Mark Scheme

Summer 2023

Pearson Edexcel GCE

In AS Mathematics (8MA0)

Paper 01 Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS
General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

   $(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

   $(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

   Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

   Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

   Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

   Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

Question	Scheme	Marks	AOs
1 (a)	$\left\{\frac{dy}{dx}=\right\}2x^2-7x-4$	M1 A1	1.1b 1.1b
		(2)	
(b)	Attempts to solve $\left\{\frac{dy}{dx}=\right\}2x^2-7x-4\ldots0$ e.g., $(2x+1)(x-4)=0$ leading to $x=\ldots$ and $x=\ldots$	M1	1.1b
	Correct critical values $x=-\frac{1}{2},4$	A1	1.1b
	Chooses inside region for their critical values	dM1	1.1b
	Accept either $-\frac{1}{2}<x<4$ **or** $-\frac{1}{2}\leqslant x\leqslant 4$	A1	1.1b
		(4)	
			(6 marks)

Notes:

(a)

M1: Decreases the power of x by one for at least one of their terms. Look for $x^n \to \ldots x^{n-1}$

Allow for $5\to 0$

A1: $\left\{\frac{dy}{dx}=\right\}2x^2-7x-4$

(b)

M1: Sets their $\frac{dy}{dx}\ldots0$ where … may be an equality or an inequality and proceeds to find two values for x from a 3TQ using the usual rules. This may be implied by their critical values.

A1: Correct critical values $x\ldots-\frac{1}{2},4$

These may come directly from a calculator and might only be seen on a sketch.

dM1: Chooses the inside region for their critical values.

A1: Accept either $-\frac{1}{2}<x<4$ **or** $-\frac{1}{2}\leqslant x\leqslant 4$ but not, e.g., $-\frac{1}{2}<x\leqslant 4$

Condone, e.g., $x>-\frac{1}{2},\quad x<4$ **or** $x>-\frac{1}{2}$ and $x<4$ **or** $\left\{x:x>-\frac{1}{2}\right\}\cap\{x:x<4\}$

or $x\in\left(-\frac{1}{2},4\right)$ **or** $x\in\left[-\frac{1}{2},4\right]$

Note: You may see $x<-\frac{1}{2},x<4$ in their initial work before $-\frac{1}{2}<x<4$. Condone this so long as it is clear that the $-\frac{1}{2}<x<4$ is their final answer.

Question	Scheme	Marks	AOs
2	Let $u=\sqrt{x}$ $6x+7\sqrt{x}-20=0\Rightarrow 6u^2+7u-20=0$ $\Rightarrow(3u-4)(2u+5)\{=0\}$	M1A1	1.1b 1.1b
	Attempts $\sqrt{x}="\frac{4}{3}","-\frac{5}{2}"\Rightarrow x=...$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	
			(4 marks)
Alt 1	$6x+7\sqrt{x}-20=0\Rightarrow 7\sqrt{x}=20-6x\Rightarrow 49x=(20-6x)^2$ $\Rightarrow 49x=400-240x+36x^2$	M1	1.1b
	$36x^2-289x+400\{=0\}$	A1	1.1b
	$(9x-16)(4x-25)=0$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	
Alt 2	$6x+7\sqrt{x}-20=0\Rightarrow(3\sqrt{x}-4)(2\sqrt{x}+5)=0$	M1 A1	1.1b 1.1b
	Attempts $\sqrt{x}="\frac{4}{3}","-\frac{5}{2}"\Rightarrow x=...$	M1	1.1b
	$x=\frac{16}{9}$ only	A1 cso	2.3
		(4)	

Notes:

M1: Attempts a valid method that enables the problem to be solved. See General Principles for Pure Mathematics Marking at the front of the mark scheme for guidance. Score for either letting $u=\sqrt{x}$ and attempting to factorise to $(au\pm c)(bu\pm d)$ with $ab=6, cd=20$

or making $7\sqrt{x}$ the subject and attempting to square both sides.

or attempting to factorise to $(a\sqrt{x}\pm c)(b\sqrt{x}\pm d)$ with $ab=6, cd=20$

or by quadratic formula or completing the square following usual rules.

A1: $(3u-4)(2u+5)\{=0\}$ **or** $36x^2-289x+400\{=0\}$ **or** $(3\sqrt{x}-4)(2\sqrt{x}+5)\{=0\}$

If they use the formula, it must be correct e.g., $u\,\{\text{or}\sqrt{x}\}=\frac{-7\pm\sqrt{7^2-4(6)(-20)}}{12}$ followed by $u\,\{\text{or}\sqrt{x}\}=\frac{4}{3}$ or equivalent e.g., $\frac{16}{12}$. Ignore if they have $u\,\{\text{or}\sqrt{x}\}=-\frac{5}{2}$ or not.

If they complete the square, they must have $\left(u+\frac{7}{12}\right)^2=\frac{529}{144}$ followed by $u\ \{\text{or}\sqrt{x}\}=\frac{4}{3}$ or equivalent e.g., $\frac{16}{12}$. Ignore if they have $u\ \{\text{or}\sqrt{x}\}=-\frac{5}{2}$ or not.

M1: Correct method from $p\sqrt{x}\pm q=0$ leading to $x=\ldots$ by squaring

In Alt 1, it is for solving their quadratic using the General Principles for Pure Mathematics Marking. There must be a method shown, i.e., the solutions should not come straight from a calculator. If attempting to factorise, it must be to $(ax\pm c)(bx\pm d)$ with $ab=36, cd=400$

In Alt 2, it is for squaring their value(s) for u to get $x=\ldots$

A1: **cso** $x=\frac{16}{9}$ only. $x=\frac{25}{4}$ must be discarded. Note 0011 is not possible.

Allow "incorrect" $x=-\frac{16}{9}$ or $x=-\frac{25}{4}$ to be seen as long as they are discarded.

Ignore any reason they give for rejecting solutions.

Note that a method to solve their quadratic must be seen – solutions must not come directly from a calculator. Simply stating the quadratic formula (without substitution) is insufficient.

Question	Scheme	Marks	AOs
3 (a)	Angle $ACB = 33°$	B1	1.1b
	Attempts $\{AB^2 =\} 8.2^2 + 15.6^2 - 2\times 8.2\times 15.6\cos 33°$	M1	1.1b
	Distance = awrt 9.8 {km}	A1	1.1b
		(3)	
(b)	• Explains that the road is not likely to be straight {and therefore the distance will be greater}. • Explains that there are likely to be objects in the way {that they must go around and therefore the distance travelled will be greater}. • The {bases of the} masts are not likely to lie in the same {horizontal} plane {and so the distance will be greater}.	B1	3.2b
		(1)	
		(4 marks)	

Notes:

(a)

B1: 33 seen anywhere but allow 72 – 39. May be indicated on a diagram (including incorrectly) or on the given Figure 1 and it might be named incorrectly.

