Question	Scheme	Marks	AOs
9	$a=\left(\frac{3}{4}\right)^2$ or $a=\frac{9}{16}$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{\frac{9}{16}}{1-\left(-\frac{3}{4}\right)}=...$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
		(3)	
	Alternative 1:		
	$\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\frac{-\frac{3}{4}}{1-\left(-\frac{3}{4}\right)}=...$ or $r=-\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=-\frac{3}{7}-\left(-\frac{3}{4}\right)$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...$ $\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+...=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ or $-\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^5-...=-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=\left(\frac{3}{4}\right)^2\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)-\left(\frac{3}{4}\right)^3\left(\frac{1}{1-\left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
	Alternative 3:		
	$\sum_{n=2}^{\infty}\left(\frac{3}{4}\right)^n\cos(180n)^\circ=S=\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^3+\left(\frac{3}{4}\right)^4-...$	B1	2.2a
	$=\left(\frac{3}{4}\right)^2\left(1-\left(\frac{3}{4}\right)+\left(\frac{3}{4}\right)^2-...\right)=\left(\frac{3}{4}\right)^2\left(\frac{1}{4}+S\right)\Rightarrow\frac{7}{16}S=\frac{9}{64}\Rightarrow S=...$	M1	3.1a
	$=\frac{9}{28}$*	A1*	1.1b
			(3 marks)
	Notes		

Question	Scheme	Marks	AOs
1(a)	$16+(21-1)\times d=24\Rightarrow d=...$	M1	1.1b
	$d=0.4$	A1	1.1b
	Answer only scores both marks.		
		(2)	
(b)	$S_n=\frac{1}{2}n\{2a+(n-1)d\}\Rightarrow S_{500}=\frac{1}{2}\times500\{2\times16+499\times\text{"}0.4\text{"}\}$	M1	1.1b
	$=57900$	A1	1.1b
	Answer only scores both marks		
		(2)	
	(b) Alternative using $S_n=\frac{1}{2}n\{a+l\}$		
	$l=16+(500-1)\times\text{"}0.4\text{"}=215.6\Rightarrow S_{500}=\frac{1}{2}\times500\{16+\text{"}215.6\text{"}\}$	M1	1.1b
	$=57900$	A1	1.1b
		(4 marks)	

Notes

(a)

M1: Correct strategy to find the common difference – must be a correct method using a = 16, and n = 21 and the 24. The method may be implied by their working.
If the AP term formula is quoted it must be correct, so use of e.g. $u_n=a+nd$ scores M0

A1: Correct value. Accept equivalents e.g. $\frac{8}{20},\frac{4}{10},\frac{2}{5}$ etc.

(b)

M1: Attempts to use a correct sum formula with a = 16, n = 500 and their numerical d from part (a)
If a formula is quoted it must be correct (it is in the formula book)

A1: Correct value

Alternative:

M1: Correct method for the 500th term and then uses $S_n=\frac{1}{2}n\{a+l\}$ with their l

A1: Correct value

Note that some candidates are showing implied use of $u_n=a+nd$ by showing the following:

(a) $d=\frac{24-16}{21}=\frac{8}{21}$ (b) $S_{500}=\frac{1}{2}\times500\left\{2\times16+499\times\frac{8}{21}\right\}=55523.80952...$

This scores (a) M0A0 (b) M1A0

Question	Scheme	Marks	AOs
3(a)	$u_2 = k-12,\ u_3 = k - \dfrac{24}{k-12}$	M1	1.1b
	$u_1 + 2u_2 + u_3 = 0 \Rightarrow 2 + 2(k-12) + k - \dfrac{24}{k-12} = 0$	dM1	1.1b
	$\Rightarrow 3k - 22 - \dfrac{24}{k-12} = 0 \Rightarrow (3k-22)(k-12) - 24 = 0$ $\Rightarrow 3k^2 - 36k - 22k + 264 - 24 = 0$ $\Rightarrow 3k^2 - 58k + 240 = 0$ *	A1*	2.1
		(3)	
(b)	$k = 6, \left(\dfrac{40}{3}\right)$	M1	1.1b
	$k = 6$ as k must be an integer	A1	2.3
		(2)	
(c)	$(u_3 =)10$	B1	2.2a
		(1)	
		(6 marks)	

Notes

(a)

M1: Attempts to apply the sequence formula once for either u_2 **or** u_3.

Usually for $u_2 = k - \dfrac{24}{2}$ o.e. but could be awarded for $u_3 = k - \dfrac{24}{\text{their "}u_2\text{"}}$

dM1: Award for

- attempting to apply the sequence formula to find both u_2 **and** u_3
- using $2 + 2"u_2" + "u_3" = 0 \Rightarrow$ an equation in k. The u_3 may have been incorrectly adapted

A1*: Fully correct work leading to the printed answer.

There must be

- (at least) one correct intermediate line between $2 + 2(k-12) + k - \dfrac{24}{k-12} = 0$ (o.e.) and the given answer that shows how the fractions are "removed". E.g. $(3k-22)(k-12) - 24 = 0$
- no errors in the algebra. The = 0 may just appear at the answer line.

