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Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE
in Core Mathematics C2 (6664/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

<u>Method mark for solving 3 term quadratic:</u>

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = …

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\ \ q \neq 0$, leading to x = …

<u>Method marks for differentiation and integration:</u>

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

May 2015
6664 Core Mathematics C2
Mark Scheme

Question Number	Scheme	Marks
1.	$\left(2-\frac{x}{4}\right)^{10}$	
Way 1	$2^{10}+\underline{\underline{\binom{10}{1}}}2^9\left(-\frac{1}{4}\underline{\underline{x}}\right)+\underline{\underline{\binom{10}{2}}}2^8\left(-\frac{1}{4}\underline{\underline{x}}\right)^2+\ldots$ For **either** the x term **or** the x^2 term including a correct binomial coefficient with a correct power of x	M1
	First term of 1024	B1
	$=\underline{1024}-1280x+720x^2$ **Either** $-1280x$ **or** $720x^2$ (Allow +-1280x here)	A1
	Both $-1280x$ and $720x^2$ (Do not allow +-1280x here)	A1 **[4]**
Way 2	$\left(2-\frac{x}{4}\right)^{10}=2^k\left(1-\underline{\underline{10}}\times\frac{\underline{\underline{x}}}{8}+\underline{\underline{\frac{10\times9}{2}}}\left(-\frac{\underline{\underline{x}}}{8}\right)^2\right)$	M1
	$1024(1\pm.......)$	
	$=\underline{1024}-1280x+720x^2$	B1A1 A1 **[4]**

Notes

M1: For either the x term or the x^2 term having correct structure i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. The powers of 2 or of ¼ may be wrong or missing.

B1: Award this for 1024 when first seen as a distinct constant term (not $1024x^0$) and not 1 + 1024

A1: For one correct term in x with coefficient simplified. Either -1280x or $720x^2$ (allow +-1280x here)

Allow $720x^2$ to come from $\left(\frac{x}{4}\right)^2$ with no negative sign. So use of + sign throughout could give M1 B1 A1 A0

A1: For both correct simplified terms i.e. -1280x and $720x^2$ (**Do not** allow +-1280x here)

Allow terms to be listed for full marks e.g. $\underline{1024}$, $-1280x$, $+720x^2$

N.B. If they follow a correct answer by a factor such as $512-640x+360x^2$ then isw

Terms may be listed. Ignore any extra terms.

Notes for Way 2

M1: Correct structure for at least one of the underlined terms. i.e. a correct binomial coefficient in any form with the correct power of x. Condone sign errors and condone missing brackets and allow alternative forms for binomial coefficients e.g. ${}^{10}C_1$ or $\binom{10}{1}$ or even $\left(\frac{10}{1}\right)$ or 10. k may even be 0 or 2^k may not be seen. Just consider the bracket for this mark.

B1: Needs 1024(1…. To become 1024

A1, A1: as before

Question Number	Scheme	Marks
2 (a)	**Way 1:** $(x \mp 2)^2 + (y \pm 1)^2 = k$, $k > 0$ **Way 2:** $x^2 + y^2 \mp 4x \pm 2y + c = 0$	M1
	Way 1: Attempts to use $r^2 = (4-2)^2 + (-5+1)^2$ **Way 2:** $4^2 + (-5)^2 - 4\times 4 + 2\times -5 + c = 0$	M1
	Way 1: Obtains $(x-2)^2 + (y+1)^2 = 20$ **Way 2:** $x^2 + y^2 - 4x + 2y - 15 = 0$	A1 **(3)**
	N.B. Special case: $(x-2)^2 - (y+1)^2 = 20$ is not a circle equation but earns M0M1A0	
(b) Way 1	Gradient of radius from centre to (4, -5) = -2 (must be correct)	B1
	Tangent gradient = $-\dfrac{1}{\text{their numerical gradient of radius}}$	M1
	Equation of tangent is $(y+5) = '\tfrac{1}{2}'(x-4)$	M1
	So equation is $x - 2y - 14 = 0$ (or $2y - x + 14 = 0$ or other integer multiples of this answer)	A1 **(4)**
b)Way 2	Quotes $xx' + yy' - 2(x+x') + (y+y') - 15 = 0$ and substitutes (4, -5)_	B1
	$4x - 5y - 2(x+4) + (y-5) - 15 = 0$ so $2x - 4y - 28 = 0$ (or alternatives as in Way 1)	M1,M1A1 **(4)**
b)Way 3	Use differentiation to find expression for gradient of circle	
	Either $2(x-2) + 2(y+1)\dfrac{dy}{dx} = 0$ **or** states $y = -1 - \sqrt{20-(x-2)^2}$ so $\dfrac{dy}{dx} = \dfrac{(x-2)}{\sqrt{20-(x-2)^2}}$	B1
	Substitute $x = 4$, $y = -5$ after valid differentiation to give gradient =	M1
	Then as Way 1 above $(y+5) = '\tfrac{1}{2}'(x-4)$ so $x - 2y - 14 = 0$	M1 A1 **(4)**
		[7]

