13.

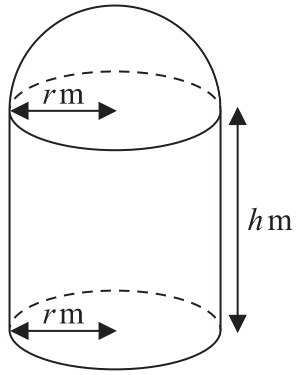

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is $6\,\text{m}^3$.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

Leave blank

14.

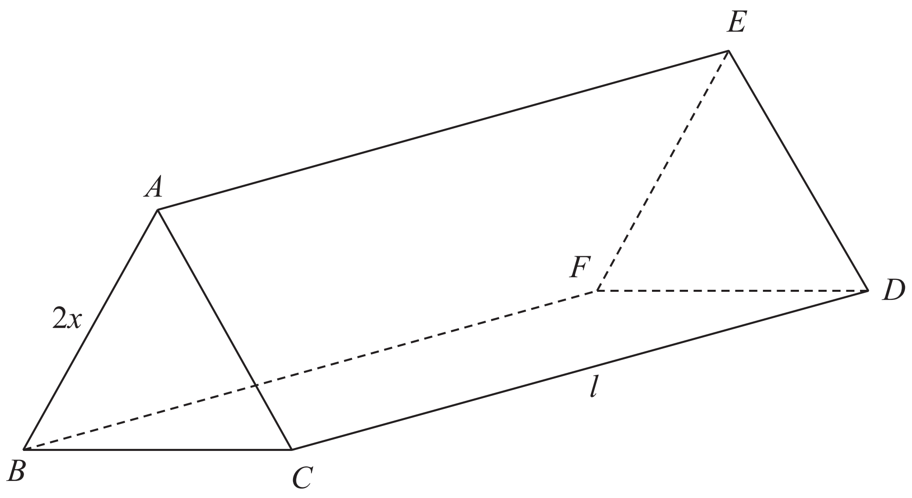

Figure 6

Figure 6 shows a solid triangular prism *ABCDEF* in which $AB = 2x$ cm and $CD = l$ cm.

The cross section *ABC* is an equilateral triangle.

The rectangle *BCDF* is horizontal and the triangles *ABC* and *DEF* are vertical.

The total surface area of the prism is S cm^2 and the volume of the prism is V cm^3.

(a) Show that $S = 2x^2\sqrt{3} + 6xl$

(3)

Given that $S = 960$,

(b) show that $V = 160x\sqrt{3} - x^3$

(5)

(c) Use calculus to find the maximum value of V, giving your answer to the nearest integer.

(5)

(d) Justify that the value of V found in part (c) is a maximum.

(2)

Leave blank

16. [*In this question you may assume the formula for the area of a circle and the following formulae:*

a **sphere** *of radius r has volume* $V = \frac{4}{3}\pi r^3$ *and surface area* $S = 4\pi r^2$

a **cylinder** *of radius r and height h has volume* $V = \pi r^2 h$ *and curved surface area* $S = 2\pi rh$]

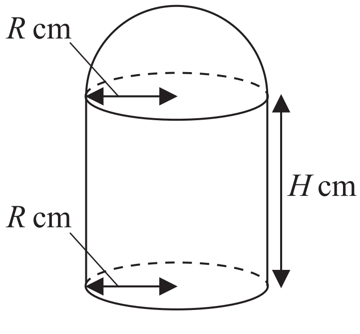

Figure 5

Figure 5 shows the model for a building. The model is made up of three parts. The roof is modelled by the curved surface of a hemisphere of radius *R* cm. The walls are modelled by the curved surface of a circular cylinder of radius *R* cm and height *H* cm. The floor is modelled by a circular disc of radius *R* cm. The model is made of material of negligible thickness, and the walls are perpendicular to the base.

It is given that the volume of the model is 800π cm^3 and that $0 < R < 10.6$

(a) Show that

$$H = \frac{800}{R^2} - \frac{2}{3}R$$

(2)

(b) Show that the surface area, A cm^2, of the model is given by

$$A = \frac{5\pi R^2}{3} + \frac{1600\pi}{R}$$

(3)

(c) Use calculus to find the value of *R*, to 3 significant figures, for which *A* is a minimum.

(5)

(d) Prove that this value of *R* gives a minimum value for *A*.

(2)

(e) Find, to 3 significant figures, the value of *H* which corresponds to this value for *R*.

(1)

Leave blank

15.