(b)

M1: Attempts to solve the quadratic which is implied by sight of $k = 6$.

This may be awarded for any of

- $3k^2 - 58k + 240 = (ak \pm c)(bk \pm d) = 0$ where $ab = 3, cd = 240$ followed by $k =$
- an attempt at the correct quadratic formula (or completing the square)
- a calculator solution giving at least $k = 6$

A1: Chooses $k = 6$ and gives a minimal reason

Examples of a minimal reason are

- 6 because it is an integer
- 6 because it is a whole number
- 6 because $\dfrac{40}{3}$ or $13.\dot{3}$ is not an integer

(c)

B1: Deduces the correct value of u_3.

Question	Scheme	Marks	AOs
4	(i) $\sum_{r=1}^{16}\left(3+5r+2^r\right)=131\,798$; (ii) $u_1, u_2, u_3, ..., : u_{n+1}=\dfrac{1}{u_n}, u_1=\dfrac{2}{3}$		
(i) Way 1	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\} \sum_{r=1}^{16}(3+5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=\dfrac{16}{2}(2(8)+15(5)) + \dfrac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=728+131\,070 = 131\,798$ *	A1*	2.1
		(4)	
(i) Way 2	$\left\{\sum_{r=1}^{16}\left(3+5r+2^r\right)=\right\} \sum_{r=1}^{16}3 + \sum_{r=1}^{16}(5r)+\sum_{r=1}^{16}\left(2^r\right)$	M1	3.1a
	$=(3\times16)+\dfrac{16}{2}(2(5)+15(5)) + \dfrac{2(2^{16}-1)}{2-1}$	M1	1.1b
		M1	1.1b
	$=48+680+131\,070 = 131\,798$ *	A1*	2.1
		(4)	
(i) Way 3	Sum = $10+17+26+39+60+97+166+299+560+1077+2106$ $+4159+8260+16457+32846+65619=131\,798$ *	M1	3.1a
		M1	1.1b
		M1	1.1b
		A1*	2.1
		(4)	
(ii)	$\left\{u_1=\dfrac{2}{3}\right\}, u_2=\dfrac{3}{2}, u_3=\dfrac{2}{3}, ...$ (***can be implied by later working***)	M1	1.1b
	$\left\{\sum_{r=1}^{100}u_r=\right\} 50\left(\dfrac{2}{3}\right)+50\left(\dfrac{3}{2}\right)$ or $50\left(\dfrac{2}{3}+\dfrac{3}{2}\right)$	M1	2.2a
	$=\dfrac{325}{3}$ $\left(\text{or } 108\dfrac{1}{3} \text{ or } 108.\dot{3} \text{ or } \dfrac{1300}{12} \text{ or } \dfrac{650}{6}\right)$	A1	1.1b
		(3)	
			(7 marks)

Question	Scheme	Marks	AOs
8 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \dots$		
	$= \dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20-17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \dfrac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40-37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \dots\dots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\dots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\} \sum_{n=1}^{48} (\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \dots\dots + \log_5 50) - (\log_5 2 + \log_5 3 + \dots\dots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

Question	Scheme	Marks	AOs
13 (a)	Uses the sequence formula $a_{n+1}=\dfrac{k\left(a_n+2\right)}{a_n}$ once with $a_1=2$	M1	1.1b
	$\left(a_1=2\right), a_2=2k, a_3=k+1, a_4=\dfrac{k(k+3)}{k+1}$ Finds four consecutive terms and sets a_4 equal to a_1 (oe)	M1	3.1a
	$\dfrac{k(k+3)}{k+1}=2 \Rightarrow k^2+3k=2k+2 \Rightarrow k^2+k-2=0$ *	A1*	2.1
		(3)	
(b)	States that when $k=1$, all terms are the same and concludes that the sequence does not have a period of order 3	B1	2.3
		(1)	
(c)	Deduces the repeating terms are $a_{1/4}=2, a_{2/5}=-4, a_{3/6}=-1,$	B1	2.2a
	$\sum_{n=1}^{80} a_k = 26\times\left(2+-4+-1\right)+2+-4$	M1	3.1a
	$=-80$	A1	1.1b
		(3)	
			(7 marks)
Notes:			

