11.

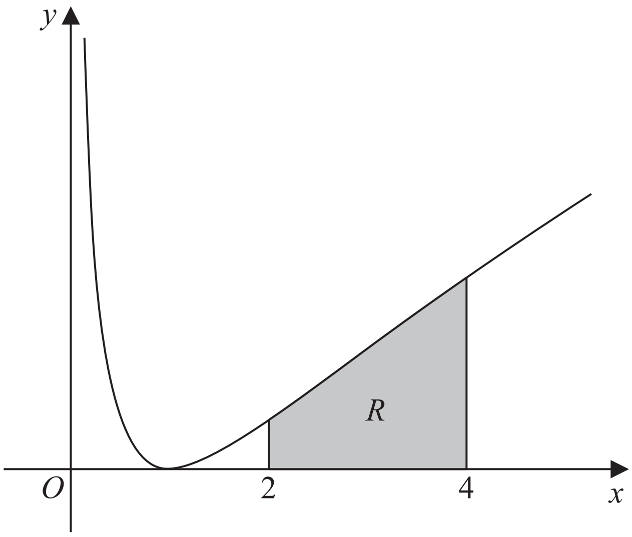

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a, b and c are integers to be found.

(5)

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2. The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table below with the time in seconds and the speed in $m s^{-1}$.

Time (s)	0	5	10	15	20	25
Speed ($m s^{-1}$)	2	5	10	18	28	42

Using all of this information,

(a) estimate the length of runway used by the jet to take off.

(3)

Given that the jet accelerated smoothly in these 25 seconds,

(b) explain whether your answer to part (a) is an underestimate or an overestimate of the length of runway used by the jet to take off.

(1)

1 The table below shows corresponding values of x and y for $y = \sqrt{\dfrac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\, dx$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\displaystyle\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, dx = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

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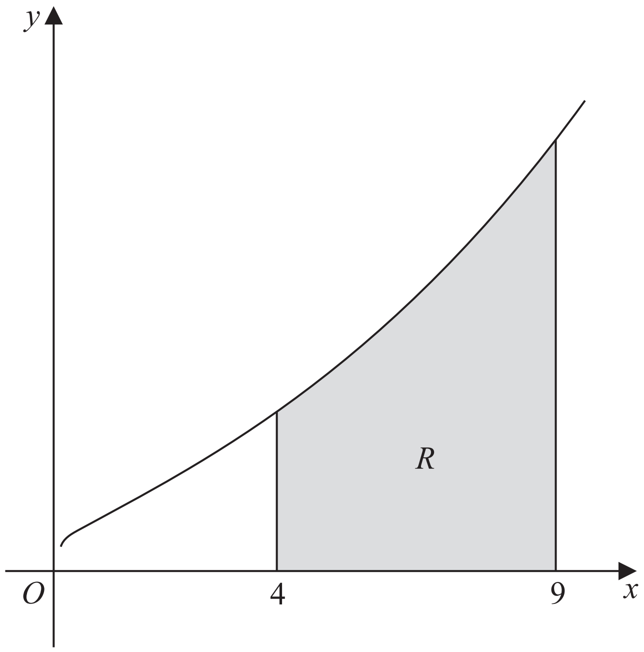

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{\sqrt{x}}, \ x > 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 4$ and $x = 9$

(a) Use the trapezium rule, with 5 strips of equal width, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(b) Use the substitution $u = \sqrt{x}$ to find, by integrating, the exact value for the area of R.

(7)

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12.

