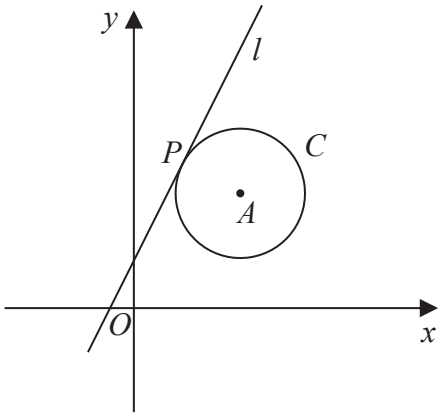
6.



Not to scale

Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

Leave blank

15.

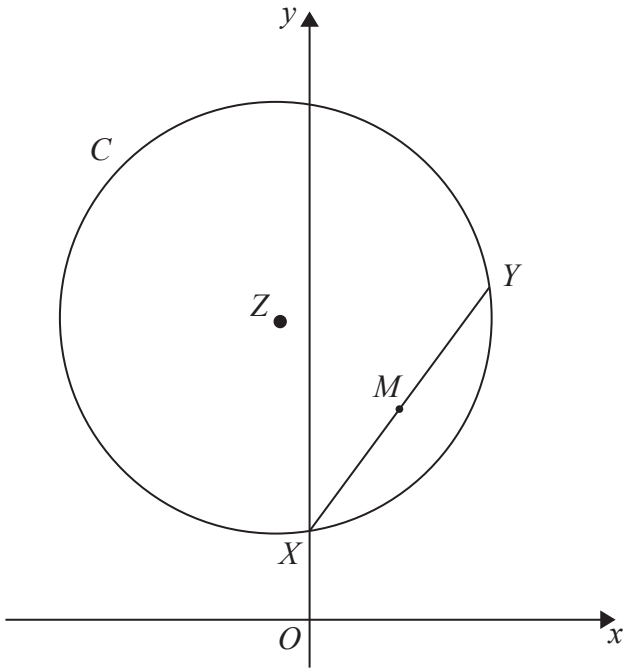


Diagram **NOT** drawn to scale

Figure 3

The points X and Y have coordinates (0, 3) and (6, 11) respectively. XY is a chord of a circle C with centre Z, as shown in Figure 3.

(a) Find the gradient of XY.

(2)

The point M is the midpoint of XY.

(b) Find an equation for the line which passes through Z and M.

(5)

Given that the y coordinate of Z is 10,

(c) find the x coordinate of Z,

(2)

(d) find the equation of the circle C, giving your answer in the form

$$x^2 + y^2 + ax + by + c = 0$$

where a, b and c are constants.

(5)

Leave blank

7. The circle C has equation

$$x^2 + y^2 + 10x - 6y + 18 = 0$$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C.

(2)

The circle C meets the line with equation $x = -3$ at two points.

(c) Find the exact values for the y coordinates of these two points, giving your answers as fully simplified surds.

(4)

Leave blank

12.

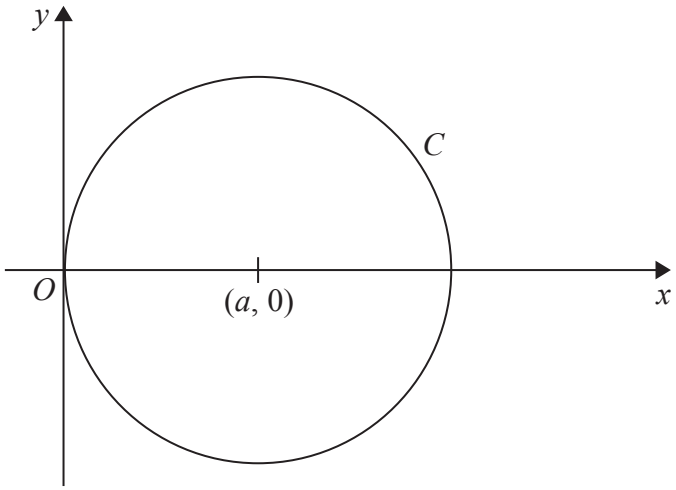


Figure 3

Figure 3 shows a circle C

C touches the y-axis and has centre at the point $(a, 0)$ where a is a positive constant.

