{\left(a + ar + ar^2 + ...ar^{n-1}\right)(1-r)}{1-r}$	Gives a minimum of '3' terms and must include the first and the nth and multiplies top and bottom by $1 - r$	M1
	$S = \dfrac{a + ar + ar^2 + ...ar^{n-1} - ar - ar^2 - ... - ar^n}{1-r}$	Expands the top with a minimum of '3' terms in each and must include the first and the nth term	M1
	$S = \dfrac{a(1-r^n)}{(1-r)}$	dM1: Dependent upon both previous M's. It is for taking out a common factor of a on top and achieving $S = \ldots$ A1*: Fully correct proof with **no errors or omissions. The use of commas instead of +'s is an error.** $S = \dfrac{a(r^n - 1)}{(r-1)}$ without reaching the printed answer is A0	dM1A1

(b)	$U = 180 \times 0.93^n$ with $n = 4$ or 5	Attempts $U = 180 \times 0.93^n$ with $n = 4$ or 5. Accept $U = 167.4 \times 0.93^n$ with $n = 3$ or 4 Allow 93% for 0.93	M1
	$U_5 = 180 \times (0.93)^5 = 125.2$ (litres)	Cso. Awrt 125.2	A1*
	Allow 93% or 1 – 7% for 0.93		
			(2)
(c)	Attempts $S_n = \dfrac{a(1-r^n)}{(1-r)}$ with any combination of: $n = 20/21$ $a = 180/167.4$ and $r = 0.93$ Allow 93% for 0.93		M1
	$S = \dfrac{167.4(1-0.93^{20})}{(1-0.93)}$ or $S = 180 \times \dfrac{0.93(1-0.93^{20})}{(1-0.93)}$ or $S = \dfrac{180(1-0.93^{21})}{(1-0.93)} - 180$ A correct numerical expression for the sum (may be implied by awrt 1831) Allow 93% or 1 – 7% for 0.93		A1
	1831 (litres)	1831 **only** (Ignore units). Do not isw here, so 1831 followed by $1831 \times 20 = \ldots$ scores A0.	A1
			(3)
			(9 marks)

Listing:

(b)	Sight of awrt 180, 167, 156, 145, 135, 125	Starts with 180 and multiplies by 0.93 either 4 or 5 times showing each result at least to the nearest litre and chooses the 5th or 6th term	M1
	$U_5 = 125.2$ (litres)	Must see all values accurate to 1dp: e.g. awrt 180, 167.4, 155.7, 144.8, (134.6 or 134.7), 125.2	A1*
			(2)
(c)	$\text{Total} = 180 \times 0.93 + 180 \times 0.93^2 + \ldots\ldots + 180 \times 0.93^{19} + 180 \times 0.93^{20} = \ldots$ Finds an expression for the sum of 20 or 21 terms		M1
	All sums accurate to awrt 1dp 167.4+155.7+144.8+134.6+125.2+.....42.2 A correct numerical expression for the sum (may be implied by awrt 1831)		A1
	1831 (litres)	1831 **only** (Ignore units). Do not isw here, so 1831 followed by $1831 \times 20 = \ldots$ scores A0.	A1
			(3)

Question Number	Scheme	Marks
11 (a)	Attempts $U_4 = 6000\times(1.015)^3 = 6274$ (tonnes)	M1A1*
		(2)
(b)	Attempts $U_N = 6000\times 1.015^{N-1} > 8000$	M1
	$1.015^{N-1} > \dfrac{8000}{6000}$ oe	A1
	$\log\left(1.015^{N-1}\right) > \log\left(\dfrac{4}{3}\right) \Rightarrow N > \dfrac{\log\left(\frac{4}{3}\right)}{\log(1.015)} + 1 = (20.3)$	M1A1
	$(N) = 21$	A1
		(5)
(c)	Attempts $S_n = \dfrac{a(1-r^n)}{(1-r)}$ with $n = 10$ $a = 6000/30000$ and $r = 1.015$	M1
	$S = 5\times\dfrac{6000(1.015^{10}-1)}{(1.015-1)}$ OR $S = \dfrac{30000(1.015^{10}-1)}{(1.015-1)}$	A1
	Awrt £321 000	A1
		(3)
		(10 marks)

(a)

M1 Attempts to use ar^3 with $a = 6000$, $r = 1.015$. Accept $r = 1+1.5\%$
Condone for this mark r =1.15 or 1.0015 Accept a list of 4 terms with the same conditions

A1* cso $6000\times(1.015)^3 = 6274$ (tonnes).

