4.

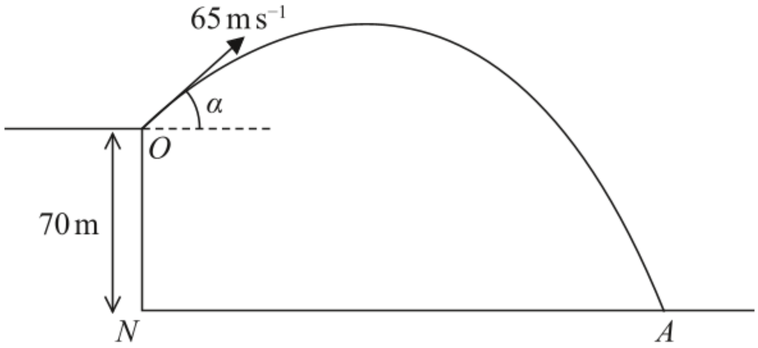

Figure 3

A small stone is projected with speed $65\,\text{m}\,\text{s}^{-1}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \dfrac{5}{12}$

The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\text{m}\,\text{s}^{-2}$

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

(5)

One limitation of the model is that it ignores air resistance.

(c) State one other limitation of the model that could affect the reliability of your answers.

(1)

10.

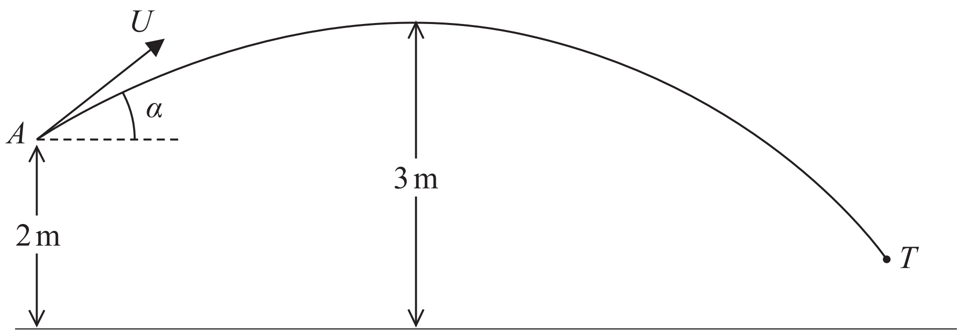

Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A, the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground.
The target is modelled as being the point T, as shown in Figure 4.
The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \dfrac{2g}{\sin^2 \alpha}$.

(2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α

(9)

(c) State one limitation of the model that could affect your answer to part (b).

(1)

(d) Find the time taken for the ball to travel from A to T.

(3)

5.

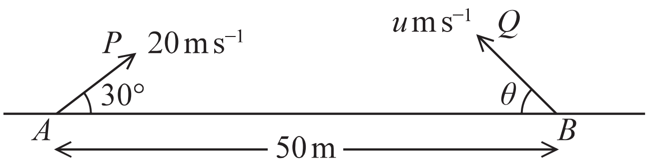

Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed $20\,\mathrm{m\,s^{-1}}$ at 30° to AB.

Ball Q is projected from B with speed $u\,\mathrm{m\,s^{-1}}$ at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

5.

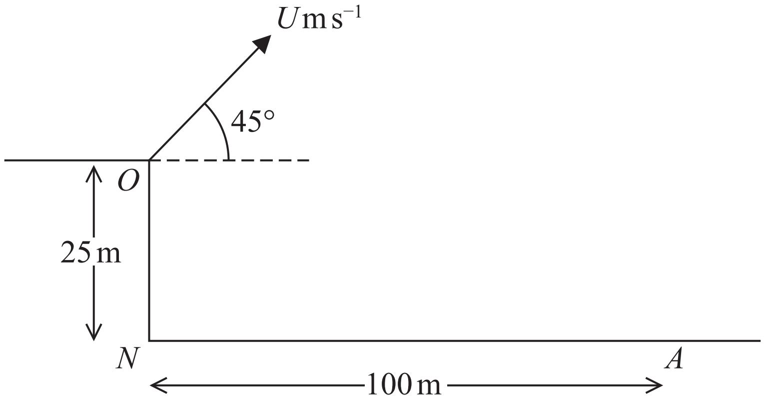

Figure 2

A small ball is projected with speed U m s^{-1} from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A, where $AN = 100$ m, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$

(6)

(b) find the greatest height of the ball above the horizontal ground NA.