M1: Uses the given model and attempts to use the cosine rule to find the distance or distance^2

Award for $8.2^2 + 15.6^2 - 2\times 8.2\times 15.6\cos\ldots$ where ... must be a value.

A1: awrt 9.8 {km} isw

(a) Alternative

B1: $\left\{\overrightarrow{AB} =\right\} \pm\begin{pmatrix}15.6\cos 51 - 8.2\cos 18\\ 15.6\sin 51 - 8.2\sin 18\end{pmatrix}$ **or** $\pm\begin{pmatrix}15.6\sin 39 - 8.2\sin 72\\ 15.6\cos 39 - 8.2\cos 72\end{pmatrix}$ o.e.

May be implied by calculation that leads to $\begin{pmatrix}\text{awrt}\pm 2.0\\ \text{awrt}\pm 9.6\end{pmatrix}$ e.g. $\begin{pmatrix}9.8\\ 12.1\end{pmatrix} - \begin{pmatrix}7.8\\ 2.5\end{pmatrix}$

Note: they may find components separately and condone, e.g., $\begin{pmatrix}\text{awrt}\pm 9.6\\ \text{awrt}\pm 2.0\end{pmatrix}$

M1: Attempts to find $\overrightarrow{AB}$ (as above) **and** uses Pythagoras to find distance or distance^2

A1: awrt 9.8 {km} isw

(b)

B1: A valid reason based on the assumptions, i.e., the plane is not really horizontal **or** the journey not being in a straight line.
Do not accept answers referencing the accuracy of the answer to part (a) being to 1d.p. or the accuracy of the values given in the question, **but** ignore if there is a separate, valid reason.

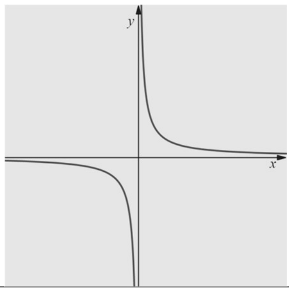
Some examples:

"Because it is unlikely the bearings are exact" – B0 see above.

"Because they may not walk in a straight line because they could take another longer or shorter route as their route could be more curved" – B0 – incorrect comment about there being a shorter route.

"Because they won't travel in one direction due to the roads" – B1 BOD

"Impossible and unrealistic to walk in a straight line" – B1

Question	Scheme	Marks	AOs
4(a)	 Shape in quadrant 1 **or** 3	M1	1.1b
	Shape and Position	A1	1.1b
		(2)	
(b)	Deduces that $x<0$	B1	2.2a
	Attempts $\frac{16}{x}...2 \Rightarrow x...\pm\frac{16}{2}$	M1	1.1b
	$x<0$ or $x \geqslant 8$	A1 cso	2.2a
		(3)	
			(5 marks)

Notes:

(a)

M1: For the correct shape in quadrant 1 **or** 3. Do not be concerned about position but it must not cross either axis. Ignore incorrect asymptotes for this mark.

A1: Correct shape and position. There should be no curve in either quadrant 2 or quadrant 4. The curve must not clearly bend back on itself but condone slips of the pen.

(b)

B1: Deduces that $x<0$ but condone $x \leqslant 0$ for this mark.

M1: Attempts $\frac{16}{x}...2 \Rightarrow x...\pm\frac{16}{2}$ where the … means any equality or inequality.

A1: cso $x<0$ or $x \geqslant 8$ (Both required)

Set notation may be seen $\{x: x<0\} \cup \{x: x \geqslant 8\}$ o.e. $x \in (-\infty, 0) \cup [8, \infty)$

Accept $x<0$, $x \geqslant 8$ but not $x<0$ and $x \geqslant 8$

Must not be combined incorrectly, e.g., $8 \leqslant x<0$ **or** $\{x: x<0\} \cap \{x: x \geqslant 8\}$

Question	Scheme	Marks	AOs
5	States or uses the upper limit is $\sqrt{5}$	B1	1.1b
	$\int 4x^2 + 3\,dx = \frac{4}{3}x^3 + 3x$	M1 A1	1.1b 1.1b
	Full method of finding the area of R e.g. $23\sqrt{5} - \left[\frac{4}{3}x^3 + 3x\right]_0^{\sqrt{5}} = ...$ e.g. $\left[20x - \frac{4}{3}x^3\right]_0^{\sqrt{5}} = ...$	M1	2.1
	$\Rightarrow$ Area $R = \frac{40}{3}\sqrt{5}$	A1	1.1b
		(5)	
		(5 marks)	

Notes:

B1: States or uses the upper limit $\sqrt{5}$ Score when seen as the solution $x = \sqrt{5}$

M1: Attempts to integrate $4x^2 + 3$ **or** $\pm\left(23 - \left(4x^2 + 3\right)\right)$ which may be simplified.

Look for one term from $4x^2 + 3$ with $x^n \to x^{n+1}$ It is not sufficient just to integrate 23.

A1: Correct integration. Ignore any $+c$ or spurious integral signs. Indices must be processed.

Look for $\int 4x^2 + 3\{dx\} = \frac{4}{3}x^3 + 3x$ **or** $\pm\int 20 - 4x^2\{dx\} = \pm\left(20x - \frac{4}{3}x^3\right)$ if (line $-$curve) or (curve – line) used.

M1: Full and complete method to find the area of R including the substitution of their upper limit. The upper limit must come from an attempt to solve $4x^2 + 3 = 23$
The lower limit might not be seen but if seen it should be 0.

See scheme for two possible ways. Condone a sign slip if (line $-$curve) or (curve – line) used.

A1: $\frac{40}{3}\sqrt{5}$ following correct algebraic integration.

If using (curve – line) then allow recovery but they must make the $-\frac{40}{3}\sqrt{5}$ positive.

Alternative using $\int x\,dy$

B1: States or uses limits 3 and 23. It must be for a clear attempt to integrate with respect to y

M1: Attempts to rearrange to $x =$ and integrate $\sqrt{\frac{y-3}{4}}$ condoning slips on the rearrangement.

Look for $...(y \pm 3)^{\frac{1}{2}} \to ...(y \pm 3)^{\frac{3}{2}}$

A1: Correct integration $\int \frac{(y-3)^{\frac{1}{2}}}{2}\{dy\} = \frac{1}{3}(y-3)^{\frac{3}{2}}$ Ignore any $+c$ or spurious integral signs.