Question	Scheme	Marks	AOs
15(a)	$S_n = a + ar + ar^2 + \ldots\ldots.. + ar^{n-1}$	B1	1.2
	$rS_n = ar + ar^2 + ar^3 + \ldots\ldots.. + ar^n \Rightarrow S_n - rS_n = \ldots$	M1	2.1
	$S_n - rS_n = a - ar^n$	A1	1.1b
	$S_n(1-r) = a(1-r^n) \Rightarrow S_n = \dfrac{a(1-r^n)}{(1-r)}$ *	A1*	2.1
		(4)	
(b)	$\dfrac{a(1-r^{10})}{1-r} = 4\times\dfrac{a(1-r^5)}{1-r}$ or $4\times\dfrac{a(1-r^{10})}{1-r} = \dfrac{a(1-r^5)}{1-r}$ Equation in r^{10} and r^5 (and possibly $1-r$)	M1	3.1a
	$1 - r^{10} = 4(1-r^5)$	A1	1.1b
	$r^{10} - 4r^5 + 3 = 0 \Rightarrow (r^5 - 1)(r^5 - 3) = 0 \Rightarrow r^5 = \ldots$ or e.g. $1 - r^{10} = 4(1-r^5) \Rightarrow (1-r^5)(1+r^5) = 4(1-r^5) \Rightarrow r^5 = \ldots$	dM1	2.1
	$r = \sqrt[5]{3}$ oe only	A1	1.1b
		(4)	
			(8 marks)

Question Number	Scheme	Marks
5.		
(a)	$u_2 = 2 - \frac{4}{3} = \frac{2}{3},\ u_3 = 2 - \frac{4}{\frac{2}{3}} = -4,\ u_4 = 2 - \frac{4}{-4} = 3$	M1 A1 A1
		[3]
(b)	$u_{61} = 3$.	B1
		[1]
(c)	$\sum_{i=1}^{99} u_i = (3 + \frac{2}{3} - 4) + (3 + \frac{2}{3} - 4) + (3 + \frac{2}{3} - 4) + \ldots$	M1
	$\sum_{i=1}^{99} u_i = 33 \times (\ldots + \ldots + \ldots)$, $= -11$	A1, A1
		[3]
(c)	**Alternative method** for part (c) Adds $n \times "3" + n \times "-4" + n \times "\frac{2}{3}"$	M1
	Uses n = 33	A1
	-11	A1
		[3]
		7 marks
	Notes	
(a)	**M1:** Attempt to use formula correctly (implied by first term correct, or given as 0.67, or third term following through from their second etc) **A1**: two correct answers **A1**: 3 correct answers (allow 0.6 recurring but not 0.667) Look for the values. Ignore the u_r label	
(b)	**B1**: cao (NB Use of AP is B0)	
(c)	**M1:** Uses sum of at least 3 terms found from part (a)) (may be implied by correct answer). Attempt to sum an AP here is M0. **A1**: obtains $33 \times$ (sum of three adjacent terms) or $11 \times$ (sum of nine adjacent terms) **A1**: - 11 cao (-11 implies both A marks) N.B. Use of n = 99 is M1A0A0	

Question Number	Scheme	Marks
11. (a)	Uses $(2p-6)-4p = 4p-60$ or $4p = \frac{60+(2p-6)}{2}$ or 60+2(4*p*-60) =2*p*-6 or etc… or two correct equations with *d*	M1
	So $p = 9$ *	A1 * **[2]**
Alternative to (a)	Use *p*=9 to give 60, 36 and 12 and deduce *d* = -24 so conclude AP when *p* = 9	M1 A1 [2]
(b)	Uses *a* + 19*d* with *a* = 60	M1
	Finds *d* = 36 – 60 = -24	B1
	So obtains -396	A1 **[3]**
(c)	Uses $\frac{n}{2}(2\times 60+(n-1)d)$	M1
	Uses $\frac{n}{2}(2\times 60-24(n-1))$	A1
	= 12*n* (6-*n*) *	A1* **[3]**
		8 marks
	Notes	
(a)	M1**: Correct** equation to enable *p* to be found or two correct equations if *d* introduced and solving simultaneous equations to eliminate *d* and enable *p* to be found NB May add three terms and use sum formula giving e.g. $60+4p+2p-6=\frac{3}{2}(60+2p-6)$ A1: cso (Do not need intermediate step)	
(b)	M1: Correct formula with their value for *d* B1: *d* = -24 seen in (a) or (b) A1: -396 If all terms are found and added 60 + 36 + 12 + -12 + .. Need 20 terms for M1, need -24 implied by first 4 terms for B1 and correct answer for A1	
(c)	M1:Uses correct formula with their value for *d* A1: Correct value for *d* A1: given answer – must be no errors to award this mark Special case: Proves formula for sum of AP M1: Correct method of proof using their d A1: For *d* = -24 A1: given answer – must be no errors to award this mark	

Question Number	Scheme	Marks
9(i)	$\sum_{r=1}^{20}(3+5r)=8+13+18+..........+103$	M1
	Use of $S_n=\frac{n}{2}\left(2a+(n-1)d\right)$ or $S_n=\frac{n}{2}\left(a+l\right)$ with a=3or 8, n=19 or 20, d=5 and l=103	M1
	$S_{20}=\frac{20}{2}\left(8+103\right)=1110$	A1
		(3)
(ii)	$\sum_{r=0}^{\infty}\frac{a}{4^r}=16\Rightarrow\frac{a}{1}+\frac{a}{4}+\frac{a}{16}....=16 \qquad r=\frac{1}{4}$ oe	B1
	Use of $S_\infty=\frac{a}{1-r}$ with $0<\lvert r\rvert<1$ and $S_\infty=16$	M1
	$16=\frac{a}{1-'r'}\Rightarrow a=..$	dM1
	$a=12$	A1
		(4)
		(7 marks)