(3)

In a refinement to the model of the motion of the ball from O to A, the effect of air resistance is included.

This refined model is used to find a new value of U.

(c) How would this new value of U compare with 28, the value given in part (a)?

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

10.

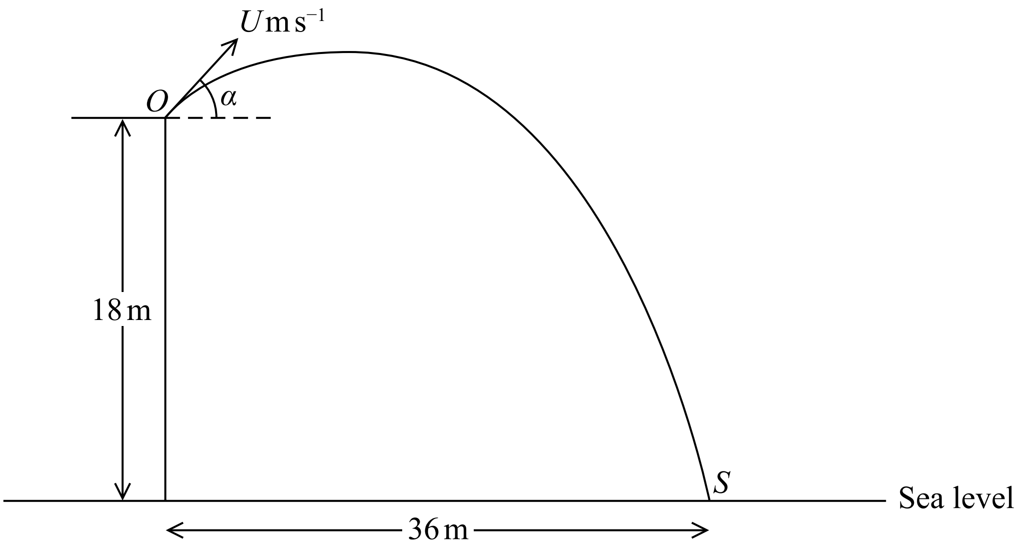

Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

(5)

(c) Suggest two improvements that could be made to the model.

(2)

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7.

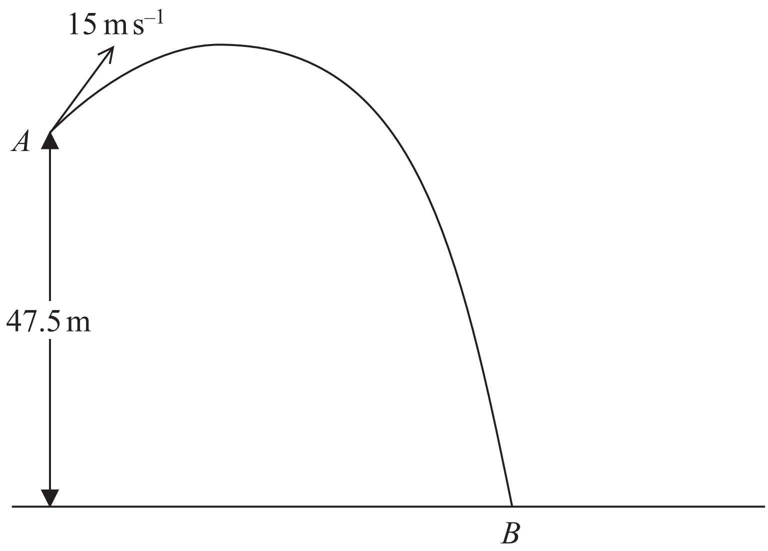

Figure 3

A small ball P is projected with speed $15\,\text{m}\,\text{s}^{-1}$ from a point A which is 47.5 m above a horizontal beach. The ball moves freely under gravity and hits the beach at the point B, as shown in Figure 3.

