Question	Scheme	Marks	AOs
4(a)	$\mathrm{f}'(x)=2x+\dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x+\dfrac{4x-4}{2x^2-4x+5}=0 \Rightarrow 2x\left(2x^2-4x+5\right)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$*	A1*	2.1
		(4)	
(b)	(i) $x_2=\dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2=0.3294$	A1	1.1b
	(ii) $x_4=0.3398$	A1	1.1b
		(3)	
(c)	$\mathrm{h}(x)=2x^3-4x^2+7x-2$ $\mathrm{h}(0.3415)=0.00366...\quad \mathrm{h}(0.3405)=-0.00130...$	M1	3.1a
	States: • there is a change of sign • $\mathrm{f}'(x)$ is continuous • $\alpha=0.341$ to 3dp	A1	2.4
		(2)	
			(9 marks)

Notes

Question	Scheme	Marks	AOs
4 (a)	Attempts $f(3)=$ and $f(4)=$ where $f(x)=\pm(2\ln(8-x)-x)$	M1	2.1
	$f(3)=(2\ln(5)-x)=(+)0.22$ and $f(4)=(2\ln(4)-4)=-1.23$ Change of sign and function continuous in interval $[3,4] \Rightarrow$ Root *	A1*	2.4
		(2)	
(b)	For annotating the graph by drawing a cobweb diagram starting at $x_1=4$ It should have at least two spirals	M1	2.4
	Deduces that the iteration formula **can be used** to find an approximation for α because **the cobweb spirals inwards** for the cobweb diagram	A1	2.2a
		(2)	
			(4 marks)

Notes:

(a)

M1: Attempts $f(3)=$ and $f(4)=$ where $f(x)=\pm(2\ln(8-x)-x)$ or alternatively **compares** $2\ln 5$ to 3 and $2\ln 4$ to 4. This is not routine and cannot be scored by substituting 3 and 4 in both functions

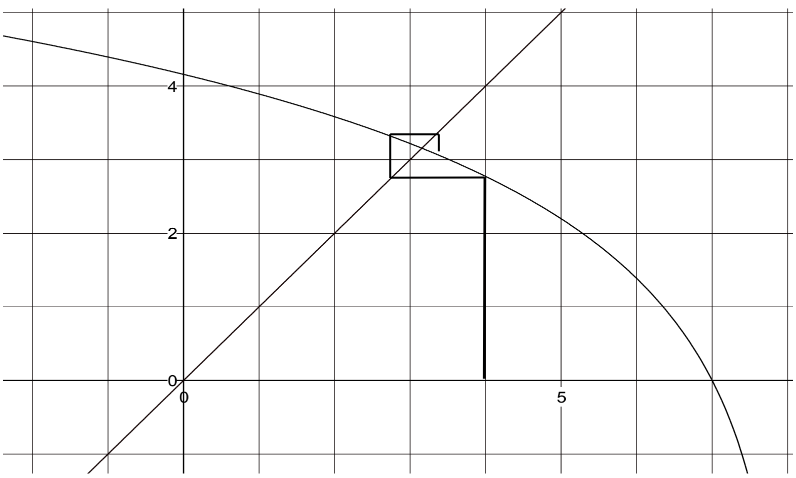
A1: Both values (calculations) correct to at least 1 sf with correct explanation and conclusion. (See underlined statements)

When comparing terms, allow reasons to be $2ln8=3.21>3,\ 2\ln4=2.77<4$ or similar

(b)

M1: For an attempt at using a cobweb diagram. Look for 5 or more correct straight lines. It may not start at 4 but it must show an understanding of the method. **If there is no graph then it is M0 A0**

A1: For a correct attempt starting at 4 and deducing that the iteration **can be used** as the iterations **converge to the root**. You must statement that it can be used with a suitable reason. Suitable reasons could be " it spirals inwards", it gets closer to the root", it converges "