M1:	Full and complete method to find the area of R including the substitution of their limits. In this case it would be for substituting 23 and 3 and subtracting either way round into their changed expression in terms of y
A1:	$\frac{40}{3}\sqrt{5}$ following correct algebraic integration.

Question	Scheme	Marks	AOs
6 (a)	$x^2+y^2-6x+10y+k=0$		
	$(x-3)^2+(y+5)^2 \pm \ldots = \ldots$	M1	1.1b
	Centre $(3,-5)$	A1	1.1b
		(2)	
(b)	Deduces that $k=9$ is a critical point	B1ft	2.2a
	Recognises that radius > 0 $"9"+"25"-k>0$	M1	3.1a
	$9<k<34$	A1	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: For sight of $(x\pm3)^2 \pm (y\pm5)^2 \pm \ldots = \ldots$ or one coordinate for centre from $(\pm3,\pm5)$

A1: Centre $(3,-5)$

(b)

B1ft: Deduces that $k \ldots 9$ is a critical point. Allow this to come from their $("5")^2$ Condone $\frac{36}{4}$

Note that this might come from setting $y = 0$ and using the discriminant on $x^2-6x+k=0$

M1: $(x\pm3)^2+(y\pm5)^2=("3")^2+("5")^2-k$ **and** recognises that the radius^2 must be positive so $("3")^2+("5")^2-k>0$ but condone $("3")^2+("5")^2-k \geqslant 0$

$k<34$ **or** $k \leqslant 34$ would imply this method mark.

Note: they may have incorrectly calculated $("3")^2+("5")^2$ in (a) so allow their value for this in place of $("3")^2+("5")^2$ as long as the intention is clear.

A1: $9<k<34$ but condone $9<k\leqslant 34$. Allow inequalities to be separate, i.e., $k>9, k<34$

Set notation may be seen $\{k:k>9\}\cap\{k:k<34\}$ **or** $k\in(9,34)$

Condone $\{k:k>9\}\cap\{k:k\leqslant 34\}$ **or** $k\in(9,34]$ **or** $k>9$ and $k\leqslant 34$

Must not be combined incorrectly, e.g., $\{k:k>9\}\cup\{k:k<34\}$

Question	Scheme	Marks	AOs
7 (a)	Uses or implies that $V = ad + b$	B1	3.3
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get either a or b	M1	3.1b
	Uses both $40 = 80a + b$ and $25 = 200a + b$ to get both a and b	dM1	1.1b
	$\Rightarrow V = -\frac{1}{8}d + 50$ o.e.	A1	1.1b
		(4)	
(b)(i)(ii)	States **either** that the initial volume was 50 {litres} **or** that the distance travelled was 400 {km}	B1 ft	3.4
	Attempts to find **both** answers by solving $0 = -\frac{1}{8}d + 50$ **and** $0 = 400 - 8V$	M1	3.4
	States **both** that the initial volume was 50 litres **and** that the distance travelled was 400 km	A1	3.2b
		(3)	
(c)	States, e.g., "Poor model" as 320km is significantly less than 400 km.	B1ft	3.5a
		(1)	
			(8 marks)

Notes:

(a)

B1: Attempts a linear model, i.e., uses or implies that $V = ad + b$ or $d = mV + c$ which may be in terms of, e.g., y and x

M1: Awarded for translating the problem in context and starting to solve.
It is scored when both $40 = 80a + b$ and $25 = 200a + b$ are written down and the candidate proceeds to find either a or b
Alternatively, scored when both $200 = 25m + c$ and $80 = 40m + c$ are written down and the candidate proceeds to find either m or c
You may just see $\pm\frac{25-40}{200-80}$ or $\pm\frac{200-80}{25-40}$ or 8km for every litre o.e. so check carefully for attempts at the gradient.

dM1: Uses $40 = 80a + b$ and $25 = 200a + b$ to find both a and b (or m and c)
Alternatively, if the gradient is found, proceeds to use one of the bullet points to find c, with the usual rules applying for straight line (coordinates must be used the correct way round, i.e., they would lead to the correct answer).

A1: $V = -\frac{1}{8}d + 50$ or exact equivalent, e.g., $d = 400 - 8V$ or $d + 8V = 400$ etc.
Withhold this mark if their answer is not stated in terms of V and d

Mark parts (b)(i) and (b)(ii) together. Note that they may restart and not use an equation.

B1ft: States **either** the initial volume was 50 {litres} **or** the distance travelled was 400 {km} but it must be clearly for the correct part, e.g., $V = 50$.
Follow through on their a and b (or m and c). This may be scored from $40 + \frac{80}{8}$ or $\frac{400}{8}$

M1: Complete attempt to find both answers. Must be from a **linear** model.

	Substitutes $V = 0$ and finds d by attempting to solve their $0 = -\frac{1}{8}d + 50$ **and** substitutes $d = 0$ and finds V by attempting to solve their $0 = 400 - 8V$ Note that one (or both) of these attempts may be implied by correct values ft their equations.
A1:	States both 50 litres and 400 km. Units are required to be correct for both values. It must be clear which answer applies to each part, which may be simply by correct units. Accept *l* or *L* for litres.
(c)	
B1ft:	**Main Scheme (comparing (b)(ii) with 320)** This mark is only available for answers from (b)(ii) if they are < 290 **or** > 350 Concludes **poor** model (o.e.) and states that 320 is **significantly** less than "400" (o.e.) Note that $320 << 400$ so it is a poor model is acceptable. It is not sufficient to say $320 \neq 400$ or $320 < 400$ so it is a poor model. Condone "the 400 is **too** far away from 320". **Alternative (finding remaining fuel after 320 km)** States **poor** model (o.e.) because after 320 km the model predicts there will be 10 litres left, which is **significantly** more than an empty tank / **much** too high compared to an empty tank (o.e.).

Question	Scheme	Marks	AOs
8	Complete method to find the RHS of an equation for l e.g., Attempts gradient = $\frac{80-60}{10}\{=2\}$ and uses intercept = 60	M1	1.1b
	$\{y=\}2x+60$	A1	1.1b
	Deduces the RHS of the equation for C is $\{y=\}ax(x-6)$ and attempts to use $(10,80)$ to find the value of a	M1	3.1a
	Equation of C is $\{y=\}2x(x-6)$	A1	1.1b
	$2x(x-6)\leqslant y\leqslant 2x+60$	B1ft	2.5
		(5)	
			(5 marks)

Notes:

M1: Complete attempt to use the given information to find an equation or inequality for l, which may be l = or have no LHS. $y - 80 = 2(x - 10)$ is acceptable for this mark.