(a) By considering energy, find the speed of P immediately before it hits the beach.

(4)

The ball was projected from A at an angle θ above the horizontal, where $\sin\theta = \frac{3}{5}$

(b) Find the greatest height above the beach of P as it moved from A to B.

(3)

(c) Find the least speed of P as it moved between A and B.

(1)

(d) Find the horizontal distance from A to B.

(6)

Leave blank

7. A particle is projected from a point O with speed U at an angle of elevation α to the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2(1+\tan^2\alpha)}{2U^2}$$

(7)

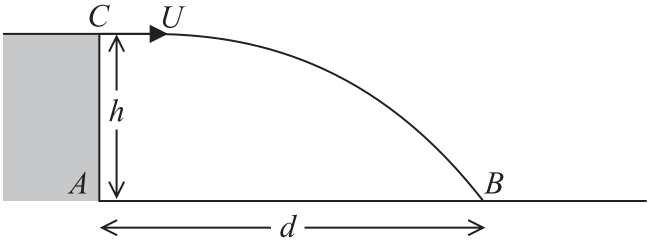

Figure 3

A small stone is projected horizontally with speed U from a point C at the top of a vertical cliff AC so as to hit a fixed target B on the horizontal ground. The point C is a height h above the ground, as shown in Figure 3. The time of flight of the stone from C to B is T, and the stone is modelled as a particle moving freely under gravity.

(b) Find, in terms of U, g and T, the speed of the stone as it hits the target at B.

(4)

It is found that, using the same initial speed U, the target can also be hit by projecting the stone from C at an angle α above the horizontal. The stone is again modelled as a particle moving freely under gravity and the distance $AB = d$.

(c) Using the result in part (a), or otherwise, show that

$$d = \frac{1}{2}gT^2\tan\alpha$$

(6)

Leave blank

8. At time $t = 0$ seconds, a golf ball is hit from a point O on horizontal ground. The horizontal and vertical components of the initial velocity of the ball are $3U \text{ m s}^{-1}$ and $U \text{ m s}^{-1}$ respectively. The ball hits the ground at the point A, where $OA = 120$ m. The ball is modelled as a particle moving freely under gravity.

(a) Show that $U = 14$

(5)

(b) Find the speed of the ball immediately before it hits the ground at A.

(2)

(c) Find the values of t when the ball is moving at an angle α to the horizontal, where $\tan \alpha = \frac{1}{4}$.

(6)

Leave blank

5. A small ball is projected vertically upwards from a point O with speed $14.7\,\text{m}\,\text{s}^{-1}$. The point O is 2.5 m above the ground. The motion of the ball is modelled as that of a particle moving freely under gravity.

Find

(a) the maximum height above the ground reached by the ball,

(4)

(b) the time taken for the ball to first reach a height of 1 m above the ground,

(4)

(c) the speed of the ball at the instant before it strikes the ground for the first time.

(3)

Leave blank

4. A small stone is released from rest from a point A which is at height h metres above horizontal ground. Exactly one second later another small stone is projected with speed 19.6 m s^{-1} vertically downwards from a point B, which is also at height h metres above the horizontal ground. The motion of each stone is modelled as that of a particle moving freely under gravity. The two stones hit the ground at the same time.

Find the value of h.

(7)

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6.