Question Number	Scheme	Marks
5. (a)	$S_n = a + (a+d) + (a+2d) + \ldots + (a+(n-1)d)$	M1
	$S_n = (a+(n-1)d) + (a+(n-2)d) + \ldots + (a+d) + a$	M1
	$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + \ldots + (2a+(n-1)d)$	M1
	$S_n = \frac{n}{2}[2a+(n-1)d]$* See notes below for those who use triangle numbers in their proof	A1* **[4]**
(b)	**Uses** either $\frac{n}{2}(2\times a+(n-1)7)$ or $\frac{n}{2}(a+497)$ or $7\times\sum_{i=1}^{71} i$	M1
	i.e $\frac{71}{2}(2\times 7+70\times 7)$ or $\frac{72}{2}(2\times 0+71\times 7)$ or $\frac{71}{2}(7+497)$ or $7\times\frac{71}{2}(72)$	A1
	= 17892	A1 **[3]**
		7 marks
	Notes	

(a) **M1**: List terms including **at least first two** and **a last term which may be *a* + *nd* or *a* + (*n* – 1)*d* or *L***
M1: List terms in reverse including **at least their last term (or correct last term) and finally their first term**
M1: The LHS should be 2*S*. The RHS must follow from at least two terms correctly matching in the addition and should include at least two terms which are each **correctly** $\{2a+(n-1)d\}$ or $(a + L)$ **or should** be $n\{2a+(n-1)d\}$ or $n(a + L)$
A1: Need some indication of at least three terms being added (i.e at least three terms and their pairs listed with terms correctly matching or three additions seen) and also need to achieve final answer with no errors and if *L* was used need to state that $L = a + (n-1)d$

NB: Some candidates use a variation of

$$\sum_{r=1}^{n}(a+(r-1)d = \sum_{r=1}^{n} a + d\sum_{r=1}^{n}(r-1) = na + d\frac{n}{2}(n+1) - dn \text{ or } na + d\frac{(n-1)}{2}(n)$$

And conclude that $S_n = \frac{n}{2}[2a+(n-1)d]$. This gains the full 4 marks M1M1M1A1, but must be completely correct.

(b) **M1:Uses** correct formula (with their *a* and *n*) with *d* =7 or with last term correct
A1: Uses consistent and correct *a* and *n*
A1: Correct answer

Question Number	Scheme	Marks
8.		
(a)	$u_2 = 3k - 12,\ u_3 = 3(u_2) - 12$	M1
	$u_2 = 3k - 12,\ u_3 = 9k - 48$	A1
	$u_4 = 3(9k - 48) - 12 = 27k - 156$ (ft their u_3) .	M1 A1ft
		[4]
(b)	$27k - 156 = 15$ so $k =$	M1
	$k = 6\frac{1}{3}$ or $\frac{19}{3}$ or 6.33 (3sf)	A1
		[2]
(c)	$\sum_{i=1}^{4} u_i = 6\frac{1}{3} + 7 + 9 + 15$ or $\sum_{i=1}^{4} u_i = k + 3k - 12 + 9k - 48 + 27k - 156$	M1
	$= 40k - 216$, $= 37\frac{1}{3}$ or $\frac{112}{3}$	A1ft, A1cao
		[3]
		9 marks
	Notes	

(a) **M1:** Attempt to use formula twice to find u_2 and u_3

A1: two correct simplified answers

M1: Attempt again to find u_4

A1ft: 4^{th} term correct and simplified - follow through their u_3

(b) **M1**: Put their 4^{th} term (not 5^{th}) equal to 15 and attempt to find $k =$

A1: accept any correct fraction or decimal answer (allow 6.33 or better here)

(c) **M1:** Uses 1^{st} term and their following 3 terms **with plus signs** (either numerical or in terms of *k*). Must be using terms from iteration and not formula for an AP or GP. May make a copying slip.

A1ft: for $40k - 216$ or follow through on their *k* so check $40k$ -216 for their *k*

A1: obtains $37\frac{1}{3}$ (must be exact) if exact answer given, then isw

Those who use 6.3 will obtain 36 They should have M1A1ftA0 – should have used exact *k* to give exact answer here.

Those who use 6.33 will obtain 37.2 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

Those who use 6.333 will obtain 37.32 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

6.3333 will obtain 37.332 This should have M1A1ftA0 – should have used exact *k* to give exact answer here.