Notes

(a) **M1:** Uses centre to write down equation of circle in one of these forms. There may be sign slips as shown.

M1: Attempts distance between two points **to establish** r^2 (independent of first M1)- allow one sign slip only using distance formula with -5 or -1, usually $(-5-1)$ in 2nd bracket. Must not identify this distance as diameter.

This mark may alternatively (e.g. way 2)be given for substituting (4, -5) into a **correct circle** equation with one unknown

Can be awarded for $r = \sqrt{20}$ or for $r^2 = 20$ stated or implied but not for $r^2 = \sqrt{20}$ or $r = 20$ or $r = \sqrt{5}$

A1: Either of the answers printed or correct equivalent e.g. $(x-2)^2 + (y+1)^2 = (2\sqrt{5})^2$ is A1 but $2\sqrt{5}^2$ (no bracket) is A0 unless there is recovery

Also $(x-2)^2 + (y-(-1))^2 = (2\sqrt{5})^2$ may be awarded M1M1A1as a correct equivalent.

N.B. $(x-2)^2 + (y+1)^2 = 40$ commonly arises from one sign error evaluating r and earns M1M1A0

(b) **Way 1:**

B1: Must be correct answer -2 if evaluated (otherwise may be implied by the following work)

M1: Uses negative reciprocal of their gradient

M1: Uses $y - y_1 = m(x - x_1)$ with (4,-5) and their **changed** gradient **or** uses $y = mx + c$ and (4, -5) with their changed gradient (not gradient of radius) to find c

A1: answers in scheme or multiples of these answers (must have " = 0"). NB Allow $1x - 2y - 14 = 0$

N.B. $(y+5) = '\tfrac{1}{2}'(x-4)$ following gradient of is ½ after errors leads to $x - 2y - 14 = 0$ but is worth B0M0M0A0

Way 2: Alternative method (b) is rare.

Way 3: Some may use implicit differentiation to differentiate- others may attempt to make y the subject and use chain rule

B1: the differentiation must be accurate and the algebra accurate too. Need to take (-) root not (+)root in the alternative

M1: Substitutes into their gradient function but must follow valid accurate differentiation

M1: Must use "their" tangent gradient and $y + 5 = m(x - 4)$ but allow over simplified attempts at differentiation for this mark.