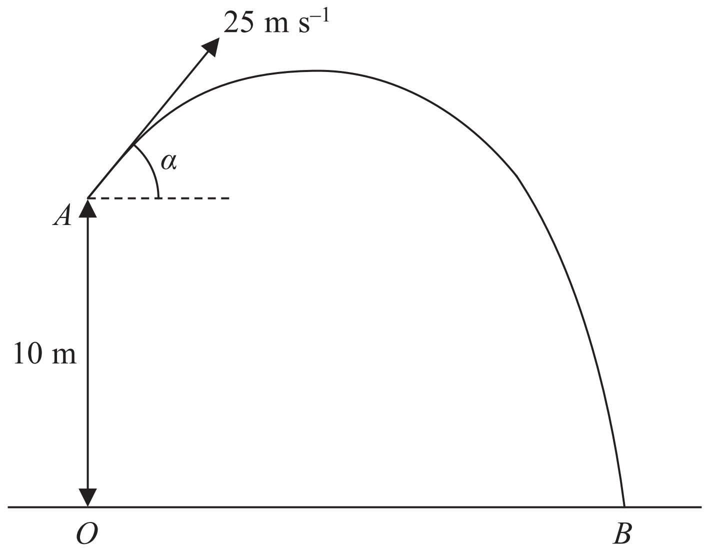

Figure 4

A particle P is projected from a point A with speed 25 m s^{-1} at an angle of elevation α, where $\sin \alpha = \frac{4}{5}$. The point A is 10 m vertically above the point O which is on horizontal ground, as shown in Figure 4. The particle P moves freely under gravity and reaches the ground at the point B.

Calculate

(a) the greatest height above the ground of P, as it moves from A to B,

(3)

(b) the distance OB.

(6)

The point C lies on the path of P. The direction of motion of P at C is perpendicular to the direction of motion of P at A.

(c) Find the time taken by P to move from A to C.

(4)

Leave blank

6.

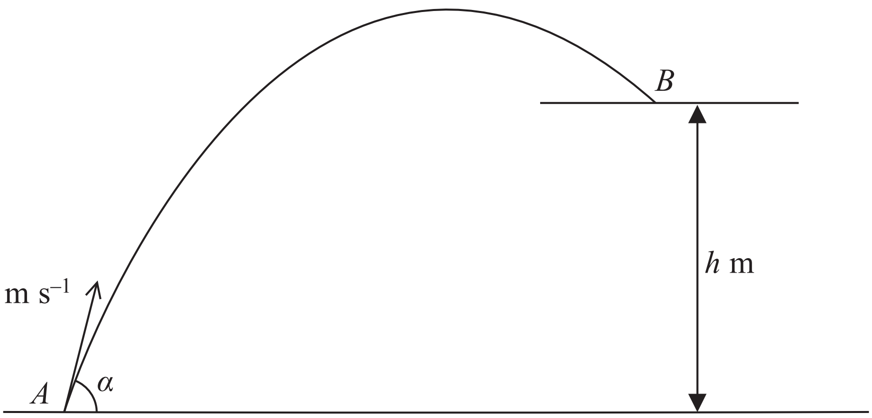

Figure 2

A small ball is projected with speed 14 m s^{-1} from a point *A* on horizontal ground. The angle of projection is α above the horizontal. A horizontal platform is at height *h* metres above the ground. The ball moves freely under gravity until it hits the platform at the point *B*, as shown in Figure 2. The speed of the ball immediately before it hits the platform at *B* is 10 m s^{-1}.

(a) Find the value of *h*.

(4)

Given that $\sin \alpha = 0.85$,

(b) find the horizontal distance from *A* to *B*.

(8)

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6.

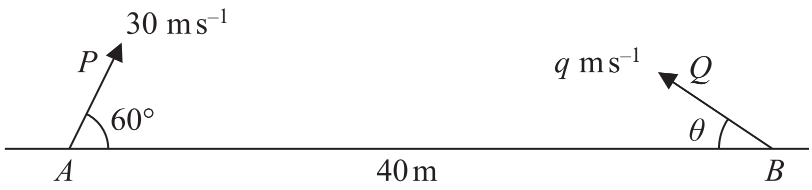

Figure 4

The points A and B lie 40 m apart on horizontal ground. At time $t = 0$ the particles P and Q are projected in the vertical plane containing AB and move freely under gravity. Particle P is projected from A with speed 30 m s^{-1} at 60° to AB and particle Q is projected from B with speed q m s^{-1} at angle θ to BA, as shown in Figure 4.