6.33333 will obtain 37.3332 etc All these answers should have M1A1ftA0 – should have used exact *k* to give exact answer here. Etc

Special case: Those who use $k = 6$ will obtain $6 + 6 + 6 + 6 = 24$ This is M1 A0 A0 in part (c) – as over simplified

Question Number	Scheme	Marks
10 (a)	$u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$	M1, A1 **[2]**
(b)	$u_{20} = 4\times\left(\frac{2}{3}\right)^{19}$;= 0.00180 or 0.0018 or exact equivalent	M1; cao A1 **[2]**
(c)	Use $\sum_{i=1}^{16} u_i = \frac{4(1-\left(\frac{2}{3}\right)^{16})}{1-\frac{2}{3}}$	M1
	Find 12 - *their* $\sum_{i=1}^{16} u_i$	dM1
	= 12 – 11.9817 = awrt 0.0183	A1 **[3]**
(d)	12 is the **sum to infinity** (and all terms are **positive)** so sum is less than 12	B1 **[1]**
	Or $\sum_{i=1}^{n} u_i = \frac{4(1-\left(\frac{2}{3}\right)^n)}{1-\frac{2}{3}} = 12-12\left(\frac{2}{3}\right)^n$ *and* $\left(\frac{2}{3}\right)^n > 0$ so is less than 12	
		[8 marks]

(a)

M1 Any one term is 2/3 the previous term. Accept for example $u_2 =$ awrt 2.67

A1 All 3 terms correct. Accept exact equivalents $u_2 = \frac{8}{3}$ or $2\frac{2}{3}$, $u_3 = \frac{16}{9}$ or $1\frac{7}{9}$, $u_4 = \frac{32}{27}$ or $1\frac{5}{27}$

(b)

M1 Uses correct nth term formula ar^{n-1} with $a = 4, n = 20$ and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7

Condone for the M mark use of ar^{n-1} with $a = \frac{8}{3}$ (awrt 2.67), $n = 20$ and $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7

Expressions such as $4\times\left(\frac{2}{3}\right)^{19}$, $\frac{8}{3}\times\left(\frac{2}{3}\right)^{18}$ and $\frac{2^{n+1}}{3^{n-1}} \to \frac{2^{21}}{3^{19}}$ are correct and sufficient for M1

A1 Accept any of $0.0018, 0.00180$, 1.80×10^{-3} or 1.8×10^{-3}

(c)

M1 Uses the correct sum formula $S = \frac{a\left(r^n - 1\right)}{(r-1)}$ or $S = \frac{a\left(1-r^n\right)}{(1-r)}$ with $a = 4, r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$

Condone the sum formula $S = \frac{a\left(r^n - 1\right)}{(r-1)}$ or $S = \frac{a\left(1-r^n\right)}{(1-r)}$ with $a = \frac{8}{3}$ (awrt 2.67), $r = \frac{2}{3}, \frac{3}{2}$ or awrt 0.7, $n = 16$

dM1 Dependent upon the previous M mark. Score for an attempt at finding $12 - \sum_{i=1}^{16} u_i$

A1 awrt 0.0183

Note: Some candidates may list all 16 terms which is acceptable provided the answer is accurate

(d)

B1 Need a reason + a minimal conclusion. Eg The sum to infinity =12 **and** sum is less than 12
Allow sum to infinity is 12, hence true.

Question Number	Scheme	Marks
5(i)		
(a)	$(U_2) = \frac{4}{4-3} = 4$	B1
		(1)
(b)	$\sum_{n=1}^{100} U_n = 100 \times 4 = 400$	M1A1
		(2)
5(ii)	$\sum_{r=1}^{n} (100-3r) < 0 \Rightarrow 97 + 94 + 91 + ...(100-3r) < 0$	
	$\sum$AP with $a = 97, d = -3, n = n, S < 0 \Rightarrow 0 = \frac{n}{2}(2\times 97 + (n-1)\times -3) < 0$	M1
	$\Rightarrow \frac{n}{2}(197 - 3n) < 0 \Rightarrow n > 65.\dot{6}$	dM1
	$\Rightarrow n = 66$	A1
		(3) **(6 marks)**
(ii) ALT I	$\sum_{r=1}^{n} (100-3r) < 0 \Rightarrow \sum_{r=1}^{n} 3r > \sum_{r=1}^{n} 100$	
	$\Rightarrow 3\frac{n(n+1)}{2} > 100n$	M1 M1A1
	$\Rightarrow n > 65.\dot{6} \Rightarrow n = 66$	

(i)(a)

B1 States that U_2 is 4. Accept $\frac{4}{1}$ but not $\frac{4}{4-3}$ and remember to isw.

Note that $U_1 = 4$ so be sure that you don't award this B1

(i)(b)

M1 Uses the method that $\sum_{n=1}^{100} U_n = k \times 4$ where $k = 100$ or 99

You may see the AP formula being used which is fine as long as $a = 4,\ d = 0$ and $n = 99/100$

Look for expression of the form $\frac{100}{2}\{2\times 4 + 99 \times 0\}$ OR $\frac{100}{2}\{4+4\}$