A1: $\{y=\}2x+60$ This is not scored by $y - 80 = 2(x - 10)$

M1: Deduces the RHS of the equation of C is $\{y=\}ax(x-6)$, $a\neq 1$, and attempts to use $(10,80)$ to find the value of a which may be implied. Again, there may be no LHS.
Other possible and more lengthy alternatives include:
1) Setting the RHS to be $\{y=\}a(x-3)^2+b$ and using $(0,0)$ and $(10,80)$ to find a and b
2) Setting the RHS to be $\{y=\}px^2+qx$ and using $(6,0)$ and $(10,80)$ to find p and q

A1: $\{y=\}2x(x-6)$ or alternative such as $\{y=\}2(x-3)^2-18$ or $\{y=\}2x^2-12x$
This may be implied by an inequality $y\ldots 2x(x-6)$ and may be seen as, e.g., $C=2x(x-6)$

B1ft: $"2x(x-6)"\leqslant y\leqslant "2x+60"$ o.e. must follow from their l and C and apply isw
Follow through only on a quadratic for C and a straight line for l
Do not allow a mixture of inequalities, i.e., $<$ with $\leqslant$
Allow $2x^2-12x<y<2x+60$ or as separate inequalities $y>2x(x-6)$, $y<2x+60$
Do not allow $2x(x-6)<R<2x+60$ **or** $2x(x-6)<\mathrm{f}(x)<2x+60$ **or** $2x(x-6)<2x+60$
Ignore any reference to $-3<x<10$
Note: $y=2x+60$ and $y=2x(x-6)$ incorrectly expanded to $y=2x^2-12$ followed by $2x^2-12\leqslant y\leqslant 2x+60$ would score 11110

Question	Scheme	Marks	AOs
9	$2\log_5(3x-2)-\log_5 x=2$		
	Uses one correct law e.g. $2\log_5(3x-2)\to\log_5(3x-2)^2$ **or** $2\to\log_5 25$ **or** $\log_5 ...=2\to ...=5^2$	B1	1.1a
	Uses two correct log laws: either $2\log_5(3x-2)\to\log_5(3x-2)^2$ **and** $2\to\log_5 25$ or $2\log_5(3x-2)-\log_5 x\to\log_5\frac{(3x-2)^2}{x}$ leading to an equation without logs	M1	3.1a
	Correct equation without logs, usually $\frac{(3x-2)^2}{x}=25$	A1	1.1b
	$\frac{(3x-2)^2}{x}=25\Rightarrow 9x^2-37x+4=0\Rightarrow(9x-1)(x-4)=0\Rightarrow x=...$	dM1	1.1b
	$x=4$ only	A1 cso	3.2a
		(5)	
			(5 marks)

Notes:

B1: Uses one correct log law. The base does not need to be seen for this mark.
This mark is independent of any other errors they make.

M1: This can be awarded for the overall strategy leading to an equation in x **not involving logs**.
It requires the correct use of two log laws as in the main scheme to reach an equation in x
This mark may **not** be awarded for correct application of two laws following incorrect log work, but numerical slips are condoned.

A1: For a correct unsimplified equation with logs removed and **no incorrect work seen.**
Ignore any incorrect simplification of their equation.

Allow recovery on missing brackets, e.g., $\log_5\frac{3x-2^2}{x}=2\to\frac{9x^2-12x+4}{x}=25$

Correct equations are likely to be $\frac{(3x-2)^2}{x}=25$ or, e.g., $(3x-2)^2=25x$ but you might see $9x-12+\frac{4}{x}=25$ Sight of a correct equation does **not** imply either the previous M1 or the A1.

Note: $\frac{\log_5(3x-2)^2}{\log_5 x}=2\to\frac{(3x-2)^2}{x}=25$ may be seen and scores B1M0A0.

dM1: For a correct method to solve their equation, **via a 3TQ set = 0**
The 3TQ may be solved by calculator - you may need to check their value(s).
Can be implied by one correct value for their 3TQ set = 0 correct to 1d.p.

A1:	cso $x = 4$ only. If $x = \frac{1}{9}$ is also given it must be rejected. $x = 0$ might also be seen and must be rejected. Ignore any reasoning for rejecting any values. Note that calculators can solve the equation at any stage and so full log work must be shown leading to a 3TQ set = 0.

Question	Scheme	Marks	AOs
10 (a)	Deduces that the gradient of line l_2 is $-\frac{5}{3}$	B1	1.1b
	Complete attempt to find the equation of line l_2 e.g., $y-0=-\frac{1}{"m_1"}(x-8)$	M1	1.1b
	$5x+3y=40$ *	A1*	2.1
		(3)	
(b)	Deduces $A(-10,0)$	B1	2.2a
	Attempts to solve $y=\frac{3}{5}x+6$ and $5x+3y=40$ simultaneously to find the y coordinate of their point of intersection	M1	1.1b
	y coordinate of C is $\frac{135}{17}$ o.e.	A1	1.1b
	Complete attempt at area $ABC = \frac{1}{2}\times\left(8+"10"\right)\times"\frac{135}{17}"$	dM1	2.1
	$=\frac{1215}{17}$	A1	1.1b
		(5)	
			(8 marks)

Notes:

(a)

B1: Deduces that the gradient of line l_2 is $-\frac{5}{3}$ (accept $-\frac{5}{3}x$)

M1: Complete attempt to find the equation of line l_2 using $B(8,0)$ and a changed gradient.

If using $y = mx + c$ they must be using a changed gradient and proceed as far as $c = \ldots$

A1*: Clear work leading to the given answer of $5x+3y=40$ with no errors seen.

There is a requirement to "show that" so the must be at least one intermediate line between $y-0=-\frac{5}{3}(x-8)$ or finding c (e.g., $y=-\frac{5}{3}x+\frac{40}{3}$) and the answer.

Condone $3y+5x=40$

(a) Alternative

B1: Rearranges $5x+3y=40$ to $y=-\frac{5}{3}x+\ldots$

M1: Complete attempt to show that the equation of line l_2 is perpendicular to l_1 and that it passes through $B(8,0)$. Requires:

- **either** $-\frac{5}{3}$ is the negative reciprocal of $\frac{3}{5}$ **or** shows $-\frac{5}{3}\times\frac{3}{5}=-1$
- evidence that l_2 passes through $(8,0)$, e.g., $5(8)+3(0)=40$ **or** $y=-\frac{5}{3}(8)+\frac{40}{3}=0$

A1*: Clear work showing all elements of $5x+3y=40$ being perpendicular to l_1 and that $(8,0)$ lies on $5x+3y=40$, as above, with no errors seen and a minimal conclusion.