At $t = 2$ seconds, P and Q collide.

(a) Find

(i) the size of angle θ,

(ii) the value of q.

(6)

(b) Find the speed of P at the instant before it collides with Q.

(5)

Leave blank

7.

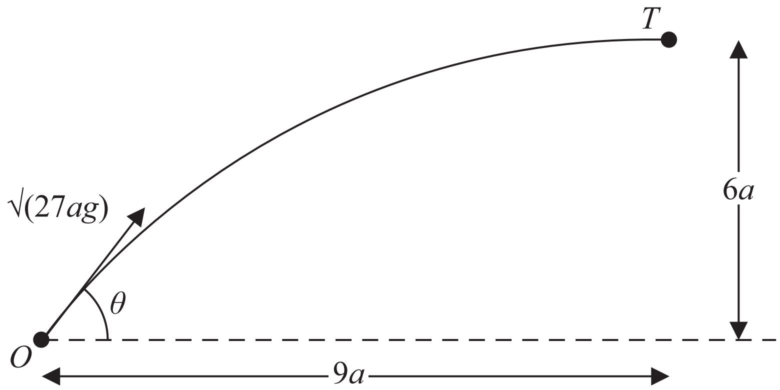

Figure 4

A small ball is projected from a fixed point O so as to hit a target T which is at a horizontal distance $9a$ from O and at a height $6a$ above the level of O. The ball is projected with speed $\sqrt{(27ag)}$ at an angle θ to the horizontal, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

(a) Show that $\tan^2\theta - 6\tan\theta + 5 = 0$

(7)

The two possible angles of projection are θ_1 and θ_2, where $\theta_1 > \theta_2$.

(b) Find $\tan\theta_1$ and $\tan\theta_2$.

(3)

The particle is projected at the larger angle θ_1.

(c) Show that the time of flight from O to T is $\sqrt{\left(\frac{78a}{g}\right)}$.

(3)

(d) Find the speed of the particle immediately before it hits T.

(3)

Leave blank

7.

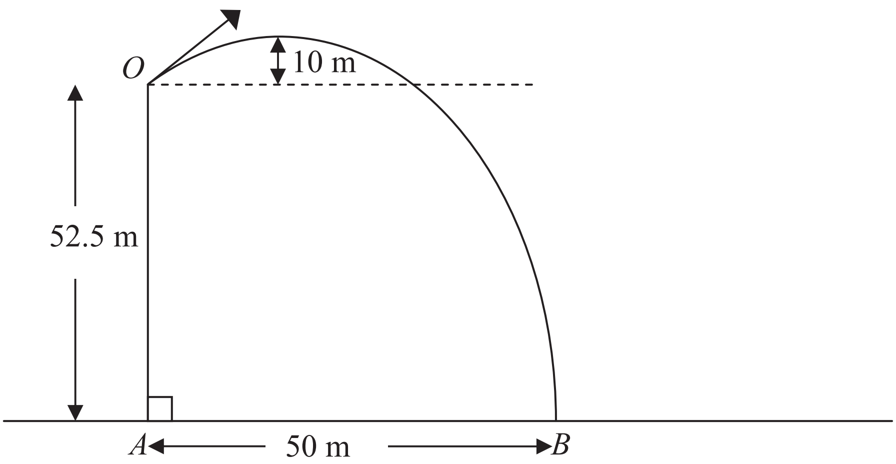

Figure 4

A small stone is projected from a point O at the top of a vertical cliff OA. The point O is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of O before hitting the sea at the point B, where AB = 50 m, as shown in Figure 4. The stone is modelled as a particle moving freely under gravity.

(a) Show that the vertical component of the velocity of projection of the stone is 14 m s^{-1}.
(3)

(b) Find the speed of projection.
(9)

(c) Find the time after projection when the stone is moving parallel to OB.
(5)