A1: As in Way 1

Question Number	Scheme	Marks
3.	$\mathrm{f}(x) = 6x^3 + 3x^2 + Ax + B$	
Way 1 (a)	Attempting $\mathrm{f}(1) = 45$ or $\mathrm{f}(-1) = 45$	M1
	$\mathrm{f}(-1) = -6 + 3 - A + B = 45$ or $-3 - A + B = 45 \Rightarrow B - A = 48$ * (allow $48 = B - A$)	A1 * **cso** **(2)**
Way 1 (b)	Attempting $\mathrm{f}(-\frac{1}{2}) = 0$	M1
	$6\left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)^2 + A\left(-\frac{1}{2}\right) + B = 0$ or $-\frac{1}{2}A + B = 0$ or $A = 2B$	A1 o.e.
	Solve to obtain $B = -48$ and $A = -96$	M1 A1 **(4)**
Way 2 (a)	Long Division $(6x^3 + 3x^2 + Ax + B) \div (x \pm 1) = 6x^2 + px + q$ and sets remainder = 45	M1
	Quotient is $6x^2 - 3x + (A + 3)$ and remainder is $B - A - 3 = 45$ so $B - A = 48$ *	A1*
Way 2 (b)	$(6x^3 + 3x^2 + Ax + B) \div (2x + 1) = 3x^2 + px + q$ and sets remainder = 0	M1
	Quotient is $3x^2 + \dfrac{A}{2}$ and remainder is $B - \dfrac{A}{2} = 0$	A1
	Then Solve to obtain $B = -48$ and $A = -96$ as in scheme above (Way 1)	M1 A1
(c)	Obtain $(3x^2 - 48)$, $(x^2 - 16)$, $(6x^2 - 96)$, $\left(3x^2 + \dfrac{A}{2}\right)$, $(3x^2 + B)$, $\left(x^2 + \dfrac{A}{6}\right)$ or $\left(x^2 + \dfrac{B}{3}\right)$ as factor or as quotient after division by $(2x + 1)$. Division by $(x+4)$ or $(x-4)$ see below	B1ft
	Factorises $(3x^2 - 48)$, $(x^2 - 16)$, $(48 - 3x^2)$, $(16 - x^2)$ or $(6x^2 - 96)$	M1
	$= 3(2x + 1)(x + 4)(x - 4)$ (if this answer follows from a wrong A or B then award A0) isw if they go on to solve to give $x = 4, -4$ and $-1/2$	A1cso **(3) [9]**

Notes

(a) Way 1: **M1:** 1 or –1 substituted into f(x) and expression put equal to ±45

A1*: Answer is given. Must have substituted -1 and put expression equal to +45.
Correct equation with powers of -1 evaluated and conclusion with no errors seen.

Way 2: **M1**: Long division as far as a remainder which is set equal to ±45

A1*: See correct quotient and correct remainder and printed answer obtained with no errors

(b) Way 1: **M1**: Must see $\mathrm{f}(-\frac{1}{2})$ and "= 0" unless subsequent work implies this.

A1: Give credit for a correct equation **even unsimplified** when first seen, then isw.
A correct equation implies M1A1.

M1: Attempts to solve the **given equation from part (a)** and their simplified or unsimplified linear equation in A and B from part (b) as far as A =... or B = ...(must eliminate one of the constants but algebra need not be correct for this mark). May just write down the correct answers.

A1: **Both** A and B correct

Way 2: **M1**: Long division as far as a remainder which is set equal to 0

A1: See correct quotient and correct remainder put equal to 0

M1A1: As in Way 1

There may be a mixture of Way 1 for (a) and Way 2 for (b) or vice versa.

(c) **B1**: May be written straight down or from long division, inspection, comparing coefficients or pairing terms

M1: Valid attempt to factorise a **listed** quadratic (see general notes) so $(3x - 16)(x + 3)$ could get M1A0

A1cso: (Cannot be awarded if A or B is wrong) Needs the answer in the scheme or $-3(2x+1)(4+x)(4 - x)$ or equivalent but factor 3 must be shown and there must be all the terms together with brackets.

Way 2: A minority might divide by (x- 4) or (x + 4) obtaining $(6x^2 + 27x + 12)$ or $(6x^2 - 21x - 12)$ for B1

They then need to factorise $(6x^2 + 27x + 12)$ or $(6x^2 - 21x - 12)$ for M1

Then A1cso as before

Special cases:

If they write down f(x) = 3 (2x+1)(x+4)(x - 4) with no working, this is B1 M1 A1

But if they give f(x) = (2x+1)(x+4)(x - 4) with no working (from calculator?) give B1M0A0

And f(x) = (2x + 1)(3x + 12)(x – 4) or f(x) = (6x + 3)(x + 4)(x – 4) or f(x) = (2x + 1)(x + 4)(3x – 12) is B1M1A0