Question	Scheme	Marks	AOs
5	The equation $2x^3 + x^2 - 1 = 0$ has exactly one real root		
(a)	$\left\{f(x) = 2x^3 + x^2 - 1 \Rightarrow\right\}$ $f'(x) = 6x^2 + 2x$	B1	1.1b
	$\left\{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} \Rightarrow\right\}$ $\{x_{n+1}\} = x_n - \dfrac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$	M1	1.1b
	$= \dfrac{x_n(6x_n^2 + 2x_n) - (2x_n^3 + x_n^2 - 1)}{6x_n^2 + 2x_n} \Rightarrow x_{n+1} = \dfrac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$ *	A1*	2.1
		(3)	
(b)	$\{x_1 = 1 \Rightarrow\}$ $x_2 = \dfrac{4(1)^3 + (1)^2 + 1}{6(1)^2 + 2(1)}$ or $x_2 = 1 - \dfrac{2(1)^3 + (1)^2 - 1}{6(1)^2 + 2(1)}$	M1	1.1b
	$\Rightarrow x_2 = \dfrac{3}{4},\ x_3 = \dfrac{2}{3}$	A1	1.1b
		(2)	
(c)	Accept any reasons why the Newton-Raphson **method** cannot be used with $x_1 = 0$ which refer or ***allude*** to either the stationary point or the tangent. E.g. • There is a stationary point at $x = 0$ • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) would not meet the x-axis • Tangent to the curve (or $y = 2x^3 + x^2 - 1$) is horizontal	B1	2.3
		(1)	
			(6 marks)

Notes for Question 5

(a)	
B1:	States that $f'(x) = 6x^2 + 2x$ or states that $f'(x_n) = 6x_n^2 + 2x_n$ (Condone $\dfrac{dy}{dx} = 6x^2 + 2x$)
M1:	Substitutes $f(x_n) = 2x_n^3 + x_n^2 - 1$ and their $f'(x_n)$ into $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
A1*:	A correct intermediate step of making a common denominator which leads to the given answer
Note:	Allow B1 if $f'(x) = 6x^2 + 2x$ is applied as $f'(x_n)$ (or $f'(x)$) in the NR formula $\{x_{n+1}\} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
Note:	Allow M1A1 for • $x_{n+1} = x - \dfrac{2x^3 + x^2 - 1}{6x^2 + 2x} = \dfrac{x(6x^2 + 2x) - (2x^3 + x^2 - 1)}{6x^2 + 2x} \Rightarrow x_{n+1} = \dfrac{4x_n^3 + x_n^2 + 1}{6x_n^2 + 2x_n}$
Note	Condone $x = x - \dfrac{2x^3 + x^2 - 1}{"6x^2 + 2x"}$ for M1
Note	Condone $x_n - \dfrac{2x_n^3 + x_n^2 - 1}{"6x_n^2 + 2x_n"}$ or $x - \dfrac{2x^3 + x^2 - 1}{"6x^2 + 2x"}$ (i.e. no $x_{n+1} = \ldots$) for M1
Note:	Give M0 for $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ followed by $x_{n+1} = 2x_n^3 + x_n^2 - 1 - \dfrac{2x_n^3 + x_n^2 - 1}{6x_n^2 + 2x_n}$
Note:	Correct notation, i.e. x_{n+1} and x_n must be seen in their final answer for A1*

Question	Scheme	Marks	AOs
11 (a) **Way 1**	$\{y = x^x \Rightarrow\}\;\; \ln y = x\ln x$	B1	1.1a
	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\;\; \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(a) **Way 2**	$\{y = x^x \Rightarrow\}\;\; y = \mathrm{e}^{x\ln x}$	B1	1.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x}{x} + \ln x\right)\mathrm{e}^{x\ln x}$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\;\; \frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(b) **Way 1**	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}\; x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) || Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
		(11 marks)	

Note	**A common solution**
	A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x\log x \Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log x$ $\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}\;\; 1 + \log x = 0 \Rightarrow x = 10^{-1}$
	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x};\; \lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\frac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...;\;\; k \neq 0$