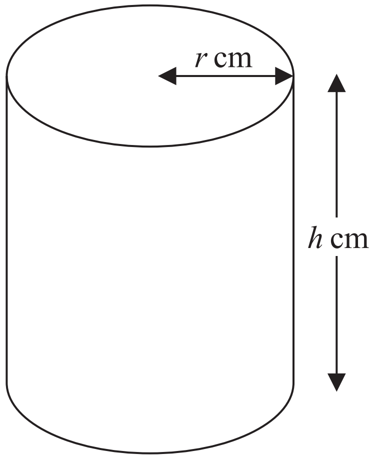

Figure 5

Figure 5 shows a design for a water barrel.

It is in the shape of a right circular cylinder with height h cm and radius r cm.

The barrel has a base but has no lid, is open at the top and is made of material of negligible thickness.

The barrel is designed to hold 60 000 cm^3 of water when full.

(a) Show that the total external surface area, S cm^2, of the barrel is given by the formula

$$S = \pi r^2 + \frac{120\,000}{r}$$

(3)

(b) Use calculus to find the minimum value of S, giving your answer to 3 significant figures.

(6)

(c) Justify that the value of S you found in part (b) is a minimum.

(2)

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15.

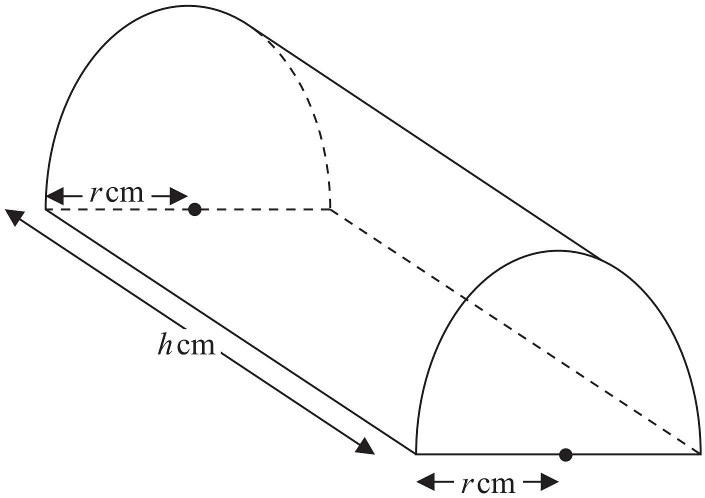

Figure 4

Figure 4 shows a solid wooden block. The block is a right prism with length h cm. The cross-section of the block is a semi-circle with radius r cm.

The total surface area of the block, including the curved surface, the two semi-circular ends and the rectangular base, is $200\,\text{cm}^2$

(a) Show that the volume $V\,\text{cm}^3$ of the block is given by

$$V = \frac{\pi r(200 - \pi r^2)}{4 + 2\pi}$$

(5)

(b) Use calculus to find the maximum value of V. Give your answer to the nearest cm^3.

(6)

(c) Justify, by further differentiation, that the value of V that you have found is a maximum.

(2)

Leave blank

9.

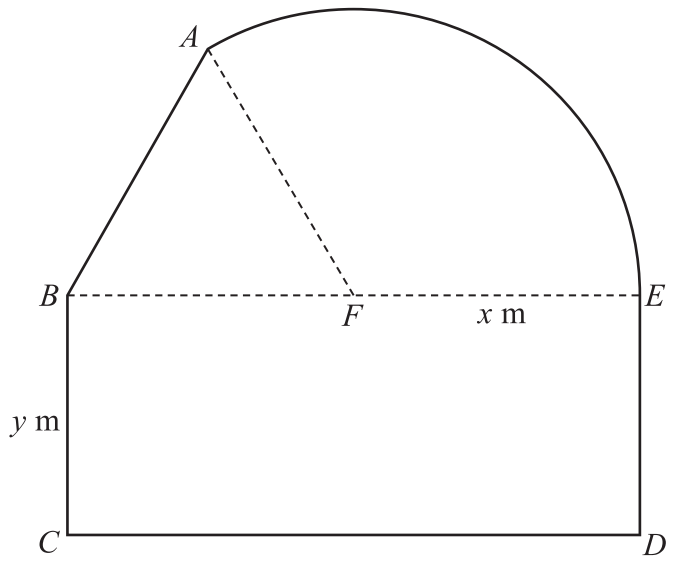

Diagram not drawn to scale

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F.*

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.
(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\,m^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre.
(5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum.
(2)

Leave blank

9. A solid glass cylinder, which is used in an expensive laser amplifier, has a volume of 75π cm^3.
The cost of polishing the surface area of this glass cylinder is £2 per cm^2 for the curved surface area and £3 per cm^2 for the circular top and base areas.