Question Number	Scheme	Marks
4.(a)	In triangle *OCD* **complete method** used to find angle *COD* so: Either $\cos COD = \frac{8^2+8^2-7^2}{2\times8\times8}$ or uses $\angle COD = 2\times\arcsin\frac{3.5}{8}$ oe **so** $\angle COD =$	M1
	($\angle COD = 0.9056(331894)$) = 0.906 (3sf) * accept awrt 0.906	A1 * **(2)**
(b)	Uses *s* = 8θ for any θ in radians or $\frac{\theta}{360}\times2\pi\times8$ for any θ in degrees	M1
	$\theta = \frac{\pi - "COD"}{2}$ $(= awrt\ 1.12)$ or $2\theta(= awrt\ 2.24)$ and Perimeter = 23+(16 $\times\theta$)	M1
	accept awrt 40.9 (cm)	A1 **(3)**
(c)	Either Way 1: (Use of Area of two sectors + area of triangle) Area of triangle = $\frac{1}{2}\times8\times8\times\sin0.906$ (or 25.1781155 accept awrt 25.2)or $\frac{1}{2}\times8\times7\times\sin1.118$ or $\frac{1}{2}\times7\times h$ after *h* calculated from correct Pythagoras or trig.	M1
	Area of sector = $\frac{1}{2}8^2\times"1.117979732"$ (or 35.77535142 accept awrt 35.8)	M1
	Total Area = Area of two sectors + area of triangle =awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 **(3)**
	Or Way 2: (Use of area of semicircle – area of segment) Area of semi-circle = $\frac{1}{2}\times\pi\times8\times8$ (or 100.5)	M1
	Area of segment = $\frac{1}{2}8^2\times("0.906"-\sin"0.906")$ (or 3.807)	M1
	So area required = awrt 96.7 or 96.8 or 96.9 (cm^2)	A1 **(3)** **[8]**

Notes

(a) **M1**: Either use correctly quoted cosine rule – may quote as $7^2 = 8^2 + 8^2 - 2\times8\times8\cos\alpha \Rightarrow \alpha = ...$
Or split isosceles triangle into two right angled triangles and use arcsin or longer methods using Pythagoras and arcos (i.e. $\pi - 2\times\arccos\frac{3.5}{8}$). **There are many ways of showing this result.**

Must conclude that $\angle COD =$

A1*: (NB this is a given answer) If any errors or over-approximation is seen this is A0. It needs correct work **leading to stated answer** of 0.906 or awrt 0.906 for A1. The cosine of *COD* is equal to 79/128 or awrt 0.617. Use of 0.62 (2sf) does not lead to printed answer. They may give 51.9 in degrees then convert to radians. This is fine.

The minimal solution $7^2 = 8^2 + 8^2 - 2\times8\times8\cos\alpha \Rightarrow \alpha = ...$ 0.906 (with no errors seen) can have M1A1 but errors rearranging result in M1A0

(b) **M1**: Uses formula for arc length with *r* = 8 and any angle i.e. *s* = 8θ if working in rads or *s* = $\frac{\theta}{360}\times2\pi\times8$ in degrees

(If the formula is quoted with *r* the 8 may be implied by the value of their $r\theta$)

M1: Uses angles on straight line (or other geometry) to find angle *BOC* or *AOD* and uses Perimeter = 23 + arc lengths *BC* and *AD* (may make a slip – in calculation or miscopying)

A1: correct work leading to awrt 40.9 not 40.8 (do not need to see cm) This answer implies M1M1A1

(c) Way 1: **M1**: Mark is given for **correct** statement of area of triangle $\frac{1}{2}\times8\times8\times\sin0.906$ (must use correct angle) or for correct answer (awrt 25.2) Accept alternative correct methods using Pythagoras and ½ base×height

M1: Mark is given for formula for area of sector $\frac{1}{2}8^2\times"1.117979732"$ with *r* = 8 and their angle *BOC* or *AOD* or (*BOC* + *AOD)* **not** *COD* . May use $A = \frac{\theta}{360}\times\pi\times8^2$ **if working in degrees**

A1: Correct work leading to awrt 96.7, 96.8 or 96.9 (This answer implies M1M1A1)

NB. Solution may combine the two sectors for part (b) and (c) and so might use $2\times\angle BOC$ rather than $\angle BOC$

Way 2: **M1**: Mark is given for **correct** statement of area of semicircle $\frac{1}{2}\times\pi\times8\times8$ or for correct answer 100.5

M1: Mark is given for formula for area of segment $\frac{1}{2}8^2\times("0.906"-\sin"0.906")$ with *r* = 8 or 3.81 **A1**: As in Way 1

