4. A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

Aliya takes a random sample of 18 beads from the sack to make a bracelet.

(a) State a suitable binomial distribution to model the number of red beads in Aliya's bracelet.

(1)

(b) Use this binomial distribution to find the probability that

(i) Aliya has just 1 red bead in her bracelet,

(ii) there are at least 4 red beads in Aliya's bracelet.

(3)

(c) Comment on the suitability of a binomial distribution to model this situation.

(1)

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(d) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

(4)

(e) Find the *p*-value in this case.

(1)

3. Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir and his friends play the game 15 times.

(a) Find the probability that Naasir wins

(i) exactly 2 games,

(ii) more than 5 games.

(3)

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

3. A fair 5-sided spinner has sides numbered 1, 2, 3, 4 and 5

The spinner is spun once and the score of the side it lands on is recorded.

(a) Write down the name of the distribution that can be used to model the score of the side it lands on.

(1)

The spinner is spun 28 times.

The random variable X represents the number of times the spinner lands on 2

(b) (i) Find the probability that the spinner lands on 2 at least 7 times.

(ii) Find $P(4 \leqslant X < 8)$

(5)

5. Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$H_0 : p = 0.15$

$H_1 : p \geqslant 0.15$

Let X = the number of customers who buy chocolate.

$X \sim B(30, 0.15)$

$P(X = 8) = 0.0420$

$0.0420 < 0.05$ so reject H_0

There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

(a) Identify the first two errors that Julie has made in her hypothesis test. **(2)**

(b) Explain whether or not these errors will affect the conclusion of her hypothesis test. Give a reason for your answer. **(1)**

(c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05 **(2)**

(d) Find the actual level of significance of this test. **(1)**

5. Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales call that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

(4)

5. (a) The discrete random variable $X \sim B(40, 0.27)$

Find $P(X \geqslant 16)$

(2)

Past records suggest that 30% of customers who buy baked beans from a large supermarket buy them in single tins. A new manager suspects that there has been a change in the proportion of customers who buy baked beans in single tins. A random sample of 20 customers who had bought baked beans was taken.

(b) Write down the hypotheses that should be used to test the manager's suspicion.

(1)

(c) Using a 10% level of significance, find the critical region for a two-tailed test to answer the manager's suspicion. You should state the probability of rejection in each tail, which should be less than 0.05

(3)

(d) Find the actual significance level of a test based on your critical region from part (c).

(1)

One afternoon the manager observes that 12 of the 20 customers who bought baked beans, bought their beans in single tins.

(e) Comment on the manager's suspicion in the light of this observation.

(1)

Later it was discovered that the local scout group visited the supermarket that afternoon to buy food for their camping trip.

(f) Comment on the validity of the model used to obtain the answer to part (e), giving a reason for your answer.

(1)

Leave blank

2. A fair coin is spun 6 times and the random variable T represents the number of tails obtained.

(a) Give two reasons why a binomial model would be a suitable distribution for modelling T.

(2)

(b) Find $P(T = 5)$

(2)

(c) Find the probability of obtaining more tails than heads.

(2)

A second coin is biased such that the probability of obtaining a head is $\frac{1}{4}$

This second coin is spun 6 times.

(d) Find the probability that, for the second coin, the number of heads obtained is greater than or equal to the number of tails obtained.

(3)

Leave blank

3. A single observation x is to be taken from $X \sim B(12, p)$

This observation is used to test $H_0: p = 0.45$ against $H_1: p > 0.45$

(a) Using a 5% level of significance, find the critical region for this test.
(2)

(b) State the actual significance level of this test.
(1)

The value of the observation is found to be 9

(c) State the conclusion that can be made based on this observation.
(1)

(d) State whether or not this conclusion would change if the same test was carried out at the

(i) 10% level of significance,

(ii) 1% level of significance.
(2)

Leave blank

5. A company claims that 35% of its peas germinate. In order to test this claim Ann decides to plant 15 of these peas and record the number which germinate.

(a) (i) State suitable hypotheses for a two-tailed test of this claim.

(ii) Using a 5% level of significance, find an appropriate critical region for this test. The probability in each of the tails should be as close to 2.5% as possible.

(4)

(b) Ann found that 8 of the 15 peas germinated. State whether or not the company's claim is supported. Give a reason for your answer.

(2)

(c) State the actual significance level of this test.

(1)

Leave blank

2. Bill owns a restaurant. Over the next four weeks Bill decides to carry out a sample survey to obtain the customers' opinions.

(a) Suggest a suitable sampling frame for the sample survey.

(1)

(b) Identify the sampling units.

(1)

(c) Give one advantage and one disadvantage of taking a census rather than a sample survey.

(2)

Bill believes that only 30% of customers would like a greater choice on the menu. He takes a random sample of 50 customers and finds that 20 of them would like a greater choice on the menu.

(d) Test, at the 5% significance level, whether or not the percentage of customers who would like a greater choice on the menu is more than Bill believes. State your hypotheses clearly.

(6)