If candidate states $U_4 = 6000\times(1.015)^3 = 6274.07$ (tonnes) or 6274.0 (or anything that rounds to 6274) they don't need to round to the given answer.

(b)

M1 Attempts to use $ar^{n-1}...8000$ or $ar^n...8000$ with $a = 6000$, $r = 1.015$ or $1+1.5\%$ condoning values of r being 1.15 or 1.0015

A1 For reaching the intermediate result $1.015^{n-1}...\frac{4}{3}$ or $1.015^n...\frac{4}{3}$.

Allow $\frac{4}{3}$ to be rounded or truncated to 1.33 (to 2dp or better)

M1 Uses logs correctly to get n or n-1 This mark may be awarded from a sum formula

A1 This is scored for a 'correct' (unrounded) answer. It may be left in log form. If the candidate has used n instead of n-1, they will not score this unless they subsequently reach a final answer of 21. Allow for N or n.

Accept versions of $n...\dfrac{\log\left(\frac{4}{3}\right)}{\log(1.015)} + 1 = (20.3)$ or $n...\log_{1.015}\left(\dfrac{4}{3}\right) + 1 = (20.3)$ or

$n...\dfrac{\log(1.33)}{\log(1.015)} + 1 = (20.15)$

A1 $(N) =$ 21 Do not accept $N > 21$ etc
The two final A marks may be implied by finding 'n' and adding 1 to reach 21

Question Number	Scheme	Marks
6. (a)	Uses $1000 = 600 + 80(N-1) \Rightarrow N = 6$	M1,A1 **[2]**
(b)	Uses $\frac{15}{2}\left(2\times 600 + (15-1)\times 80\right) = (£)17400$	M1 A1 **[2]**
(c)	Total for Saima = $\frac{15}{2}\left(2A + 14A\right) = \left(120A\right)$	B1
	Sets $120A = 17400 \Rightarrow A = 145$	M1A1 **[3]**
		(7 marks)

(a)

M1 Attempts to use the formula $u_n = a + (n-1)d$ to find the value of 'n'.
Evidence would be $1000 = 600 + 80(N-1)$
Alternatively attempts $\frac{1000-600}{80} + 1$ or repeated addition of £80 onto £600 until £1000 is reached

A1 $N = 6$ or accept the 6th year (or similar). The answer alone would score both marks.

(b)

M1 Uses a correct sum formula $S = \frac{n}{2}\left(2a + (n-1)d\right)$ with $n = 15, a = 600, d = 80$
Alternatively uses $S = \frac{n}{2}\left(a + l\right)$ with $n = 15$, $a = 600$, $l = 600 + 14\times 80$ or 1720
Accept the sum of 15 terms starting 600 + 680+ 760+ 840+....

A1 cao (£)17400

(c)

B1 Finds the sum for Saima.
Accept unsimplified forms such as $\frac{15}{2}\left(2A + 14A\right)$ or $\frac{15}{2}\left(A + 15A\right)$ or the simplified answer of $120A$
Remember to isw following a correct answer

M1 Sets their $120A$ equal to their answer to (b) and proceeds to find a value for A.
They must be attempting to calculate sums rather than terms to score this mark.
Condone slips on the sum of an AP formula and award for a valid attempt from GP formula.