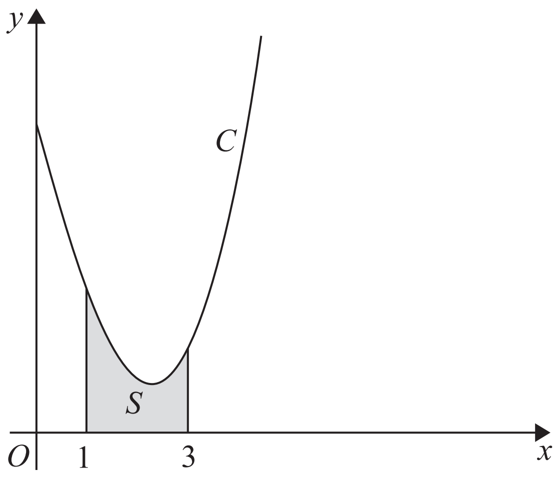

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the lines with equations $x = 1$ and $x = 3$

(a) Complete the table below with the value of y corresponding to $x = 2$. Give your answer to 4 decimal places.

x	1	1.5	2	2.5	3
y	2	1.3041		0.9089	1.2958

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(c) Use calculus to find the exact area of S.

Give your answer in the form $\frac{a}{b} + \ln c$, where a, b and c are integers.

(6)

(d) Hence calculate the percentage error in using your answer to part (b) to estimate the area of S. Give your answer to one decimal place.

(2)

(e) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

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13.

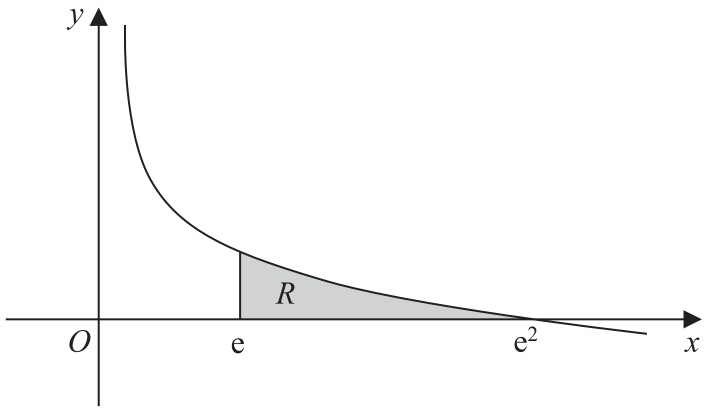

Figure 5

Figure 5 shows a sketch of part of the curve with equation $y = 2 - \ln x, \quad x > 0$

The finite region R, shown shaded in Figure 5, is bounded by the curve, the x-axis and the line with equation $x = \mathrm{e}$.

The table below shows corresponding values of x and y for $y = 2 - \ln x$

x	e	$\dfrac{\mathrm{e}+\mathrm{e}^2}{2}$	e^2
y	1		0

(a) Complete the table giving the value of y to 4 decimal places.
(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 3 decimal places.
(3)

(c) Use integration by parts to show that $\int (\ln x)^2 \, \mathrm{d}x = x(\ln x)^2 - 2x\ln x + 2x + c$
(4)

The area R is rotated through 360° about the x-axis.

(d) Use calculus to find the exact volume of the solid generated.

Write your answer in the form $\pi\mathrm{e}(p\mathrm{e} + q)$, where p and q are integers to be found.
(6)

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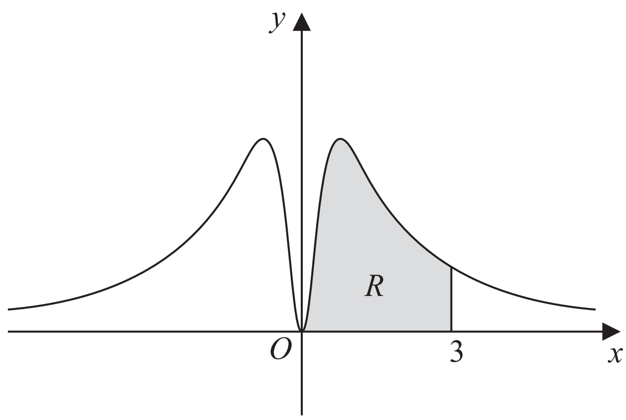

Figure 3

Figure 3 shows part of the curve C with equation

$$y = \frac{3\ln(x^2 + 1)}{(x^2 + 1)}, \quad x \in \mathbb{R}$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Using your answer to (a), find the exact coordinates of the stationary point on the curve C for which $x > 0$. Write each coordinate in its simplest form.