Question Number	Scheme	Marks
5.(i)	**Mark (a) and (b) together**	
(a)	$a+ar=34$ or $\dfrac{a(1-r^2)}{(1-r)}=34$ or $\dfrac{a(r^2-1)}{(r-1)}=34$; $\quad \dfrac{a}{1-r}=162$	B1; B1
(Way 1)	Eliminate a to give $(1+r)(1-r)=\dfrac{17}{81}$ or $1-r^2=\dfrac{34}{162}$.. (not a cubic)	aM1
	(and so $r^2=\frac{64}{81}$ and) $r=\frac{8}{9}$ only	aA1 **(4)**
(b)	Substitute their $r=\frac{8}{9}$ ($0<r<1$) to give $a=$	bM1
	$a = 18$	bA1 **(2)**
(Way 2) Part (b) first	Eliminate r to give $\dfrac{34-a}{a}=1-\dfrac{a}{162}$	bM1
	gives $a = 18$ or 306 and rejects 306 to give $a = 18$	bA1
Then part (a) again	Substitute a = 18 to give $r =$	aM1
	$r=\frac{8}{9}$	aA1
(ii)	$\dfrac{42(1-\frac{6}{7}^n)}{1-\frac{6}{7}}>290$ (For trial and improvement approach see notes below)	M1
	to obtain So $(\frac{6}{7})^n<(\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n>(\frac{294}{4})$ or $(\frac{6}{7})^n<(\frac{2}{147})$	A1
	So $n>\dfrac{\log"(\frac{4}{294})"}{\log(\frac{6}{7})}$ or $\log_{\frac{6}{7}}"(\frac{4}{294})"$ or equivalent but must be log of positive quantity	M1
	(i.e. $n>$ 27.9) so $n = 28$	A1 **(4)**

Notes

(i) (a) **B1**: Writes **a correct** equation connecting a and r and 34 (allow equivalent equations – may be implied)
B1: Writes a **correct** equation connecting a and r and 162 (allow equivalent equation – may be implied)

Way 1: a**M1**: Eliminates a correctly for these two equations to give $(1+r)(1-r)=\dfrac{17}{81}$ or $(1+r)(1-r)=\dfrac{34}{162}$ or equivalent –
not a cubic – should have factorized (1 - r) to give a correct quadratic
a**A1:** Correct value for r. Accept 0.8 recurring or 8/9 (not 0.889) Must only have positive value.

b**M1**: Substitutes their r ($0<r<1$) into a correct formula to give value for a. Can be implied by $a = 18$
b**A1**: must be 18 (not answers which round to 18)

Way 2: Finds *a* first - B1, B1: As before then award the (b) M and A marks before the (a) M and A marks

b**M1**: Eliminates r correctly to give $\dfrac{34-a}{a}=1-\dfrac{a}{162}$ or $a^2-324a+5508=0$ or equivalent
b**A1:** Correct value for a so $a = 18$ only. (Only award after 306 has been rejected)
a**M1**: Substitutes their 18 to give $r =$
a**A1**: $r=\frac{8}{9}$ only

(ii) **M1:** Allow n **or** n – 1 and any symbols from ">", "<", or "=" etc **A1 :** Must be power n (**not** n – 1) with any symbol
M1: Uses logs correctly on $(\frac{6}{7})^n$ or $(\frac{7}{6})^n$ **not on** $(36)^n$ to get as far as n Allow any symbol
A1: n = 28 cso (any errors with inequalities earlier e.g. failure to reverse the inequality when dividing by the negative $\log(\frac{6}{7})$ or any contradictory statements must be penalised here) Those with equals throughout may gain this mark if they follow 27.9 by n=28. Just n = 28 without mention of 27.9 is only allowed following correct inequality work.