(b)

B1: Deduces $A(-10, 0)$ May be awarded on the diagram as -10 or within a calculation.

M1: For the attempt to solve $y = \frac{3}{5}x + 6$ (or e.g., $5y - 3x = 30$) and $5x + 3y = 40$ simultaneously to find the ***y* coordinate** of their point of intersection.
May be implied, i.e., from a calculator solution which must be correct to 1d.p.
They should be using the given equations but allow slips in rearranging.

A1: *y* coordinate of *C* is $\frac{135}{17}$ (Accept awrt 7.9 for this mark)

dM1: Scored for a complete and correct attempt to find the **exact** area of triangle *ABC*.
There may be numerical slips, e.g., in finding the *x* coordinates of *A*, but, e.g., the *x* and *y* coordinates should not be used the wrong way round.
Do not allow the use of decimals in place of exact values as they cannot meet the demand of the question.
See scheme using just the *y* coordinate of *C*.
Another option is to use Pythagoras' theorem to find *AC* and *BC* lengths using $A(-10, 0)$, $B(8, 0)$ and their $C\left(\frac{55}{17}, \frac{135}{17}\right)$ Note: $AC = \frac{45\sqrt{34}}{17}$ and $BC = \frac{27\sqrt{34}}{17}$

A1: Proceeds correctly to area $ABC = \frac{1215}{17}$

(b) Alternative – you might see the following from Further Maths candidates:
B1M1A1 as above.

dM1: $$\frac{1}{2}\begin{vmatrix} 8 & "\frac{55}{17}" & "-10" & 8 \\ 0 & "\frac{135}{17}" & 0 & 0 \end{vmatrix} = \frac{1}{2}\left(8 \times "\frac{135}{17}" - "-10" \times "\frac{135}{17}"\right)$$

or $$\frac{1}{2}\begin{vmatrix} 8 & 0 & 1 \\ "\frac{55}{17}" & "\frac{135}{17}" & 1 \\ "-10" & 0 & 1 \end{vmatrix} = \frac{1}{2}\left(8 \times "\frac{135}{17}" - "-10" \times "\frac{135}{17}"\right)$$

A1: Proceeds correctly to area $ABC = \frac{1215}{17}$

Question	Scheme	Marks	AOs
11(a)	$h = 2.3 - 1.7e^0$	M1	3.4
	Either 0.6 {m} or 60 cm	A1	1.1b
		(2)	
(b)	$\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$	M1	3.1b
	At $t = 4 \Rightarrow$ Rate of growth is $0.34e^{-0.2\times4} = 0.15277...\{m / year\}$	dM1	3.4
	0.153 {m per year} = 15.3 cm {per year} *	A1*	1.1b
		(3)	
(c)	2.3 (m)	B1	2.2a
		(1)	
			(6 marks)

Notes:

(a)

M1: Substitutes $t = 0$ into $h = 2.3 - 1.7e^{-0.2t}$ Implied by e.g., $h = 2.3 - 1.7e^{-0}$ or $h = 0.6$

A1: Allow 0.6, 0.6 m, or 60 cm and isw after a correct height. Allow $\frac{3}{5}$

The M mark may be implied by A1.

(b)

M1: Links rate of change to gradient and differentiates $h = 2.3 - 1.7e^{-0.2t}$ to $k\,e^{-0.2t}$, $k \neq -1.7$

Accept, e.g., $-0.2 \times -1.7e^{-0.2t}$ Must be seen in (b).

dM1: Substitutes $t = 4$ into $k\,e^{-0.2t}$, $k \neq -1.7$ and calculates its value.

A1*: Fully correct. Requires

- sight of $\left\{\frac{dh}{dt} =\right\} 0.34e^{-0.2t}$ o.e., e.g., $\left\{\frac{dh}{dt} =\right\} \frac{17}{50}e^{-0.2t}$ or $\left\{\frac{dh}{dt} =\right\} -0.2 \times -1.7e^{-0.2t}$
- $\left\{\frac{dh}{dt} =\right\}$ awrt 0.153 {metres per year}
- changing to awrt 15.3 cm {per year}.

Note: Substituting $t = 4$ into $h = 2.3 - 1.7e^{-0.2t}$ gives $h = 1.536...$ scores M0dM0A0 unless differentiation and further correct work is seen separately.

(c)

B1: Allow 2.3, 2.3 m, or 230 cm

$2.2\dot{9}$ and 2.2999… which clearly continues are both acceptable, but 2.29999999 is not.

Question	Scheme	Marks	AOs
12(a)	States or uses $\tan x = \dfrac{\sin x}{\cos x}$	B1	1.2
	$4\sin x = 5\cos^2 x \Rightarrow 4\sin x = 5\left(1-\sin^2 x\right)$	M1	1.1b
	$5\sin^2 x + 4\sin x - 5 = 0$ *	A1*	2.1
		(3)	
(b)	Attempts to solve $5\sin^2 x + 4\sin x - 5 = 0 \Rightarrow \sin x = ...$	M1	1.1b
	$\sin x = \dfrac{-2 \pm \sqrt{29}}{5}$ $\left(\sin x = \text{awrt } 0.677\right)$	A1	1.1b
	Takes $\sin^{-1}$ leading to at least one answer in the range	dM1	1.1b
	$x = \text{awrt } 42.6\{°\}$ and $x = \text{awrt } 137.4\{°\}$ only	A1	1.1b
		(4)	
(c)	$15 \times "2" = 30$ following through on their "2"	B1ft	2.2a
	Explains either "mathematically" by stating $3 \times 5 \times$ their number in range 0 to 360° or 'in words" e.g., stating 3 ×"2" values every 360° and 5 lots of 360°	B1ft	2.4
		(2)	
		(9 marks)	

Notes:

(a) **Allow use of e.g. θ but the final mark requires the equation to be in terms of x**

B1: States or uses $\tan x = \dfrac{\sin x}{\cos x}$ e.g., $4\tan x = 5\cos x \Rightarrow 4\dfrac{\sin x}{\cos x} = 5\cos x$ Allow e.g. $\tan x = \dfrac{\sin\theta}{\cos\theta}$

M1: Multiplies by $\cos x$ and uses $\cos^2 x = 1 - \sin^2 x$ to set up a quadratic equation in just $\sin x$
Condone mixed arguments here.