Question	Scheme	Marks	AOs
7(a)	$\ln x \to \frac{1}{x}$	B1	1.1a
	Method to differentiate $\frac{4x^2+x}{2\sqrt{x}}$ – see notes	M1	1.1b
	E.g. $2\times\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}\times\frac{1}{2}x^{-\frac{1}{2}}$	A1	1.1b
	$\frac{dy}{dx}=3\sqrt{x}+\frac{1}{4\sqrt{x}}-\frac{4}{x}=\frac{12x^2+x-16\sqrt{x}}{4x\sqrt{x}}$ *	A1*	2.1
		(4)	
(b)	$12x^2+x-16\sqrt{x}=0 \Rightarrow 12x^{\frac{3}{2}}+x^{\frac{1}{2}}-16=0$	M1	1.1b
	E.g. $12x^{\frac{3}{2}}=16-\sqrt{x}$	dM1	1.1b
	$x^{\frac{3}{2}}=\frac{4}{3}-\frac{\sqrt{x}}{12} \Rightarrow x=\left(\frac{4}{3}-\frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$ *	A1*	2.1
		(3)	
(c)	$x_2=\sqrt[3]{\left(\frac{4}{3}-\frac{\sqrt{2}}{12}\right)^2}$	M1	1.1b
	x_2 = awrt 1.13894	A1	1.1b
	x = 1.15650	A1	2.2a
		(3)	
			(10 marks)

Question Number	Scheme	Marks
7 (a)	Applies $vu'+uv'$ with u=2x+2x^2 and v=ln$x$ or vice versa	
	$$\mathrm{f}'(x)=\ln(x)\left(2+4x\right)+\left(2x+2x^2\right)\times\frac{1}{x}$$	M1A1A1
		(3)
(b)	Sets $\ln(x)\left(2+4x\right)+\left(2x+2x^2\right)\times\frac{1}{x}=0$ and makes ln$x$ the subject	M1
	$$\ln(x)=-\frac{1+x}{1+2x}\Rightarrow x=\mathrm{e}^{-\frac{1+x}{1+2x}}$$	dM1A1*
		(3)
(c)	Subs $x_0=0.46$ into $x=\mathrm{e}^{-\frac{1+x}{1+2x}}$	M1
	x_1= awrt 0.4675, x_2= awrt 0.4684 x_3= awrt 0.4685	A1,A1
		(3)
(d)	A= (0.47, -1.04)	M1A1
		(2)
		(11 marks)
Alt 7 (a)	Writes $\mathrm{f}(x)=2x\ln x+2x^2\ln x$ and applies $vu'+uv'$	
	$\mathrm{f}'(x)=2\ln(x)+2x\times\frac{1}{x}+2x^2\times\frac{1}{x}+4x\ln x$	M1A1A1
		(3)

Question Number	Scheme	Marks
1. (a)	$f(1.5) = -1.75$, $f(2) = 8$	M1
	Sign change (and $f(x)$ is continuous) therefore there is a root α {lies in the interval $[1.5, 2]$}	A1
		[2]
(b)	$x_1 = \left(5 - \frac{1}{2}(1.5)\right)^{\frac{1}{3}}$	M1
	$x_1 = 1.6198$, $\quad x_1 = 1.6198$ cao	A1cao
	$x_2 = 1.612159576...$, $x_3 = 1.612649754...$ $\quad x_2 =$ awrt 1.6122 and $x_3 =$ awrt 1.6126	A1
		[3]
(c)	$f(1.61255) = -0.001166022687...$, $f(1.61265) = 0.0004942645692...$	
	Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[1.61255, 1.61265] \Rightarrow \alpha = 1.6126$ (4 dp)	M1A1
		[2] **7**

Notes

(a) **M1:** Attempts to evaluate both $f(1.5)$ and $f(2)$ and finds at **least one** of $f(1.5) =$ awrt -1.8 or truncated -1.7 **or** $f(2) = 8$ **Must be using this interval or a sub interval e.g.[1.55, 1.95] not interval which goes outside the given interval such as [1.6, 2.1]**

A1: both $f(1.5) =$ awrt -1.8 or truncated -1.7 **and** $f(2) = 8$, **states sign change** { or $f(1.5) < 0 < f(2)$ or $f(1.5)\ f(2) < 0$ } or $f(1.5) < 0$ and $f(2) > 0$; **and conclusion** e.g. therefore a root α [lies in the interval $[1.5, 2]$]or "so result shown" or qed or "tick" etc…