(a) Write down an equation for C in terms of a

(2)

Given that the point $P(4, -3)$ lies on C,

(b) find the value of a

(3)

Leave blank

13. The circle C has centre $A(1, -3)$ and passes through the point $P(8, -2)$.

(a) Find an equation for the circle C.

(4)

The line l_1 is the tangent to C at the point P.

(b) Find an equation for l_1, giving your answer in the form $y = mx + c$

(4)

The line l_2, with equation $y = x + 6$, is the tangent to C at the point Q.

(c) Find the coordinates of the point Q.

(5)

Leave blank

13. (a) On separate axes sketch the graphs of

(i) $y = c^2 - x^2$

(ii) $y = x^2(x - 3c)$

where c is a positive constant.

Show clearly the coordinates of the points where each graph crosses or meets the x-axis and the y-axis.

(5)

(b) Prove that the x coordinate of any point of intersection of

$$y = c^2 - x^2 \text{ and } y = x^2(x - 3c)$$

where c is a positive constant, is given by a solution of the equation

$$x^3 + (1 - 3c)x^2 - c^2 = 0$$

(2)

Given that the graphs meet when $x = 2$

(c) find the exact value of c, writing your answer as a fully simplified surd.

(4)

Leave blank

14.

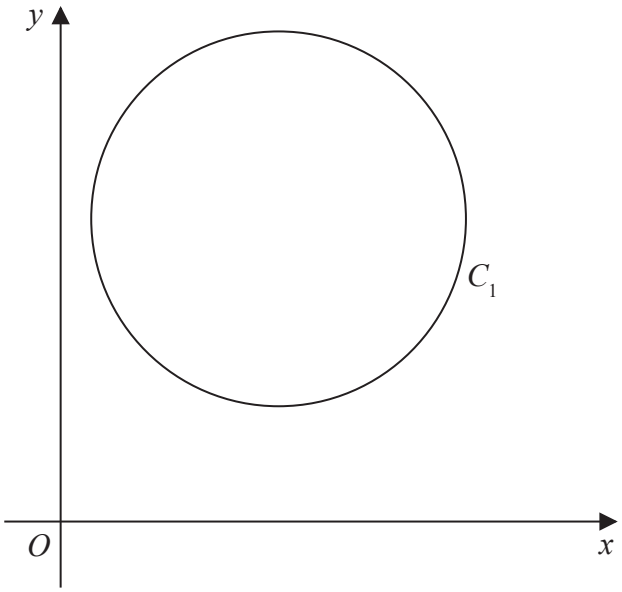


Figure 5

Figure 5 shows a sketch of the circle C_1

The points $A(1, 4)$ and $B(7, 8)$ lie on C_1

Given that AB is a diameter of the circle C_1

(a) find the coordinates for the centre of C_1

(2)

(b) find the exact radius of C_1 simplifying your answer.

(2)

Two distinct circles C_2 and C_3 each have centre (0, 0).

Given that each of these circles touch circle C_1

(c) find the equation of circle C_2 and the equation of circle C_3

(4)

Leave blank

13. The circle C has equation

$$(x - 3)^2 + (y + 4)^2 = 30$$

Write down

(a) (i) the coordinates of the centre of C,

(ii) the exact value of the radius of C.

(2)

Given that the point P with coordinates $(6, k)$, where k is a constant, lies inside circle C,

(b) show that

$$k^2 + 8k - 5 < 0$$

(3)

(c) Hence find the exact set of values of k for which P lies inside C.

(4)

Leave blank

3.