A1*: Proceeds to $5\sin^2 x + 4\sin x - 5 = 0$ with correct notation and algebra, showing all key steps.
The = 0 must be present in the final answer line.
Condone a single slip in notation, e.g., $\sin x^2$ or $\sin\theta$ seen once.

(b)

M1: Attempts to solve $5\sin^2 x + 4\sin x - 5 = 0 \Rightarrow$ **$\sin x = ...$** using the usual rules.
$\sin x =$ may be implied later.
Allow solution(s) from a calculator but one must be correct (0.6 or 0.7 or –1.4 or –1.5)

A1: Achieves $\sin x = \dfrac{-4 \pm \sqrt{116}}{10}$ $\left(\sin x = \text{awrt } 0.677\right)$ $\sin x =$ may be implied later.

dM1: Finds one value of x in the range 0 to 360° from their $\sin x =$
May be scored for working in radians. If using $\sin x =$ 0.677 they should have awrt 0.744 or awrt 2.40
If they have made a slip in solving the quadratic, e.g., by the formula, then their values will need checking both in degrees and radians to see if this mark can be implied.

A1:	$x =$ awrt $42.6\{°\}$ and $x =$ awrt $137.4\{°\}$ only. Ignore any values outside of 0 to 360° isw if they round their values to e.g., 3sf after stating acceptable answers. There must be some evidence that the quadratic has been solved.
(c)	
B1ft:	Follow through on 15 multiplied by the number of solutions in (b) in the range 0 to 360° If working in radians in (b), they must state 30 (solutions).
B1ft:	Explains either mathematically **or** in words. See scheme. Note that you might see arguments expanding the range from 1800 to 5400 to account for the stretch parallel to the x axis. $\frac{5400}{360} = 15$ and $15 \times 2 = 30$ which is also acceptable.
Note:	If candidates list 30 values and conclude that there are 30 solutions, score B1ftB1ft There is no need to check their 30 values are correct, but there must be 30.

Question	Scheme	Marks	AOs
13 (a)	$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(-8\mathbf{i}+9\mathbf{j})-(10\mathbf{i}-3\mathbf{j})$	M1	1.1b
	$=-18\mathbf{i}+12\mathbf{j}$	A1	1.1b
		(2)	
(b)	$\lvert\overrightarrow{AB}\rvert=\sqrt{"18"^2+"12"^2}\ \{=\sqrt{468}\}$	M1	1.1b
	$=6\sqrt{13}$	A1	1.1b
		(2)	
(c)	For the key step in using the fact that *BCA* forms a straight line in an attempt to find "*p*" $\overrightarrow{AB}=\lambda\overrightarrow{BC}\Rightarrow-18\mathbf{i}+12\mathbf{j}=6\lambda\mathbf{i}+\lambda(p-9)\mathbf{j}$ with components equated leading to a value for λ and to $p=\ldots$	M1	2.1
	(i) $p=5$	A1	1.1b
	(ii) ratio = 2: 3	B1 (A1 on EPEN)	2.2a
		(3)	
		(7 marks)	

Notes:

(a) **Must be seen in (a)**

M1: Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component.
Allow as coordinates for this mark. Condone missing brackets, e.g., $-8\mathbf{i}+9\mathbf{j}-10\mathbf{i}-3\mathbf{j}$

A1: cao $-18\mathbf{i}+12\mathbf{j}$ o.e. $\begin{pmatrix}-18\\12\end{pmatrix}$ Condone $\begin{matrix}-18\\12\end{matrix}$

Do not allow $\begin{pmatrix}-18\mathbf{i}\\12\mathbf{j}\end{pmatrix}$ or $(-18,\ 12)$ or $\left(\dfrac{-18}{12}\right)$ for the A1.

(b)

M1: Attempts to use Pythagoras' theorem on their vector from part (a). Allow restarts.
$\lvert\overrightarrow{AB}\rvert=\sqrt{"18"^2+"12"^2}\ \{=\sqrt{468}\}$ Note that -18 will commonly be squared as 18
May be implied by awrt 21.6 This will need checking if (a) is incorrect.

A1: cao $6\sqrt{13}$ May come from $\begin{pmatrix}\pm18\\\pm12\end{pmatrix}$

(c)

M1: For the key step in using the fact that *BCA* forms a straight line in an attempt to find "*p*"
Condone sign slips. Award, for example, for $\pm\dfrac{p-9}{6}=\pm\dfrac{2}{3}$ leading to $p=\ldots$
It is implied by $p=5$ unless it comes directly from work that is clearly incorrect.

e.g., award for an attempt to use

- $\overrightarrow{AB} = \alpha\overrightarrow{AC} \Rightarrow -18\mathbf{i} + 12\mathbf{j} = -12\alpha\mathbf{i} + \alpha(p+3)\mathbf{j}$ with components equated leading to a value for α and to $p = \ldots$
- gradient BC = gradient BA = $-\frac{2}{3}$ e.g., $\frac{p-9}{6} = \frac{9--3}{-8-10}$ leading to $p = \ldots$
- triangles BCM and BAN are similar with lengths in a ratio 1:3. e.g., $p = 9 - \frac{1}{3}\times 12$ **or** $p = -3 + \frac{2}{3}\times 12$
- attempt to find the equation of line AB using both points (FYI line AB has equation $y = -\frac{2}{3}x + \frac{11}{3}$) and then sub in $x = -2$ leading to $p = \ldots$
- $\frac{p+3}{12} = \frac{2}{3}$ **or** $\frac{p+3}{2} = 9 - p$ leading to $p = \ldots$

A1: $p = 5$ Correct answer implies both marks, unless it comes directly from work that is clearly incorrect.

B1: States ratio = 2: 3 or equivalent such as 1: 1.5 or 22:33
Note that 3:2 is incorrect but condone {Area}AOB : {Area}AOC = 3: 2
This might follow incorrect work or even incorrect p
For reference, area AOC = 22, area AOB = 33 and area BOC = 11

Question	Scheme	Marks	AOs
14	Attempts the term in x^3 or the term in x^5 of $\left(3-\frac{1}{2}x\right)^6$ Look for ${}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ or ${}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$	M1	3.1a
	Correct term in x^3 or correct term in x^5 of $\left(3-\frac{1}{2}x\right)^6$ $-\frac{135}{2}x^3$ **or** $-\frac{9}{16}x^5$	A1	1.1b
	Attempts one of the required terms in x^5 of $\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$ Either $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ **or** $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$	M1	1.1b
	Attempts the sum of $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ and $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$	dM1	2.1
	Coefficient of x^5 $=-\frac{45}{16}-540=-\frac{8685}{16}$	A1	1.1b
		(5)	
			(5 marks)

Notes:

M1: For the key step in attempting to find one of the required terms in the expansion of $\left(3-\frac{1}{2}x\right)^6$ to enable the problem to be solved.