(5)

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis and the line $x = 3$

(c) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3
y	0		$\frac{3}{5}\ln 5$	$\frac{3}{10}\ln 10$

(1)

(d) Use the trapezium rule with all the y values in the completed table to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

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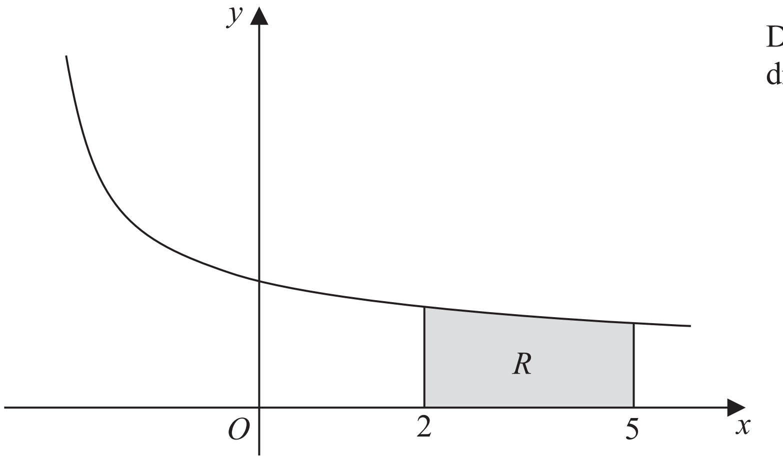

Diagram not drawn to scale

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{1}{\sqrt{2x+5}}, \quad x > -2.5$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = 2$ and $x = 5$

(a) Use the trapezium rule with three strips of equal width to find an estimate for the area of R, giving your answer to 3 decimal places.

(4)

(b) Use calculus to find the exact area of R.

(4)

(c) Hence calculate the magnitude of the error of the estimate found in part (a), giving your answer to one significant figure.

(1)

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5.

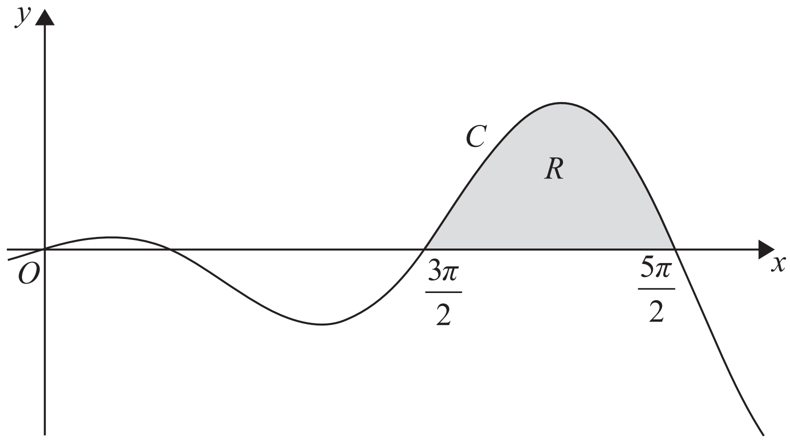

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = x\cos x, \quad x \in \mathbb{R}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the x-axis for $\frac{3\pi}{2} \leqslant x \leqslant \frac{5\pi}{2}$

(a) Complete the table below with the exact value of y corresponding to $x = \frac{7\pi}{4}$ and with the exact value of y corresponding to $x = \frac{9\pi}{4}$

x	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π	$\frac{9\pi}{4}$	$\frac{5\pi}{2}$
y	0		2π		0

(1)

(b) Use the trapezium rule, with all five y values in the completed table, to find an approximate value for the area of R, giving your answer to 4 significant figures.

(3)

(c) Find

$$\int x\cos x \, dx$$

(3)

(d) Using your answer from part (c), find the exact area of the region R.