(b) M1: An attempt to substitute $x_0 = 1.5$ into the iterative formula

e.g. see $\left(5 - \frac{1}{2}(1.5)\right)^{\frac{1}{3}}$. Or can be implied by $x_1 =$ awrt 1.6

A1: $x_1 = 1.6198$ This **exact answer to 4 decimal** places is required for this mark

A1: $x_2 =$ awrt 1.6122 **and** $x_3 =$ awrt 1.6126 (so e.g. 1.61216 and 1.6126498 would be acceptable here)

(c) M1: Choose suitable interval for x, e.g. $[1.61255, 1.61265]$ and at least one attempt to evaluate $f(x)$.

A minority of candidate may choose a tighter range which should include1.61262 (alpha to 5dp), e.g. $[1.61259, 1.61263]$ This would be acceptable for both marks, provided the conditions for the A mark are met.

A1: needs (i) both evaluations correct to 1 sf, (either rounded or truncated) e.g. -0.001 and 0.0005 or 0.0004

(ii) sign change stated and

(iii)some form of conclusion which may be :

$\Rightarrow \alpha = 1.6126$ or "so result shown" or qed or tick or equivalent

N.B. f(1.61264)=0.0003 (to 1 sf)

Question Number	Scheme	Marks
10(a)	$y=\frac{x^2\ln x}{3}-2x+4\Rightarrow\frac{dy}{dx}=\underbrace{\frac{2x\ln x}{3}+\frac{x^2}{3x}},\underbrace{-2}$	M1A1, B1
	$\frac{2x\ln x}{3}+\frac{x^2}{3x}-2=0\Rightarrow x(2\ln x+1)=6\Rightarrow x=..$	dM1
	$\Rightarrow x=\frac{6}{1+\ln x^2}$	A1*
		(5)
(b)	$x_1=\frac{6}{1+\ln(2.27^2)}=$ awrt 2.273	M1A1
	x_2 = awrt 2.271 and x_3 =awrt 2.273	A1
		(3)
(c)	A= (2.3, 0.9)	M1 A1
		(2)
		(10 marks)

Question Number	Scheme	Marks
5. (a)	$\mathrm{f}(1) = -3\,,\quad \mathrm{f}(2) = 2$	M1
	Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α lies in the interval $[1,\ 2]$	A1
		[2]
(b)	$\mathrm{f}(x) = -x^3 + 4x^2 - 6 = 0 \Rightarrow x^2(4-x) = 6$	M1
	$\Rightarrow x^2 = \left(\dfrac{6}{4-x}\right)$ and so $x = \sqrt{\left(\dfrac{6}{4-x}\right)}$ *	A1*
		[2]
(c)	$x_2 = \sqrt{\left(\dfrac{6}{4-1.5}\right)}$	M1
	$x_2 = \text{awrt } 1.5492$,	A1
	$x_3 = \text{awrt } 1.5647$, **and** $x_4 = \text{awrt } 1.5696 / 1.5697$	A1
		[3]
(d)	$\mathrm{f}(1.5715) = -0.00254665...,\quad \mathrm{f}(1.5725) = 0.0026157969$	
	Sign change (and as $\mathrm{f}(x)$ is continuous) therefore a root α lies in the interval $[1.5715,\ 1.5725] \Rightarrow \alpha = 1.572$ (3 dp)	M1A1
		[2]
		(9 marks)

(a)

M1 Attempts to evaluate **both** $\mathrm{f}(1)$ and $\mathrm{f}(2)$ and achieves at least one of $\mathrm{f}(1) = -3$ **or** $\mathrm{f}(2) = 2$

If a smaller interval is chosen, eg 1.57 and 1.58, the candidate must refer back to the region 1 to 2

A1 Requires (i) both $\mathrm{f}(1) = -3$ **and** $\mathrm{f}(2) = 2$ correct,

(ii) sign change stated or equivalent Eg $\mathrm{f}(1) \times \mathrm{f}(2) < 0$ and

(iii) some form of conclusion which may be : or "so result shown" or qed or tick or equivalent

(b)