Look for ${}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ **or** ${}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ but condone missing brackets and slips in signs.

May be part of a complete expansion but only one of the required terms needs to be of the correct form.

A1: For $-\frac{135}{2}\left\{x^3\right\}$ **or** $-\frac{9}{16}\left\{x^5\right\}$ which may be unsimplified but the ${}^6C_3$ or ${}^6C_5$ must be processed. May be implied by $-540\left\{x^5\right\}$ **or** $-\frac{45}{16}\left\{x^5\right\}$

M1: Attempts one of the required terms in x^5 of the expansion of $\left(5+8x^2\right)\left(3-\frac{1}{2}x\right)^6$

Look for $5\times{}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ **or** $8x^2\times{}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$ which would also imply the previous M.

The x^5 may be missing as just the coefficient is required.

May be implied by $-540\left\{x^5\right\}$ **or** $-\frac{45}{16}\left\{x^5\right\}$

Condone missing brackets and signs.

You might see candidates make a slip in, e.g., their binomial coefficients, but have an (essentially) correct method to solve the problem.

Note that this M mark is not dependent on the first, so you may be able to award it even if they have made a slip in finding their x^3 or x^5 term in the expansion.

dM1: Attempts the sum of $5\times {}^6C_5 3^1\left(-\frac{1}{2}x\right)^5$ and $8x^2\times {}^6C_3 3^3\left(-\frac{1}{2}x\right)^3$

Dependent on the previous M but may be scored at the same time.

The x^5 may be missing as just the coefficients are required.
Condone missing brackets and signs.

A1: $-\frac{8685}{16}$ or exact equivalent, -542.8125 and apply isw

Condone $-\frac{8685}{16}x^5$ for A1

Note that rounded decimals, e.g., -542.81 will not score the last mark.

Note that full marks can be scored for concise solutions such as:

$$5\times {}^6C_5\times 3\times\left(-\frac{1}{2}\right)^5 \quad + \quad 8\times {}^6C_3\times 3^3\times\left(-\frac{1}{2}\right)^3 \quad = -\frac{8685}{16}$$

Alternative
Attempts via the taking out of the common factor can be scored in the same way.

$$\left(3-\frac{1}{2}x\right)^6 = 3^6\left\{1+6\times\left(-\frac{1}{6}x\right)^1+\frac{6\times5}{2}\left(-\frac{1}{6}x\right)^2+\frac{6\times5\times4}{3!}\left(-\frac{1}{6}x\right)^3+\frac{6\times5\times4\times3}{4!}\left(-\frac{1}{6}x\right)^4+\frac{6\times5\times4\times3\times2}{5!}\left(-\frac{1}{6}x\right)^5+\left(-\frac{1}{6}x\right)^6\right\}$$

For M1 A1 look for $3^6\times\frac{6\times5\times4}{3!}\left(-\frac{1}{6}x\right)^3$ **or** $3^6\times\frac{6\times5\times4\times3\times2}{5!}\left(-\frac{1}{6}x\right)^5$

Score the remaining marks as per the main scheme.

Question	Scheme	Marks	AOs
15 (a)	Attempts both $y = 8 - 10\times1 + 6\times1^2 - 1^3$ and $y = 1^2 - 12\times1 + 14$	M1	1.1b
	Achieves $y = 3$ for both equations and gives a minimal conclusion / statement, e.g., $(1, 3)$ lies on both curves so they intersect at $x = 1$	A1	1.1b
		(2)	
(b)	(Curves intersect when) $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ $\Rightarrow x^3 - 5x^2 - 2x + 6 = 0$	M1	1.1b
	For the key step in dividing by $(x-1)$ $x^3 - 5x^2 - 2x + 6 = (x-1)(x^2 + px \pm 6)$	dM1	3.1a
	$x^3 - 5x^2 - 2x + 6 = (x-1)(x^2 - 4x - 6)$	A1	1.1b
	Solves $x^2 - 4x - 6 = 0$ $(x-2)^2 = 10 \Rightarrow x = \ldots$	ddM1	1.1b
	$x = 2 - \sqrt{10}$ only	A1	1.1b
		(5)	
			(7 marks)

Notes:

(a) **Must be seen in (a)**

M1: As scheme.
For M1 A0, allow a statement that $(1,3)$ lies on both curves without sight of the calculation.
Amongst various alternatives are:

- Setting $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ and attempting to rearrange to $x^3 - 5x^2 - 2x + 6 = 0$ before substituting in $x = 1$
- Setting $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ and attempting to divide $x^3 - 5x^2 - 2x + 6$ by $(x-1)$ either by long division or inspection

A1: For the complete mathematical argument.
Requires both correct calculations with a minimal conclusion, which may be as a preamble.
e.g., in the alternatives

- as $1^3 - 5\times1^2 - 2\times1 + 6 = 0$, hence curves meet when $x = 1$
- $x^3 - 5x^2 - 2x + 6 = (x-1)(x^2 - 4x - 6)$ so the curves intersect when $x = 1$

(b) **Allow the use of *x* or *k* throughout this part.**

M1: Sets $x^2 - 12x + 14 = 8 - 10x + 6x^2 - x^3$ and proceeds to a cubic equation set = 0
Must be seen or used in (b)

dM1: For the key step in realising that $(x-1)$ is a factor of the cubic.
It is for dividing by $(x-1)$ to get the quadratic factor.

For division look for their first two terms, i.e., $x^2 \pm 4x$

(This will need checking if they have made an error in rearranging the cubic.)

$$\begin{array}{r} x^2 \pm 4x\ldots\ldots\ldots\ldots \\ x-1\overline{)\,x^3-5x^2-2x+6} \\ \underline{x^3-1x^2} \\ -4x^2 \end{array}$$

By inspection look for the first and last term $x^3-5x^2-2x+6=(x-1)(x^2+px\pm 6)$

A1: $x^3-5x^2-2x+6=(x-1)(x^2-4x-6)$ or just x^2-4x-6 or k^2-4k-6 as their quadratic factor following algebraic division.

ddM1: Attempts to solve their $x^2-4x-6=0$, which must be a 3TQ, by completing the square or the quadratic formula, leading to an exact solution. Their quadratic factor must **not** factorise. Their quadratic "factor" may come from algebraic division that has a remainder but we will still allow them to score this mark.