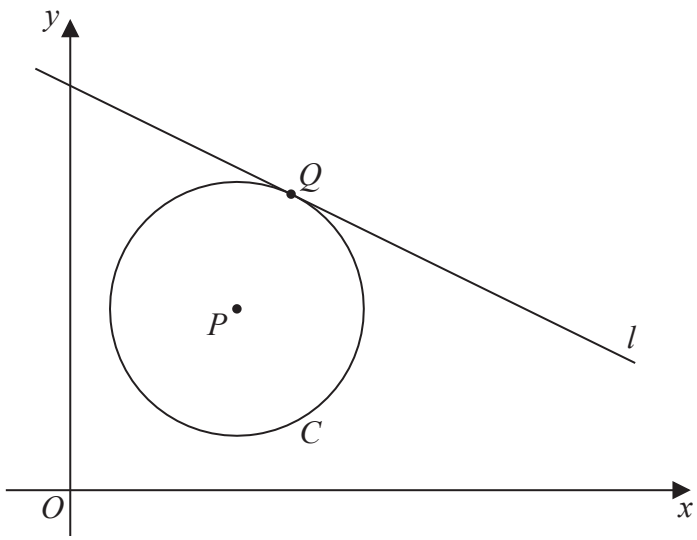


Diagram not drawn to scale

Figure 2

The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$, as shown in Figure 2.

(a) Find the length PQ, giving your answer as an exact value.

(2)

(b) Hence write down an equation for C.

(2)

The line l is a tangent to C at the point Q, as shown in Figure 2.

(c) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

Leave blank

9.

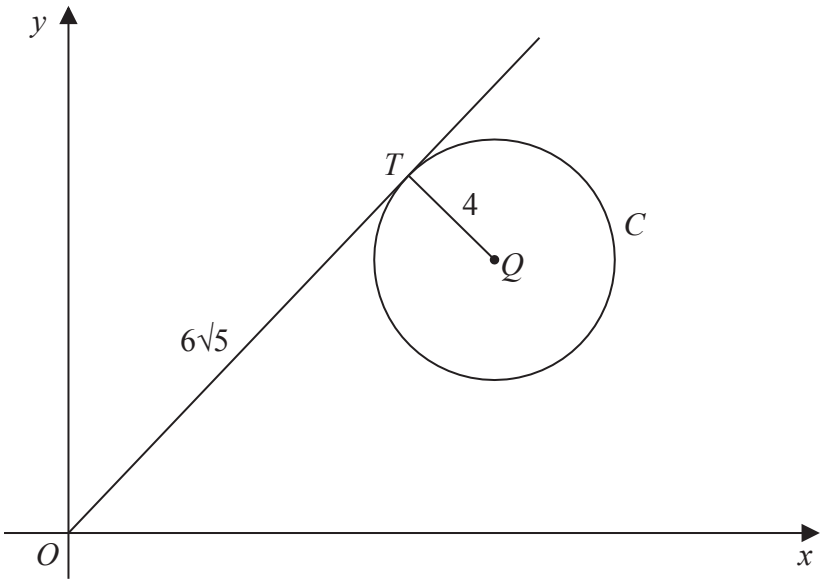


Figure 3

Figure 3 shows a circle C with centre Q and radius 4 and the point T which lies on C.

The tangent to C at the point T passes through the origin O and $OT = 6\sqrt{5}$

Given that the coordinates of Q are $(11, k)$, where k is a positive constant,

(a) find the exact value of k,

(3)

(b) find an equation for C.

(2)

Leave blank

5. The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M.

(a) Find

(i) the coordinates of the point M,

(ii) the radius of the circle C.

(5)

N is the point with coordinates (25, 32).

(b) Find the length of the line MN.

(2)

The tangent to C at a point P on the circle passes through point N.

(c) Find the length of the line NP.

(2)

Leave blank

3.