M1 Must either state f(x) = 0 or set $-x^3 + 4x^2 - 6 = 0$ before writing down at least the line equivalent to $\pm x^2(x-4) = \pm 6$

A1* Completely correct with all signs correct. There is no requirement to show $\dfrac{-6}{4-x} \to \dfrac{6}{x-4}$

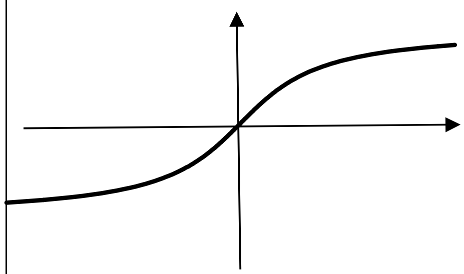
Expect to see a minimum of the equivalent to $x^2 = \left(\dfrac{-6}{4-x}\right)$ and $x = \sqrt{\left(\dfrac{6}{x-4}\right)}$

Alternative working backwards

M1 **S**tarts with answer and squares, multiplies across and expands

$$x = \sqrt{\left(\frac{6}{4-x}\right)} \Rightarrow x^2 = \frac{6}{4-x} \Rightarrow x^2(4-x) = 6 \Rightarrow 4x^2 - x^3 = 6$$

A1 Completely correct $-x^3 + 4x^2 - 6 = 0$ **and** states "therefore f(x) = 0" or similar

Question Number	Scheme		Marks
10(a)		M1: Curve not a straight line through (0, 0) in quadrants 1 and 3 only. A1: Grad $\to 0$ as $x \to \pm\infty$	M1A1
			(2)
(b)	$3\arctan(x+1)-\pi=0$ $\Rightarrow \arctan(x+1)=\frac{\pi}{3}$	Substitutes $g(x+1)=\arctan(x+1)$ in $3g(x+1)-\pi=0$ and makes $\arctan(x+1)$ the subject. Do not condone missing brackets unless later work implies their presence.	M1
	$\Rightarrow x=\tan\left(\frac{\pi}{3}\right)-1=\sqrt{3}-1$	dM1: Takes tan and makes x the subject e.g. allow $x=\sqrt{3}\pm1$. Note that $\tan\left(\frac{\pi}{3}\right)$ does not need to be evaluated for this mark. May be implied by e.g. $x=0.732\ldots$ A1: $\sqrt{3}-1$	dM1A1
			(3)
(c)	Sub $x=5$ **and** $x=6$ into $\pm\left(\arctan x-4+\frac{1}{2}x\right) \Rightarrow -0.126..,+0.405...$ **and obtains at least one answer correct to 1sf**		M1
	Both values correct (to one sig fig), change of sign + conclusion Allow equivalent statements e.g. positive, negative therefore root etc. but this mark may be withheld if there are any contradictory statements e.g. therefore root lies between g(5) and g(6)		A1
	If $-\left(\arctan x-4+\frac{1}{2}x\right)$ is used to give $0.126..,-0.405...$, allow both marks if a conclusion is given.		
			(2)
(d)	$x_1=8-2\arctan 5$	Score for $x_1=8-2\arctan 5=\ldots$ This may be implied by awrt 5.3 (radians) or awrt -149 (degrees) for x_1	M1
	$x_1=5.253,\quad x_2=5.235$	$x_1=$ awrt 5.253, $x_2=$ awrt 5.235 Ignore any subsequent iterations and ignore labelling if answers are clearly the second and third terms.	A1
			(2)
			(9 marks)

Qu	Scheme	Marks
2(a)	$f(x) = x^3 - 5x + 16 = 0$ so $x^3 = 5x - 16$	M1
	$\Rightarrow x = \sqrt[3]{5x-16}$	A1
		(2)
(b)	$x_2 = \sqrt[3]{5\times -3 - 16}$	M1
	$x_2 = -3.141$ awrt	A1
	$x_3 = -3.165$ awrt **and** $x_4 = -3.169$ awrt	A1
		(3)
(c)	$f(-3.175) = -0.130984375..,\ f(-3.165) = 0.120482875$ Sign change (and as $f(x)$ is continuous) therefore a root α lies in the interval $[-3.175, -3.165] \Rightarrow \alpha = -3.17$ (2 dp)	M1A1
		(2)
		(7 marks)