Given that the radius of the cylinder is r cm,

(a) show that the cost of the polishing, £C, is given by

$$C = 6\pi r^2 + \frac{300\pi}{r}$$

(4)

(b) Use calculus to find the minimum cost of the polishing, giving your answer to the nearest pound.

(5)

(c) Justify that the answer that you have obtained in part (b) is a minimum.

(1)

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10.

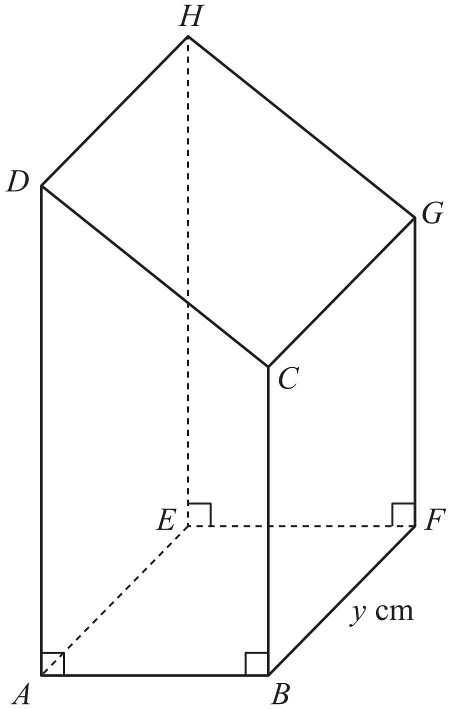

Figure 4

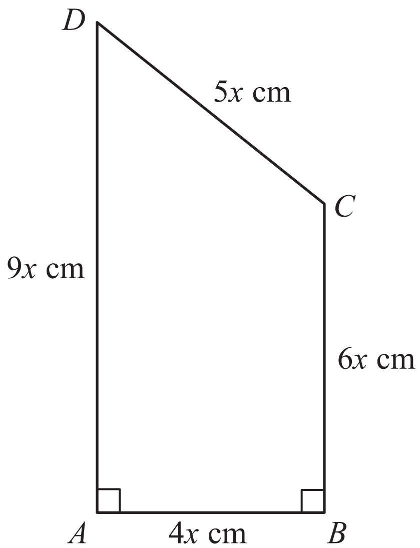

Figure 5

Figure 4 shows a closed letter box *ABFEHGCD,* which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 4. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 5.
The angle $DAB = 90°$ and the angle $ABC = 90°$.

The volume of the letter box is 9600 cm^3.

(a) Show that

$$y = \frac{320}{x^2}$$

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by

$$S = 60x^2 + \frac{7680}{x}$$

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

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8.

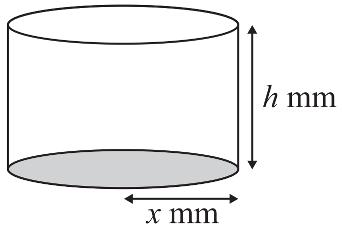

Figure 3

A manufacturer produces pain relieving tablets. Each tablet is in the shape of a solid circular cylinder with base radius x mm and height h mm, as shown in Figure 3.

Given that the volume of each tablet has to be 60 mm^3,

(a) express h in terms of x,

(1)

(b) show that the surface area, A mm^2, of a tablet is given by $A = 2\pi x^2 + \dfrac{120}{x}$

(3)

The manufacturer needs to minimise the surface area A mm^2, of a tablet.

(c) Use calculus to find the value of x for which A is a minimum.

(5)

(d) Calculate the minimum value of A, giving your answer to the nearest integer.

(2)

(e) Show that this value of A is a minimum.

(2)

Leave blank

8.

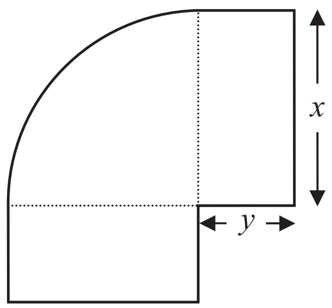

Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2,

(a) show that

$$y = \frac{16 - \pi x^2}{8x}$$

(3)

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x$$

(3)

(c) Use calculus to find the minimum value of P.

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)