A1 400

Question	Scheme	Marks
6.		
(a)	$u_2 = 24,\ u_3 = 16$ and $u_4 = \dfrac{32}{3}$	M1, A1 **[2]**
(b)	$r = \dfrac{2}{3}$	B1 **[1]**
(c)	$u_{11} = ar^{10} = 36 \times (r)^{10}$.	M1
	$u_{11} = ar^{10} = 36 \times \left(\dfrac{2}{3}\right)^{10} = \left(\dfrac{4096}{6561}\right)$	
	$= 0.6243$	A1 **[2]**
(d)	$\sum_{i=1}^{6} u_i = \dfrac{36(1-(\frac{2}{3})^6)}{1-\frac{2}{3}}$ or $\sum_{i=1}^{6} u_i = 36+24+16+\dfrac{32}{3}+u_5+u_6$	M1
	$= 98\frac{14}{27}$	A1cao **[2]**
(e)	$\sum_{i=1}^{\infty} u_i = \dfrac{36}{1-\frac{2}{3}} = 108$	M1 A1 **[2]**
		9 marks
	Notes	

(a)

M1: Attempt to use formula correctly at least twice. It may be seen for example in u_3 and u_4

A1: All three correct exact simplified answers. Allow $10.\dot{6}$

(b)

B1: Accept $\dfrac{2}{3}$ or equivalent such as $\dfrac{24}{36}$ Allow awrt 0.667

(c)

M1: Uses $u_{11} = ar^{10} = 36 \times (r)^{10}$ with their r

A1: Accept awrt 0.6243 or $\dfrac{4096}{6561}$

(d)

M1: Uses correct sum formula with $a = 36$ and their r **or** alternatively for adding their first six terms.
FYI Sight of 36, 24, 16, 10.7, 7.1, 4.7 followed by 98.5 implies this mark. (You may only see the first 4 terms in part a)

A1: Obtains $= 98\frac{14}{27}$ (must be exact). For information $\dfrac{2660}{27} = 98\frac{14}{27}$ Allow $98.\dot{5}1\dot{8}$

(e)

M1: Uses correct sum to infinity formula with $a = 36$ and either $r = \dfrac{2}{3}$ or their r as long as $|r| < 1$

A1: Obtains 108 (must be exact)

Question Number	Scheme		Marks
4 (a)	$S_9 = 54$ $\Rightarrow 54 = \frac{9}{2}(2a+8d)$ or $\Rightarrow 54 = \frac{9}{2}(a+a+8d)$	Uses a correct sum formula with $n = 9$ and $S_9 = 54$	M1
	$\Rightarrow a+4d = 6$*	cso	A1*
	Listing:		
	$a+a+d+a+2d+....+a+8d = 54$ $\Rightarrow 9a+36d = 54$ Scores M1 for attempting to sum 9 terms (both lines needed) or $a+a+d+a+2d+a+3d+a+4d+a+5d+a+6d+a+7d+a+8d = 54$ Scores M1 on its own and then A1 if they complete correctly.		
			(2)
(b)	$a+7d = \frac{1}{2}(a+6d)$ or $\frac{1}{2}(a+7d) = a+6d$	Uses $t_8 = \frac{1}{2}t_7$ or $\frac{1}{2}t_8 = t_7$ to produce one of these equations.	M1
	$\Rightarrow 6-4d+7d = \frac{1}{2}(6-4d+6d)$ $\Rightarrow d = ...$	Uses the **given equation from (a)** and their second linear **<u>equation</u>** in a and d and proceeds to find a value for either a or d.	M1
	$\Rightarrow d = -1.5, a = 12$	A1: Either $d = -1.5$ (oe) or $a = 12$ A1: Both $d = -1.5$(oe) and $a = 12$	A1A1
	Note that use of $\frac{1}{2}t_8 = t_7$ in (b) gives a = 30 and d = -6		
			(4)
			(6 marks)

Question Number	Scheme	Marks
9.(a)	$a = 7k - 5,\ ar = 5k - 7$ and $ar^2 = 2k + 10$	B1
	(So $r =$) $\dfrac{5k-7}{7k-5} = \dfrac{2k+10}{5k-7}$ or $(7k-5)(2k+10) = (5k-7)^2$ or equivalent	M1
	See $(5k-7)^2 = 25k^2 - 70k + 49$	M1
	$14k^2 + 60k - 50 = 25k^2 - 70k + 49 \rightarrow\ 11k^2 - 130k + 99 = 0$ *	A1cso * **(4)**
(b)	$(k-11)(11k-9)$ so $k =$	M1
	$k = 9/11$ only* (after rejecting 11)	A1*
	N.B. Special case $k = 9/11$ can be verified in (b) (1 mark only)	
	$11 \times \left(\dfrac{9}{11}\right)^2 - 130 \times \left(\dfrac{9}{11}\right) + 99 = \dfrac{81}{11} - \dfrac{1170}{11} + \dfrac{1089}{11} = 0$ M1A0	**(2)**
(c)	$a = \dfrac{8}{11}$	B1
	$\dfrac{5 \times \frac{9}{11} - 7}{7 \times \frac{9}{11} - 5}$ *or* $\dfrac{2 \times \frac{9}{11} + 10}{5 \times \frac{9}{11} - 7}$ so $r = -4$	B1
	(i) Fourth term = $ar^3 = -\dfrac{512}{11}$	M1A1
	(ii) $S_{10} = \dfrac{a(1-r^{10})}{(1-r)} = \dfrac{\frac{8}{11}(1-(-4)^{10})}{(1-(-4))} = -152520$	M1A1
		(6) **[12]**

