1. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

12.

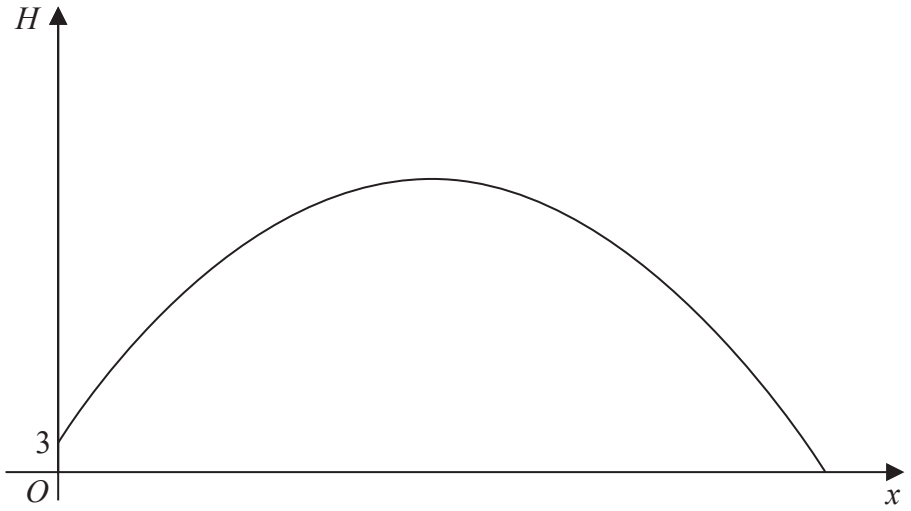

Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express $f(x)$ in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2)

6.

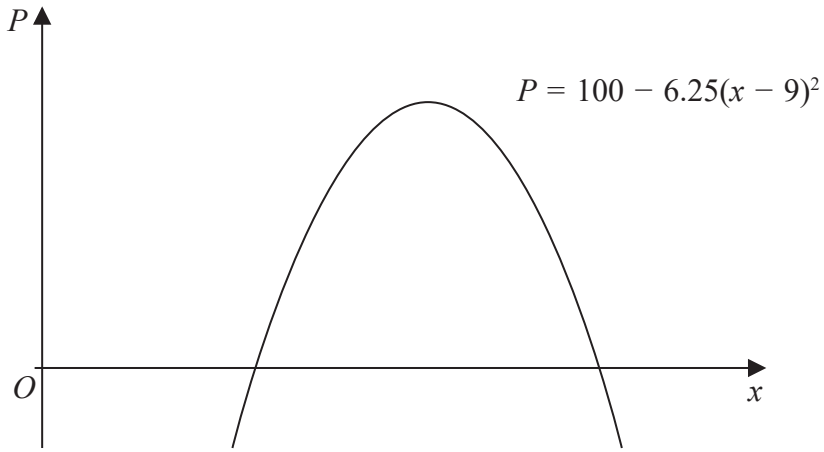

Figure 1

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2$$

where P is the profit measured in thousands of pounds and x is the selling price of the toy in pounds.

A sketch of P against x is shown in Figure 1.

Using the model,

(a) explain why £15 is not a sensible selling price for the toy.

(2)

Given that the company made an annual profit of more than £80 000

(b) find, according to the model, the least possible selling price for the toy.

(3)

The company wishes to maximise its annual profit.

State, according to the model,

(c) (i) the maximum possible annual profit,

(ii) the selling price of the toy that maximises the annual profit.

(2)

9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020,

(1)

(b) deduce the maximum total mass of tin that could be mined,

(1)

(c) calculate the mass of tin that will be mined in 2023.

(2)

(d) State, giving reasons, the limitation on the values of n.

(2)

10. The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that

$$0 \leqslant k < \frac{3}{4}$$

(4)

(Total for Question 10 is 4 marks)

Leave blank

11. The equation $5x^2 + 6 = k(13x^2 - 12x)$, where k is a constant, has two distinct real roots.

(a) Show that k satisfies the inequality

$$6k^2 + 13k - 5 > 0$$

(4)

(b) Find the set of possible values for k.

(4)

Leave blank

2. Find the range of values of x for which

(a) $4(x - 2) \leqslant 2x + 1$

(2)

(b) $(2x - 3)(x + 5) > 0$

(3)

(c) **both** $4(x - 2) \leqslant 2x + 1$ **and** $(2x - 3)(x + 5) > 0$

(1)

Leave blank

13. The equation $k(3x^2 + 8x + 9) = 2 - 6x$, where k is a real constant, has no real roots.

(a) Show that k satisfies the inequality

$$11k^2 - 30k - 9 > 0$$

(4)

(b) Find the range of possible values for k.

(4)

Leave blank

9. The equation $x^2 + (6k + 4)x + 3 = 0$, where k is a constant, has no real roots.

(a) Show that k satisfies the inequality

$$9k^2 + 12k + 1 < 0$$

(3)

(b) Find the range of possible values for k, giving your boundaries as fully simplified surds.

(4)

Leave blank

13. The curve C has equation

$$y = 3x^2 - 4x + 2$$

The line l_1 is the normal to the curve C at the point $P(1, 1)$

(a) Show that l_1 has equation

$$x + 2y - 3 = 0$$

(5)

The line l_1 meets curve C again at the point Q.

(b) By solving simultaneous equations, determine the coordinates of the point Q.

(4)

Another line l_2 has equation $kx + 2y - 3 = 0$, where k is a constant.

(c) Show that the line l_2 meets the curve C once only when

$$k^2 - 16k + 40 = 0$$

(4)

(d) Find the two exact values of k for which l_2 is a tangent to C.

(2)

Leave blank

10. The equation

$$kx^2 + 4x + k = 2, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(a) Show that k satisfies

$$k^2 - 2k - 4 < 0$$

(4)

(b) Hence find the set of all possible values of k.

(3)

Leave blank

8. Find the range of values of k for which the quadratic equation

$$kx^2 + 8x + 2(k + 7) = 0$$

has no real roots.

(7)

Leave blank

5. $f(x) = x^2 - 8x + 19$

(a) Express $f(x)$ in the form $(x + a)^2 + b$, where a and b are constants.

(2)

The curve C with equation $y = f(x)$ crosses the y-axis at the point P and has a minimum point at the point Q.

(b) Sketch the graph of C showing the coordinates of point P and the coordinates of point Q.

(3)

(c) Find the distance PQ, writing your answer as a simplified surd.

(3)

Leave blank

6. (a) Given $y = 2^x$, show that

$$2^{2x+1} - 17(2^x) + 8 = 0$$

can be written in the form

$$2y^2 - 17y + 8 = 0$$

(2)

(b) Hence solve

$$2^{2x+1} - 17(2^x) + 8 = 0$$

(4)

Leave blank

8. The straight line with equation $y = 3x - 7$ does not cross or touch the curve with equation $y = 2px^2 - 6px + 4p$, where p is a constant.

(a) Show that $4p^2 - 20p + 9 < 0$

(4)

(b) Hence find the set of possible values of p.

(4)