If using the quadratic formula, they need to have, e.g., $\dfrac{4-\sqrt{4^2-4(-6)}}{2}$

or $\dfrac{4-\sqrt{40}}{2}$ as a minimum (i.e., they must not jump straight to $2-\sqrt{10}$ from a calculator).

A1: $k=2-\sqrt{10}$ or exact equivalent but allow the use of x e.g., $x=\dfrac{4-\sqrt{40}}{2}$

If using the quadratic formula, the discriminant must be processed.
Must come from a correct quadratic factor.
They must have discarded $2+\sqrt{10}$ if seen.

Question	Scheme	Marks	AOs
16	Sets $f'(4) = 0 \Rightarrow 16 + 2a + b = 0$	M1	2.1
	Integrates $f'(x) = 4x + a\sqrt{x} + b \Rightarrow \{f(x) =\} 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx \ \{+c\}$	M1 A1ft	1.1b 1.1b
	Deduces that $c = -5$	B1	2.2a
	Full and complete method using the given information $f'(4) = 0$ and $f(4) = 3$ in order to find values for a and b Note: $a = -15$ and $b = 14$	ddM1	3.1a
	$\{f(x) =\} 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$	A1	1.1b
		(6)	
		(6 marks)	

Notes:

M1: For the key step in setting $f'(4) = 0 \Rightarrow 16 + 2a + b = 0$ to set up an equation in a and b. Condone slips.

M1: For attempting to integrate $f'(x)$. Award for $x^n \to x^{n+1}$ or $b \to bx$
This may come after finding values for a or b or both.

A1ft: $\{f(x) =\} 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx \ \{+c\}$ or, e.g., $\{f(x) =\} 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + (-16 - 2a)x \ \{+c\}$
Allow ft on their b in terms of a if they substituted in from their $f'(4) = 0 \Rightarrow 16 + 2a + b = 0$
Do not ft if they have a value(s) for a or b
This may be left unsimplified but the indices must be processed.
isw once the mark is awarded. Condone the omission of the $+ c$
This accuracy mark requires only the previous M mark to be scored.

B1: Deduces that the constant term in $f(x)$ is -5.
Note that deducing $b = -5$ is B0. It must be the constant in a changed function.

ddM1: For a complete strategy to find values for both a and b.
Do not be concerned about the logistics of how they solve the simultaneous equations – this may be done on a calculator.
Note: $a = -15$ and $b = 14$
This is dependent on **both** previous method marks and so must include use of both

- $f'(4) = 0$ (their $16 + 2a + b = 0$ o.e.)
- $f(4) = 3$ (their $32 + \frac{16}{3}a + 4b - 5 = 3$ o.e.)

A1: $\{f(x) =\} 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$ or exact simplified equivalent, e.g., use of $x\sqrt{x}$ in place of $x^{\frac{3}{2}}$
Apply isw once a correct expression is seen.

Question	Scheme	Marks	AOs
17 (a)	Provides a counter example with a reason. e.g., $6^3 - 1^3 = 215$ which is a multiple of 5	B1	2.4
		(1)	
(b)	States or uses, e.g., $2n$ and $2n+2$ **or** $2n+2$ and $2n+4$	M1	2.1
	Attempts $(2n+2)^3 - (2n)^3 = 8n^3 + 24n^2 + 24n + 8 - 8n^3$ leading to a quadratic.	dM1	1.1b
	$= 24n^2 + 24n + 8$	A1	1.1b
	$24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ So $q^3 - p^3$ is a multiple of 8	A1	2.1
		(4)	
		(5 marks)	

Notes:

(a)

B1: Provides a counter example with a reason. There is no need to state "not true".
e.g., $7^3 - 2^3 = 335$ which divides by 5 {exactly}.
It is sufficient to have, e.g., $9^3 - 4^3 = 665$ and $\frac{665}{5} = 133$
Here q must be greater than p and both must be natural numbers, not 0 or negatives.
Note that any pair of positive integers n and $n+5k$ will provide a counter example, but $q^3 - p^3$ must be evaluated correctly, and if they divide by 5 this also needs to be correct.

(b)

M1: For the key step in stating the algebraic form of consecutive even numbers.
See main scheme for examples. They might be used either way round for this mark.

dM1: Attempts $(2n+2)^3 - (2n)^3 = ...$ condoning slips but must lead to a quadratic.
Alternatively, $(2n+2)^3 - (2n)^3 = 2^3\{(n+1)^3 - n^3\}$
May be subtracted the wrong way round for this mark as below.
$(2n)^3 - (2n+2)^3 = ...$ but this will score M1dM1A0A0

A1: e.g., $(2n+2)^3 - (2n)^3 = 24n^2 + 24n + 8$ **or** $(2n+4)^3 - (2n+2)^3 = 24n^2 + 72n + 56$
or $(2n+2)^3 - (2n)^3 = 8\{(n+1)^3 - n^3\}$ **or** $(2n)^3 - (2n-2)^3 = 24n^2 - 24n + 8$ etc.
Must come from correct work and the algebra will need checking carefully.

A1: For a full and rigorous proof showing all necessary steps including:

- correct quadratic expression for $q^3 - p^3$ for their even numbers, e.g., $24n^2 + 24n + 8$
- reason e.g., $24n^2 + 24n + 8 = 8(3n^2 + 3n + 1)$ **or**, e.g., in $24n^2 + 24n + 8$ the coefficients are all multiples of 8
- minimal conclusion, "hence true"

Alt 1:
If the even numbers are set as n and $n + 2$ there must be sufficient work seen before marks can be awarded.
e.g.,

M1dM1: $n = 2k \Rightarrow (n+2)^3 - n^3 = ...n^2 + ...n + ... = ...(2k)^2 + ...(2k) + ...$

A1: $= 24k^2 + 24k + 8$

A1: $= 8(3k^2 + 3k + 1)$ so $q^3 - p^3$ is a multiple of 8

Alt 2:
If they just use any two even numbers, e.g., $2a$ and $2b$, **or** $2m$ and $2n + 2$ then they will score as follows:

M1: $(2a)^3 - (2b)^3$ Condone missing brackets if recovered.

dM1: $= ...a^3 - ...b^3$

A1: $= 8a^3 - 8b^3$ Note $8(a^3 - b^3)$ would imply this mark.

A1: $= 8(a^3 - b^3)$ so $q^3 - p^3$ is a multiple of 8 if q and p are {any two} even {numbers}

and hence $q^3 - p^3$ is a multiple of 8 if q and p are *consecutive* even numbers

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