(a) Way 1:

M1: Must state f(x) = 0 (or imply by writing $x^3 - 5x + 16 = 0$) and reach $x^3 = \pm 5x \pm 16$

A1: completely correct with all lines including f(x) = 0 stated or implied (see above), $x^3 = 5x - 16$ and $x = \sqrt[3]{5x-16}$ oe with or without a = 5, b = −16. Isw after a correct answer

If a candidate writes $x^3 = 5x - 16 \Rightarrow x = (5x-16)^{1/3}$ then they can score 1 0 for a correct but incomplete solution.

Similarly if a candidate writes $x^3 - 5x + 16 = 0 \Rightarrow x = (5x-16)^{1/3}$

Way 2:

M1: starts with answer, cubes and reaches a =.., b = .

A1: Completely correct reaching equation and stating hence f(x) = 0

(b)

Ignore subscripts in this part, just mark as the first, second and third values given.

M1: An attempt to substitute $x_1 = -3$ into **their** iterative formula. E.g. Sight of $\sqrt[3]{-31}$, or can be implied by $x_2 = awrt -3.14$

A1: $x_2 =$ awrt -3.141

A1: $x_3 =$ awrt -3.165 **and** $x_4 =$ awrt -3.169

(c)

M1: Choose suitable interval for x, e.g. $[-3.175, -3.165]$ and at least one attempt to evaluate f(x). Evidence would be the values embedded within an expression or one value correct. A minority of candidates may choose a tighter range which should include -3.1698 (alpha to 4dp). This would be acceptable for both marks, provided the conditions for the A mark are met. Some candidates may use an adapted $f(x) = 0$, for example $g(x) = x - \sqrt[3]{(5x-16)}$ This is also acceptable even if it is called f, but you must see it defined. For your information $g(-3.175) = -0.004, g(-3.165) = (+)0.004$ If the candidate states an f (without defining it) it must be assumed to be $f(x) = x^3 - 5x + 16$

A1: needs (i) both evaluations correct to 1 sf, (either rounded or truncated)
(ii) sign change stated (>0, <0 acceptable as would a negative product) and
(iii) some form of conclusion which may be $\Rightarrow \alpha = -3.17$ or "so result shown" or qed or tick or equivalent

Question Number	Scheme	Marks
11.(a)	$\sqrt{\frac{3}{2}}$ or $\frac{\sqrt{3}}{\sqrt{2}}$ or $\sqrt{1.5}$ or $\frac{\sqrt{6}}{2}$	B1
		(1)
(b)	$y=(2x^2-3)\tan\left(\frac{1}{2}x\right)\Rightarrow\frac{dy}{dx}=4x\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$	M1A1A1
	When $x=\alpha$ $\quad 4\alpha\tan\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)\times\frac{1}{2}\sec^2\left(\frac{1}{2}\alpha\right)=0$	
	$8\alpha\frac{\sin\left(\frac{1}{2}\alpha\right)}{\cos\left(\frac{1}{2}\alpha\right)}+(2\alpha^2-3)\times\frac{1}{\cos^2\left(\frac{1}{2}\alpha\right)}=0$	M1
	$8\alpha\sin\left(\frac{1}{2}\alpha\right)\cos\left(\frac{1}{2}\alpha\right)+(2\alpha^2-3)=0$	
	$4\alpha\sin\alpha+(2\alpha^2-3)=0$	dM1
	$2\alpha^2-3+4\alpha\sin\alpha=0$	A1*
		(6)
(c)	$x_2=\frac{3}{(2\times0.7+4\sin0.7)}$	M1
	$x_2=0.7544,\ x_3=0.7062$	A1
		(2)
(d)	Chooses interval $[0.72825, 0.72835]$	M1
	$2\times0.72825^2-3+4\times0.72825\sin0.72825=-0.0005<0$ $2\times0.72835^2-3+4\times0.72835\sin0.72835=0.00026>0$ + Reason +conclusion	A1
		(2)
		(11 marks)

(a)

B1: $x=\sqrt{\frac{3}{2}}$ or exact equivalent and no others **inside** the range. Ignore any solution outside the range so allow e.g. $x=\pm\sqrt{\frac{3}{2}}$. $\sqrt{\frac{3}{2}}$ seen unless seen in an incorrect statement e.g. $x^2=\sqrt{\frac{3}{2}}$.