(2)

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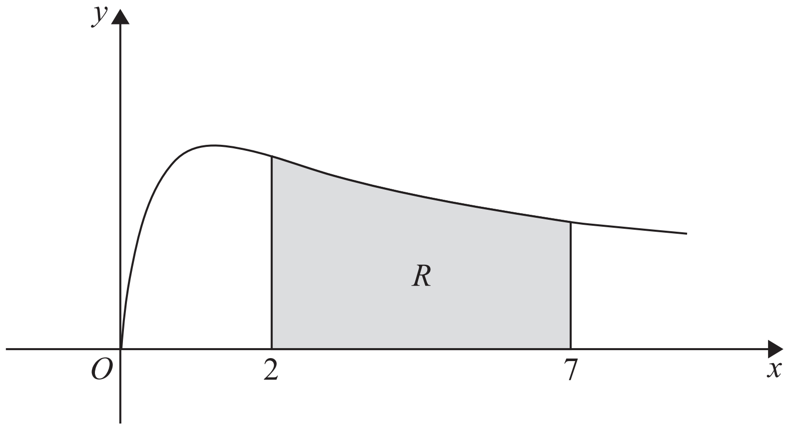

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{\dfrac{x}{x^2+1}}, \quad x \geqslant 0$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 7$

The table below shows corresponding values of x and y for $y = \sqrt{\dfrac{x}{x^2+1}}$

x	2	3	4	5	6	7
y	0.6325	0.5477	0.4851	0.4385	0.4027	0.3742

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 3 decimal places.

(3)

The region R is rotated 360° about the x-axis to form a solid of revolution.

(b) Use calculus to find the exact volume of the solid of revolution formed. Write your answer in its simplest form.

(4)

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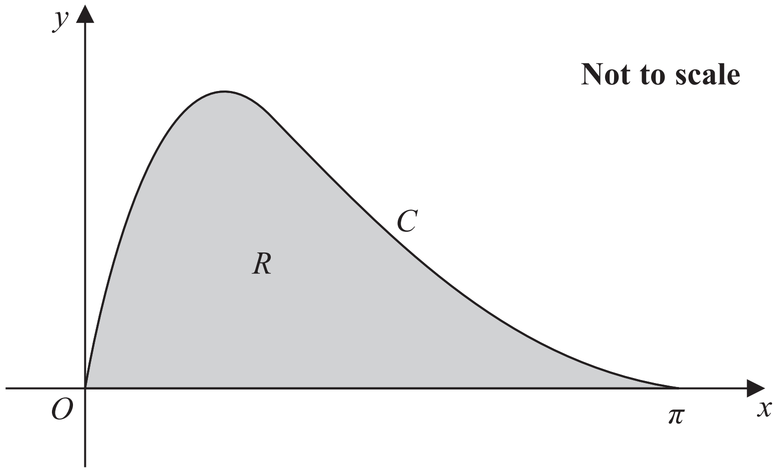

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 2e^{-x}\sqrt{\sin x}$, $0 \leqslant x \leqslant \pi$.
The finite region R, shown shaded in Figure 1, is bounded by the curve and the x-axis.

(a) Complete the table below with the value of y corresponding to $x = \frac{\pi}{2}$, giving your answer to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	0.76679		0.15940	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(3)

(c) Given $y = 2e^{-x}\sqrt{\sin x}$, find $\frac{dy}{dx}$ for $0 < x < \pi$.

(3)

The curve C has a maximum turning point when $x = a$.

(d) Use your answer to part (c) to find the value of a, giving your answer to 3 decimal places.

(3)

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9. (a) Given that a is a constant, $a > 1$, sketch the graph of

$$y = a^x, \qquad x \in \mathbb{R}$$

On your diagram show the coordinates of the point where the graph crosses the y-axis.
(2)

The table below shows corresponding values of x and y for $y = 2^x$

x	–4	–2	0	2	4
y	0.0625	0.25	1	4	16

(b) Use the trapezium rule, with all of the values of y from the table, to find an approximate value, to 2 decimal places, for

$$\int_{-4}^{4} 2^x \, \mathrm{d}x$$

(4)

(c) Use the answer to part (b) to find an approximate value for

(i) $\int_{-4}^{4} 2^{x+2} \, \mathrm{d}x$

(ii) $\int_{-4}^{4} (3 + 2^x) \, \mathrm{d}x$

(4)

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