A1 cao $A = 145$

Question Number	Scheme	Marks
14 (a)	$u_6 = 8000\times(0.85)^5 = 3549.6 \approx 3550$	M1, A1
		[2]
(b)	States $\lvert r \rvert < 1$ **or** 0.85 < 1 **and makes no reference to terms**	B1
		[1]
(c)	$S_\infty = \dfrac{a}{1-r} = \dfrac{8000}{1-0.85} =$ awrt $53333 \quad 53334 \quad \dfrac{160\,000}{3}$	M1A1
		[2]
(d)	Uses $S_N = \dfrac{8000(1-0.85^N)}{1-0.85}$	M1
	$\dfrac{8000(1-0.85^N)}{1-0.85} = 40000 \Rightarrow 0.85^N = 0.25$	dM1 A1
	$\Rightarrow N = \dfrac{\log 0.25}{\log 0.85}(=8.53) \Rightarrow N = 9$	M1 A1
		[5] **[10 marks]**

(a)

M1 Attempts $u_6 = 8000\times(r)^5$ with $r = 0.85$ or 85% or $1-0.15$ or 1-15%

A1* Completes proof. States $u_6 = 8000\times(0.85)^5$ oe (see above) and shows answer is awrt 3549.6 or 3550

(b)

B1 States $|r| < 1$ or 0.85 < 1 **and makes no reference to terms**

Allow $r < 1 \quad -1 < r < 1$ **and makes no reference to terms**

Allow for an understanding of why S_∞ exists. Accept $0.85^n \to 0$ as $n \to \infty$ or $r^n \to 0$ as $n \to \infty$

Do not allow from an incorrect statement... if they give $r = 0.15$

Do not allow on an explanation that is based around terms.

Eg Do not allow $8000\times 0.85^{n-1} \to 0$ as $n \to \infty$

Do not allow as $r < 1$ $u_n \to 0$ and so a limit exists

Do not allow if they state 85% is less than 100%

If you feel that a candidate deserves this mark then please seek advice.

(c)

M1 Attempts $S_\infty = \dfrac{8000}{1-r}$ with $r = 0.85$ oe

A1 $\dfrac{8000}{1-0.85}$ with an answer of awrt 53333 or $53\,334$ or $\dfrac{160\,000}{3}$

3.			
(a)	$120000\times(1.05)^3=138915$ *	Or $120000\times1.05\times1.05\times1.05=138915$ Or 120000, 126000, 132300, 138915 Or a = 120000 and $a\times\left(1.05\right)^3=138915$	B1
			(1)
(b)	$120000\times(1.05)^{n-1}>200000$	Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$\log1.05^{n-1}>\log\left(\frac{5}{3}\right)$	Takes logs correctly Allow n or $n-1$ and ">", "<", or "=" etc.	M1
	$(n-1>)\frac{\log\left(\frac{5}{3}\right)}{\log1.05}$ or equivalent e.g $(n>)\frac{\log\left(\frac{7}{4}\right)}{\log1.05}$	Allow n or $n-1$ and ">", "<", or "=" etc. Allow $1.\dot{6}$ or awrt 1.67 for 5/3.	A1
	2024	M1: Identifies a calendar year using their value of n or n - 1 A1: 2024	M1A1
			(5)
(c)	$\frac{a(1-r^n)}{1-r}=\frac{120000\left(1-1.05^{11}\right)}{1-1.05}$	M1: Correct sum formula with n = 10, 11 or 12 A1: Correct numerical expression with n = 11	M1 A1
	1704814	Cao (Allow 1704814.00)	A1
			(3)
			[9]
	Listing or trial/improvement in (b)		
	U_{10} = 186 159.39, U_{11} = 195 467.36, U_{12} = 205 240.72		
	Attempt to find at least the 10th or 11th or 12th terms correctly using a common ratio of 1.05 (all the terms need **not** be listed)		M1
	Forms the geometric progression correctly to reach a term > 200 000		M1
	Obtains an "11th" term of awrt 195 500 **and** a "12th" term of awrt 205 200		A1
	Uses their number of terms to identify a calendar year		M1
	2024		A1
			(5)