(b)

M1: Attempts product rule on $y=(2x^2-3)\tan\left(\frac{1}{2}x\right)$ or $y=2x^2\tan\left(\frac{1}{2}x\right)$ if they multiply out first so look for

$\frac{d(2x^2-3)}{dx}\times\tan\left(\frac{1}{2}x\right)+(2x^2-3)\times\frac{d\tan\left(\frac{1}{2}x\right)}{dx}$ or $\frac{d\left(2x^2\right)}{dx}\times\tan\left(\frac{1}{2}x\right)+2x^2\times\frac{d\tan\left(\frac{1}{2}x\right)}{dx}$ or e.g.

$Ax\tan\left(\frac{1}{2}x\right)+Bx^2\sec^2\frac{1}{2}x$

Question Number	Scheme	Marks
1(a)	$x^5+x^3-12x^2-8=0 \Rightarrow x^5+x^3=12x^2+8$	M1
	$x^3\left(x^2+1\right)=12x^2+8 \Rightarrow x^3=\dfrac{12x^2+8}{(x^2+1)}$ or e.g. $x^3=\dfrac{4\left(3x^2+2\right)}{(x^2+1)}$	A1
	Note that going straight from $x^5+x^3=12x^2+8$ to $x^3=\dfrac{12x^2+8}{(x^2+1)}$ is acceptable for the first 2 marks but the final mark should be withheld for not explicitly showing the factorisation of the lhs	
	$\Rightarrow x=\sqrt[3]{\dfrac{4\left(3x^2+2\right)}{(x^2+1)}}$ or $x=\sqrt[3]{\dfrac{4\left(2+3x^2\right)}{(x^2+1)}}$	A1*
		(3)
(b)	$x_1=\sqrt[3]{\dfrac{4\left(3\times2^2+2\right)}{2^2+1}}=2.237$	M1A1
	$x_2=2.246,\quad x_3=2.247$	A1
		(3)
(c)	Interval $\left[2.2465,2.2475\right]\Rightarrow \mathrm{f}\left(2.2465\right)=\ldots,\mathrm{f}\left(2.2475\right)=\ldots$	M1
	$\mathrm{f}(2.2465)=-0.0057,\ \mathrm{f}(2.2475)=(+)0.083$ +Reason + Conclusion	A1
		(2)
		(8 marks)
Alt (a)	$x=\sqrt[3]{\dfrac{4\left(3x^2+2\right)}{(x^2+1)}}\Rightarrow x^3(x^2+1)=12x^2+8$	M1
	$x^5+x^3-12x^2-8=0$	A1
	Statement Hence $\mathrm{f}(x)=0$	A1*
		(3)

(a)

M1: Attempts to write equation in the form $x^5\pm x^3=12x^2\pm8$ or $x^3(x^2\pm1)=12x^2\pm8$.

A1: Intermediate line of $x^3=\dfrac{12x^2+8}{(x^2+1)}$ seen

A1*: cso with the factorisation of the lhs seen explicitly and a statement at the start that f(x) = 0 or $x^5+x^3-12x^2-8=0$ oe e.g. $x^3\left(x^2+1\right)-4\left(3x^2+2\right)=0$

Do not be overly concerned about the cube root encompassing the whole fraction but do not allow if it is only unambiguously the numerator that has the cube root e.g. $\Rightarrow x=\dfrac{\sqrt[3]{4\left(3x^2+2\right)}}{(x^2+1)}$

Beware of other algebraic methods of establishing the result in (a) – if in doubt send to review.

Alternative for part (a):

M1: Cubes the printed result and multiplies up

A1: Obtains the required equation with no errors

A1*: Makes a conclusion (may be minimal e.g. tick, QED, # etc.) and $x^3(x^2+1)=x^5+x^3$ seen explicitly in the working

(b)

M1: Substitutes x_0