8. The curve C has equation

$$px^3 + qxy + 3y^2 = 26$$

where p and q are constants.

(a) Show that

$$\frac{dy}{dx} = \frac{apx^2 + bqy}{qx + cy}$$

where a, b and c are integers to be found.

(4)

Given that

- the point $P(-1, -4)$ lies on C
- the normal to C at P has equation $19x + 26y + 123 = 0$

(b) find the value of p and the value of q.

(5)

9.

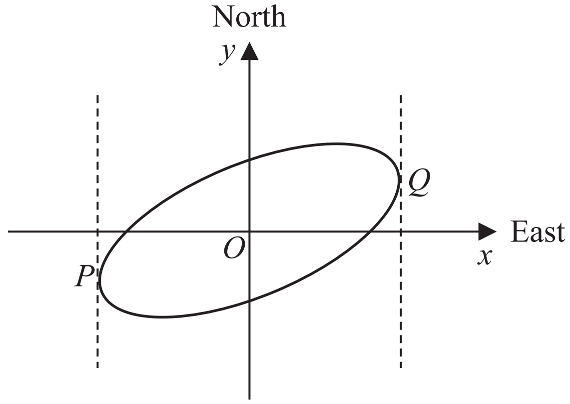

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

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5. (a) Prove, by using logarithms, that

$$\frac{d}{dx}(2^x) = 2^x \ln 2$$

(3)

The curve C has the equation

$$2x + 3y^2 + 3x^2y + 12 = 4 \times 2^x$$

The point P, with coordinates (2, 0), lies on C.

(b) Find an equation of the tangent to C at P.

(6)

Leave blank

2. A curve C has the equation

$$x^3 - 3xy - x + y^3 - 11 = 0$$

Find an equation of the tangent to C at the point $(2, -1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

1. A curve has equation

$$4x^2 - y^2 + 2xy + 5 = 0$$

The points P and Q lie on the curve.

Given that $\dfrac{dy}{dx} = 2$ at P and at Q,

(a) use implicit differentiation to show that $y - 6x = 0$ at P and at Q.

(6)

(b) Hence find the coordinates of P and Q.

(3)

Leave blank

3. A curve C has equation

$$3^x + 6y = \frac{3}{2}xy^2$$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates (2, 3). Give your answer in the form $\frac{a + \ln b}{8}$, where a and b are integers.

(7)

Leave blank

2. The point P with coordinates $\left(\frac{\pi}{2}, 1\right)$ lies on the curve with equation

$$4x\sin x = \pi y^2 + 2x, \qquad \frac{\pi}{6} \leqslant x \leqslant \frac{5\pi}{6}$$

Find an equation of the normal to the curve at P.

(6)

Leave blank

1. Find an equation of the tangent to the curve

$$x^3 + 3x^2y + y^3 = 37$$

at the point (1, 3). Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

Leave blank

1. A curve C has equation

$$3x^2 + 2xy - 2y^2 + 4 = 0$$

Find an equation for the tangent to C at the point (2, 4), giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

Leave blank

2. The curve C has equation

$$y^3 + x^2y - 6x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence find the exact coordinates of the points on C for which $\frac{dy}{dx} = 0$

(6)

Leave blank

4. The curve C has equation

$$4x^2 - y^3 - 4xy + 2^y = 0$$

The point P with coordinates $(-2, 4)$ lies on C.

(a) Find the exact value of $\frac{dy}{dx}$ at the point P.

(6)

The normal to C at P meets the y-axis at the point A.

(b) Find the y coordinate of A, giving your answer in the form $p + q\ln 2$, where p and q are constants to be determined.

(3)

Leave blank

3. The curve C has equation

$$2x^2y + 2x + 4y - \cos(\pi y) = 17$$

(a) Use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

The point P with coordinates $\left(3, \frac{1}{2}\right)$ lies on C.

The normal to C at P meets the x-axis at the point A.

(b) Find the x coordinate of A, giving your answer in the form $\frac{a\pi + b}{c\pi + d}$, where a, b, c and d are integers to be determined.

(4)

Leave blank

2. The curve C has equation

$$3^{x-1} + xy - y^2 + 5 = 0$$

Show that $\dfrac{dy}{dx}$ at the point (1, 3) on the curve C can be written in the form $\dfrac{1}{\lambda}\ln(\mu e^3)$, where λ and μ are integers to be found.

(7)