Special case: Trial and improvement: Gives n = 28 as S = awrt 290.1 (M1A1)and when n = 27 S = (awrt) 289 so n = 28 (M1A1) – n = 28 with no working is M1A0M0A0 and insufficient accuracy is M1A0M1A0
Uses nth term instead of sum of n terms – over simplified – do not treat as misread – award 0/4

Question Number	Scheme	Marks
	May mark (a) and (b) together	
6. (a)	Expands to give $10x^{\frac{3}{2}} - 20x$	B1
	Integrates to give $\frac{10}{"\frac{5}{2}"}x^{"\frac{5}{2}"} + \frac{-"20"x^2}{2}$ (+ c)	M1 A1ft
	Simplifies to $4x^{\frac{5}{2}} - 10x^2 (+ c)$	A1cao **(4)**
(b)	Use limits 0 and 4 either way round on their integrated function (may only see 4 substituted)	M1
	Use limits 4 and 9 either way round on their integrated function	dM1
	Obtains either $\pm$ –32 or $\pm$ 194 needs at least one of the previous M marks for this to be awarded	A1
	(So area = $\left\lvert\int_0^4 ydx\right\rvert + \int_4^9 ydx$) i.e. 32 + 194, = 226	ddM1,A1 **(5)** **[9]**

Notes

(a) **B1**: Expands the bracket correctly

M1: Correct integration process **on at least one term** after attempt at multiplication. (Follow correct expansion or one slip resulting in $10x^k - 20x$ where k may be $\frac{1}{2}$ or $\frac{5}{2}$ or resulting in $10x^{\frac{3}{2}} - Bx$, where B may be 2 or 5)

So $x^{\frac{3}{2}} \to \frac{x^{\frac{5}{2}}}{5/2}$ or $x^{\frac{1}{2}} \to \frac{x^{\frac{3}{2}}}{3/2}$ or $x^{\frac{5}{2}} \to \frac{x^{\frac{7}{2}}}{7/2}$ and/or $x \to \frac{x^2}{2}$.

A1: Correct unsimplified follow through for both terms of their integration. Does not need (+ c)

A1: Must be simplified and correct– allow answer in scheme or $4x^{2\frac{1}{2}} - 10x^2$. Does not need (+ c)

(b) **M1**: (does not depend on first method mark) Attempt to substitute 4 into their integral (however obtained but must not be differentiated) or seeing their evaluated number (usually 32) is enough – do not need to see minus zero.

d**M1:** (depends on first method mark in (a)) Attempt to subtract either way round using the limits 4 and 9 $A\times 9^{\frac{5}{2}} - B\times 9^2$ with $A\times 4^{\frac{5}{2}} - B\times 4^2$ is enough – or seeing 162 –(-32) {but not 162 – 32 }

A1: At least one of the values (32 and 194) correct (needs just one of the two previous M marks in (b)) or may see 162 + 32 + 32 or 162 + 64 or may be implied by correct final answer if not evaluated until last line of working

ddM1: Adds 32 and 194 (may see 162 + 32 + 32 or may be implied by correct final answer if not evaluated until last line of working). This depends on everything being correct to this point.

A1cao: Final answer of 226 not (- 226)

Common errors: $4\times 4^{\frac{5}{2}} - 10\times 4^2 + 4\times 9^{\frac{5}{2}} - 10\times 9^2 - 4\times 4^{\frac{5}{2}} - 10\times 4^2 = \pm$ 162 obtains M1 M1 A0 (neither 32 nor 194 seen and final answer incorrect) then M0 A0 so 2/5

Uses correct limits to obtain -32 +162 +32 = +/-162 is M1 M1 A1 (32 seen) M0 A0 so 3/5

Special case: In part (b) Uses limits 9 and 0 = 972 – 810 - 0 = 162 M0 M1 A0 M0A0 scores 1/5
This also applies if 4 never seen.