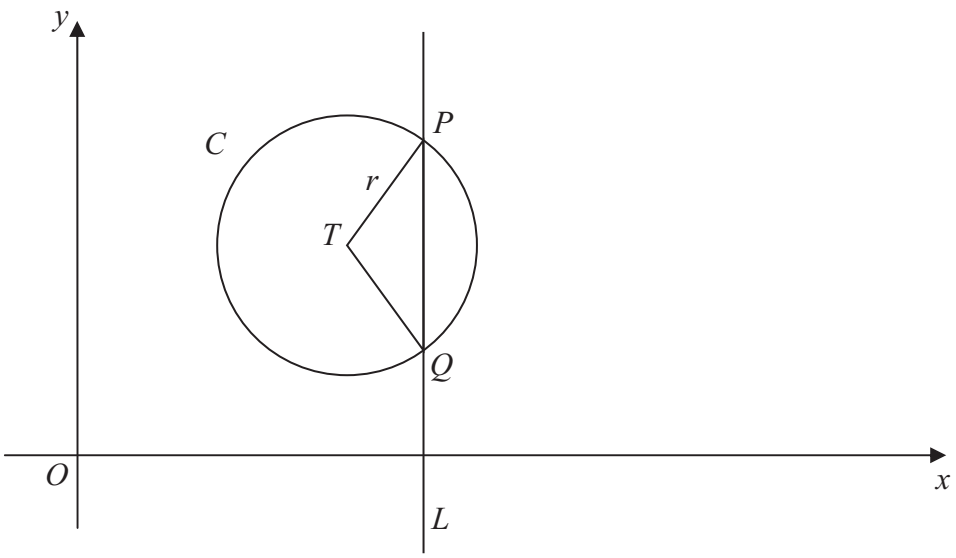


Figure 1

The circle C with centre T and radius r has equation

$$x^2 + y^2 - 20x - 16y + 139 = 0$$

(a) Find the coordinates of the centre of C.

(3)

(b) Show that $r = 5$

(2)

The line L has equation $x = 13$ and crosses C at the points P and Q as shown in Figure 1.

(c) Find the y coordinate of P and the y coordinate of Q.

(3)

Given that, to 3 decimal places, the angle PTQ is 1.855 radians,

(d) find the perimeter of the sector PTQ.

(3)

8.

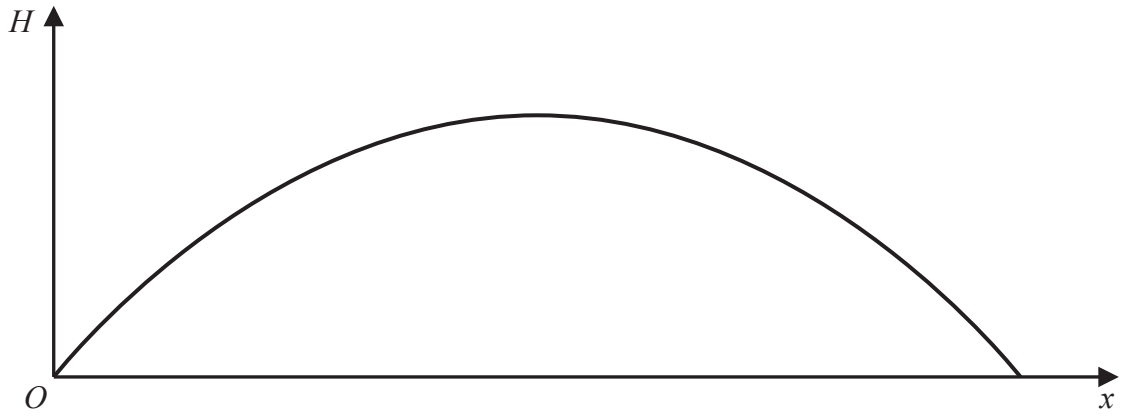


Figure 1

Figure 1 is a graph showing the trajectory of a rugby ball.

The height of the ball above the ground, H metres, has been plotted against the horizontal distance, x metres, measured from the point where the ball was kicked.

The ball travels in a vertical plane.

The ball reaches a maximum height of 12 metres and hits the ground at a point 40 metres from where it was kicked.

(a) Find a quadratic equation linking H with x that models this situation.

(3)

The ball passes over the horizontal bar of a set of rugby posts that is perpendicular to the path of the ball. The bar is 3 metres above the ground.

(b) Use your equation to find the greatest horizontal distance of the bar from O.

(3)

(c) Give one limitation of the model.

(1)

7. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost
- a cost that is proportional to the number of bars of soap that are made that day

(a) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80

Using the above information,

(b) show that $y = 0.84x + 428$

(3)

(c) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(d) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)