Notes

(a) Mark parts (a) and (b) together

B1: Correct statement (needs all three terms)**– this may be omitted and implied** by correct statement in k only, as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately)

M1: Valid Attempt to eliminate a and r and to obtain equation in k only

M1: Correct expansion of $(5k-7)^2 = 25k^2 - 70k + 49$ - may have four terms $(5k-7)^2 = 25k^2 - 35k - 35k + 49$

A1cso: No incorrect work seen. The printed answer is obtained including "=0".

(b) M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula – see notes at start of mark scheme) **or** see 9/11 substituted and given as "=0" for M1A0

A1*: 9/11 **only and** 11 should be seen and rejected. Accept 9/11 underlined or k= 9/11 written on following line. Alternatively (k − 11) may be seen in the factorisation and a statement 'k not integer' given with k=9/11 stated.

(c) Mark parts (i) and (ii) together

B1: $a = \dfrac{8}{11}$ or any equivalent (If not stated explicitly or used in formula may be implied by correct answer to (ii))

B1:Substitutes k = 9/11 completely and obtain $r = -4$ (If not stated explicitly, may be implied by correct answer to (i) or (ii))

(i) M1: Use of correct formula with $n = 4$ a and/or r may still be in terms of k or uses $(2k+10)\times r$. May assume $r = k$.

A1: Correct exact answer

(ii) M1: Use of correct formula with $n = 10$ a and/or r may still be in terms of k May assume $r = k$ A1 : -152520 cao

NB Correct formula **with negative sign** in numerator followed by the incorrect $(8/11)(1+4^{10})/(1-(-4))$ usually found equal to 152520.2909 with no negative sign can be allowed M1A0 but if the incorrect numerical expression appears on its own with no formula then M0A0

Listing terms can get: B1 (first term) B1 M1A1 (implied by correct 4th term) M1A1 (implied by −152520)

Question Number	Scheme	Marks
5.(i)	**Mark (a) and (b) together**	
(a)	$a+ar=34$ or $\frac{a(1-r^2)}{(1-r)}=34$ or $\frac{a(r^2-1)}{(r-1)}=34$; $\frac{a}{1-r}=162$	B1; B1
(Way 1)	Eliminate a to give $(1+r)(1-r)=\frac{17}{81}$ or $1-r^2=\frac{34}{162}$ .. (not a cubic)	aM1
	(and so $r^2=\frac{64}{81}$ and) $r=\frac{8}{9}$ only	aA1 **(4)**
(b)	Substitute their $r=\frac{8}{9}$ ($0<r<1$) to give $a=$ $a=18$	bM1 bA1 **(2)**
(Way 2) Part (b) first	Eliminate r to give $\frac{34-a}{a}=1-\frac{a}{162}$	bM1
	gives $a=18$ or 306 and rejects 306 to give $a=18$	bA1
Then part (a) again	Substitute a = 18 to give $r=$	aM1
	$r=\frac{8}{9}$	aA1
(ii)	$\frac{42(1-\frac{6}{7}^n)}{1-\frac{6}{7}}>290$ (For trial and improvement approach see notes below)	M1
	to obtain So $(\frac{6}{7})^n<(\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n>(\frac{294}{4})$ or $(\frac{6}{7})^n<(\frac{2}{147})$	A1
	So $n>\frac{\log"(\frac{4}{294})"}{\log(\frac{6}{7})}$ or $\log_{\frac{6}{7}}"(\frac{4}{294})"$ or equivalent but must be log of positive quantity	M1
	(i.e. $n>$ 27.9) so $n=28$	A1 **(4)**

Notes

(i) (a) **B1**: Writes **a correct** equation connecting a and r and 34 (allow equivalent equations – may be implied)
B1: Writes a **correct** equation connecting a and r and 162 (allow equivalent equation – may be implied)

Way 1: a**M1**: Eliminates a correctly for these two equations to give $(1+r)(1-r)=\frac{17}{81}$ or $(1+r)(1-r)=\frac{34}{162}$ or equivalent – **not a cubic** – should have factorized (1 - r) to give a correct quadratic

a**A1:** Correct value for r. Accept 0.8 recurring or 8/9 (not 0.889) Must only have positive value.

b**M1**: Substitutes their r ($0<r<1$) into a correct formula to give value for a. Can be implied by $a=18$

b**A1**: must be 18 (not answers which round to 18)

Way 2: Finds *a* first - B1, B1: As before then award the (b) M and A marks before the (a) M and A marks

b**M1**: Eliminates r correctly to give $\frac{34-a}{a}=1-\frac{a}{162}$ or $a^2-324a+5508=0$ or equivalent

b**A1:** Correct value for a so $a=18$ only. (Only award after 306 has been rejected)

a**M1**: Substitutes their 18 to give $r=$

a**A1**: $r=\frac{8}{9}$ only

(ii) **M1:** Allow n **or** $n-1$ and any symbols from ">", "<", or "=" etc **A1 :** Must be power n (**not** $n-1$) with any symbol

