OCR
Oxford Cambridge and RSA

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Wednesday 23 May 2018 – Morning

Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, Mean of X is np, Variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$s = \tfrac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \tfrac{1}{2}at^2$$

Section A: Pure Mathematics
Answer **all** the questions

1 In triangle ABC, $AB = 20\,\text{cm}$ and angle $B = 45°$.

(i) Given that $AC = 16\,\text{cm}$, find the two possible values for angle C, correct to 1 decimal place. **[4]**

(ii) Given instead that the area of the triangle is $75\sqrt{2}\,\text{cm}^2$, find BC. **[2]**

2 **(i)** The curve $y = \frac{2}{3+x}$ is translated by four units in the positive x-direction. State the equation of the curve after it has been translated. **[2]**

(ii) Describe fully the single transformation that transforms the curve $y = \frac{2}{3+x}$ to $y = \frac{5}{3+x}$. **[2]**

3 In each of the following cases choose one of the statements

$$P \Rightarrow Q \qquad P \Leftarrow Q \qquad P \Leftrightarrow Q$$

to describe the relationship between P and Q.

(i) P: $y = 3x^5 - 4x^2 + 12x$
Q: $\frac{\mathrm{d}y}{\mathrm{d}x} = 15x^4 - 8x + 12$ **[1]**

(ii) P: $x^5 - 32 = 0$ where x is real
Q: $x = 2$ **[1]**

(iii) P: $\ln y < 0$
Q: $y < 1$ **[1]**

4 **(i)** Express $4x^2 - 12x + 11$ in the form $a(x+b)^2 + c$. **[3]**

(ii) State the number of real roots of the equation $4x^2 - 12x + 11 = 0$. **[1]**

(iii) Explain fully how the value of r is related to the number of real roots of the equation $p(x+q)^2 + r = 0$ where p, q and r are real constants and $p > 0$. **[2]**

5 **In this question you must show detailed reasoning.**

The line $x + 5y = k$ is a tangent to the curve $x^2 - 4y = 10$. Find the value of the constant k. **[5]**

6 A pan of water is heated until it reaches 100 °C. Once the water reaches 100 °C, the heat is switched off and the temperature T °C of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + a\mathrm{e}^{-kt},$$

where t denotes the time, in minutes, after the heat is switched off and a and k are positive constants.

(i) Write down the value of a. **[1]**

(ii) Explain what the value of 25 represents in the equation $T = 25 + a\mathrm{e}^{-kt}$. **[1]**

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15 °C per minute.

(iii) Calculate the value of k. **[3]**

(iv) Find the time taken for the temperature of the water to drop from 100 °C to 45 °C. **[3]**

(v) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than 100 °C. Suggest how the equation for the temperature as the water cools would be modified by this. **[1]**

7 **(i)** Show that the equation

$$2\sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0.$$ **[3]**

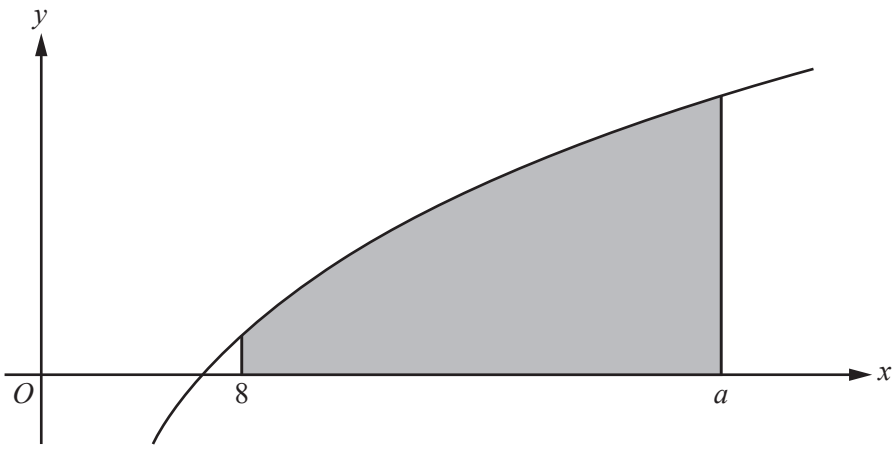
(ii) Hence solve the equation

$$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of θ between 0° and 180°, correct to 1 decimal place. **[5]**

8 In this question you must show detailed reasoning.

The diagram shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x-axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a. **[9]**

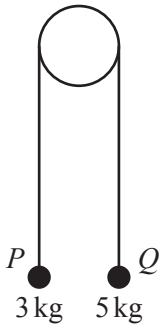
Section B: Mechanics
Answer **all** the questions

9 In this question the horizontal unit vectors **i** and **j** are in the directions east and north respectively.

A model ship of mass 2 kg is moving so that its acceleration vector $\mathbf{a}\,\text{m}\,\text{s}^{-2}$ at time t seconds is given by $\mathbf{a} = 3(2t-5)\mathbf{i}+4\mathbf{j}$. When $t = T$, the magnitude of the horizontal force acting on the ship is 10 N.

Find the possible values of T. **[4]**

10 Particles P and Q, of masses 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and P and Q are above a horizontal plane (see diagram).

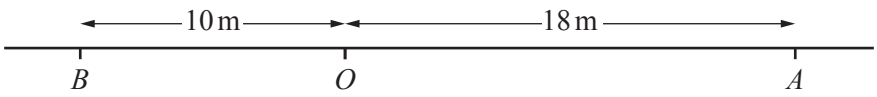


(i) Find the tension in the string immediately after the particles are released. **[4]**

After descending 2.5 m, Q strikes the plane and is immediately brought to rest. It is given that P does not reach the pulley in the subsequent motion.

(ii) Find the distance travelled by P between the instant when Q strikes the plane and the instant when the string becomes taut again. **[4]**

11



A particle P is moving along a straight line with constant acceleration. Initially the particle is at O. After 9 s, P is at a point A, where $OA = 18$ m (see diagram) and the velocity of P at A is $8\,\text{m}\,\text{s}^{-1}$ in the direction $\overrightarrow{OA}$.

(i) **(a)** Show that the initial speed of P is $4\,\text{m}\,\text{s}^{-1}$. **[2]**

(b) Find the acceleration of P. **[2]**

B is a point on the line such that $OB = 10$ m, as shown in the diagram.

(ii) Show that P is never at point B. **[4]**

A second particle Q moves along the same straight line, but has variable acceleration. Initially Q is at O, and the displacement of Q from O at time t seconds is given by

$$x = at^3 + bt^2 + ct,$$

where a, b and c are constants.

It is given that
- the velocity and acceleration of Q at the point O are the same as those of P at O,
- Q reaches the point A when $t = 6$.

(iii) Find the velocity of Q at A. **[5]**

END OF QUESTION PAPER