M1: Uses logs correctly on $(\frac{6}{7})^n$ or $(\frac{7}{6})^n$ **not on** $(36)^n$ to get as far as n Allow any symbol

A1: $n=28$ cso (any errors with inequalities earlier e.g. failure to reverse the inequality when dividing by the negative $\log(\frac{6}{7})$ or any contradictory statements must be penalised here) Those with equals throughout may gain this mark if they follow 27.9 by n=28. Just $n=28$ without mention of 27.9 is only allowed following correct inequality work.

Special case: Trial and improvement: Gives $n=28$ as $S=$ awrt 290.1 (M1A1)and when $n=27$ $S=$ (awrt) 289 so $n=28$ (M1A1) – $n=28$ with no working is M1A0M0A0 and insufficient accuracy is M1A0M1A0

Uses nth term instead of sum of n terms – over simplified – do not treat as misread – award 0/4

Question Number	Scheme		Marks
6(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $= 160$	M1: Use of a correct S_∞ formula A1: 160	M1A1
	Accept correct answer only (160)		
			[2]
(b)	$S_{12} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}}$; $= 127.77324...$	M1: Use of a correct S_n formula with $n = 12$ (condone missing brackets around 7/8) A1: **awrt** 127.8	M1A1
	T & I in (b) requires all 12 terms to be calculated correctly for M1 and A1 for awrt 127.8		
			[2]
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working. (condone missing brackets around 7/8)(Allow $=, <, >, \geq, \leq$) but see note below.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $+\left(\frac{7}{8}\right)^N$ oe (Allow $=, <, >, \geq, \leq$) but see note below. **Dependent on the previous M1**	**d**M1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the power law of logarithms or takes logs base 0.875 correctly to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ (Allow $=, <, >, \geq, \leq$) but see note below.	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but not $N > 44$	A1 **cso**
	An incorrect **<u>inequality</u>** statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			[4]
			Total 8
	<u>Trial & Improvement Method in (c):</u>		
	1st M1: Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		
	2nd M1: Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		
	3rd M1: For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		
	A1: $N = 44$ **cso**		
	Answer of $N = 44$ only with no working scores no marks		

Question Number	Scheme	Marks
5.(a)	$a = 4p,\ ar = (3p+15)$ and $ar^2 = 5p+20$	B1
	(So $r =$) $\dfrac{5p+20}{3p+15} = \dfrac{3p+15}{4p}$ or $4p(5p+20) = (3p+15)^2$ or equivalent	M1
	See $(3p+15)^2 = 9p^2 + 90p + 225$	M1
	$20p^2 + 80p = 9p^2 + 90p + 225 \rightarrow\ 11p^2 - 10p - 225 = 0$ *	A1 *
		(4)
(b)	$(p-5)(11p+45)$ so $p =$	M1
	p = 5 only (after rejecting - 45/11)	A1
	<u>N.B. Special case p = 5 can be verified in (b) (1 mark only)</u>	
	$11\times 5^2 - 10\times 5 - 225 \quad = 275 - 50 - 225 = 0$ M1A0	
		(2)
(c)	$\dfrac{3\times 5+15}{4\times 5}$ or $\dfrac{5\times 5+20}{3\times 5+15}$	M1
	$r = \dfrac{3}{2}$	A1
		(2)
(d)	$S_{10} = \dfrac{20\left(1-\left("\frac{3}{2}"\right)^{10}\right)}{\left(1-"\frac{3}{2}"\right)}$	M1A1ft
	(= 2266.601568…) = 2267	A1
		(3) **Total 11**

	Notes for Question 5
(a)	B1: Correct statement (needs all three terms)– **this may be omitted and implied** by correct statement in *p* only as candidates may use geometric mean, or may use ratio of terms being equal and give a correct line 2 without line 1. (This would earn the B1M1 immediately) M1: Valid Attempt to eliminate *a* and *r* and to obtain equation in *p* only M1: Correct expansion of $(3p+15)^2 = 9p^2 + 90p + 225$ A1cso: No incorrect work seen. The printed answer is obtained. **NB Those who show *p* = 5 in part (a) obtain no credit for this**
(b)	M1: Attempt to solve quadratic by usual methods (factorisation, completion of square or formula) Must appear in part (b) – not part (a) A1: 5 **only and** -45/11 should be seen and rejected or (11*p* + 45) seen and statement $p > 0$
(c)	M1: Substitutes *p* = 5 completely and attempt ratio (correct way up) A1: 1.5 or any equivalent
(d)	M1: Use of correct formula with *n* = 10 *a* and/or *r* may still be in terms of *p* A1ft: Correct expression ft on their *r* only – must have *a* = 20 and power = 10 here A1 2267 (accept awrt 2267)