Question Number	Scheme	Marks
7. (i)	$8^{2x+1}=24$ $(2x+1)\log 8 = \log 24$ or $(2x+1)=\log_8 24$	or $8^{2x}=3$ and so $(2x)\log 8 = \log 3$ or $(2x)=\log_8 3$	M1
	$x=\frac{1}{2}\left(\frac{\log 24}{\log 8}-1\right)$ or $x=\frac{1}{2}\left(\log_8 24-1\right)$	$x=\frac{1}{2}\left(\frac{\log 3}{\log 8}\right)$ or $x=\frac{1}{2}\left(\log_8 3\right)$ o.e.	dM1
	=0.264	A1 **(3)**
(ii)	$\log_2(11y-3)-\log_2 3-2\log_2 y=1$	
	$\log_2(11y-3)-\log_2 3-\log_2 y^2=1$.	M1
	$\log_2\frac{(11y-3)}{3y^2}=1$ or $\log_2\frac{(11y-3)}{y^2}=1+\log_2 3=2.58496501$	dM1
	$\log_2\frac{(11y-3)}{3y^2}=\log_2 2$ or $\log_2\frac{(11y-3)}{y^2}=\log_2 6$ (allow awrt 6 if replaced by 6 later)	B1
	Obtains $6y^2-11y+3=0$ o.e. i.e. $6y^2=11y-3$ for example	A1
	Solves quadratic to give $y=$	ddM1
	$y=\frac{1}{3}$ and $\frac{3}{2}$ (need both- one should not be rejected)	A1 **(6)** **[9]**
Notes (i)	**M1**: Takes logs and uses law of powers correctly. (Any log base may be used) Allow lack of brackets. **dM1**: Make *x* subject of their formula correctly (may evaluate the log before subtracting 1 and calculate e.g. (1.528 -1)/2) **A1**: Allow answers which round to 0.264	
(ii)	**M1**: Applies power law of logarithms replacing $2\log_2 y$ by $\log_2 y^2$ **dM1**: Applies quotient or product law of logarithms correctly to the three log terms including term in y^2. (dependent on first M mark) or applies quotient rule to two terms and collects constants (allow "triple" fractions) $1+\log_2 3$ on RHS is not sufficient – need $\log_2 6$ or 2.58… e.g. $\log_2(11y-3)=\log_2 3+\log_2 y^2+\log_2 2$ becoming $\log_2(11y-3)=\log_2 6y^2$ **B1**: States or uses $\log_2 2=1$ or $2^1=2$ at any point in the answer so may be given for $\log_2(11y-3)-\log_2 3-2\log_2 y=\log_2 2$ or for $\frac{(11y-3)}{3y^2}=2$, for example (Sometimes this mark will be awarded before the second M mark, and it is possible to score M1M0B1in some cases) Or may be given for $\log_2 6=2.584962501..$ or $2^{2.584962501..}=6$ **A1**: This or equivalent quadratic equation (does not need to be in this form but should be equation) **ddM1:** (dependent on the two previous M marks) Solves their quadratic equation following reasonable log work using factorising, completion of square, formula or implied by both answers correct. **A1:** Any equivalent correct form – need both answers- allow awrt 0.333 for the answer 1/3 *NB: If "=0" is missing from the equation but candidate continues correctly and obtains correct answers then allow the penultimate A1 to be implied (Allow use of *x* or other varable instead of *y* throughout)	

Question Number	Scheme	Marks
8. (i)	Way 1: Divides by $\cos 3\theta$ to give $\tan 3\theta = \sqrt{3}$ so $(3\theta) = \frac{\pi}{3}$ — Or Way 2: Squares both sides, uses $\cos^2 3\theta + \sin^2 3\theta = 1$, obtains $\cos 3\theta = \pm\frac{1}{2}$ or $\sin 3\theta = \pm\frac{\sqrt{3}}{2}$ so $(3\theta) = \frac{\pi}{3}$	M1
	Adds π or 2π to previous value of angle (to give $\frac{4\pi}{3}$ or $\frac{7\pi}{3}$)	M1
	So $\theta = \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}$ (all three, no extra in range)	A1 **(3)**
(ii)(a)	$4(1-\cos^2 x) + \cos x = 4 - k$ Applies $\sin^2 x = 1 - \cos^2 x$	M1
	Attempts to solve $4\cos^2 x - \cos x - k = 0$, to give $\cos x =$	dM1
	$\cos x = \frac{1 \pm \sqrt{1+16k}}{8}$ or $\cos x = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{k}{4}}$ or other correct equivalent	A1 **(3)**
(b)	$\cos x = \frac{1 \pm \sqrt{49}}{8} = 1$ and $-\frac{3}{4}$ (see the note below if errors are made)	M1
	Obtains two solutions from 0 , 139 , 221 (0 or 2.42 or 3.86 in radians)	dM1
	x = 0 and 139 and 221 (allow awrt 139 and 221) must be in degrees	A1 **(3)**
		[9]

Notes

(i) **M1**: Obtains $\frac{\pi}{3}$. Allow $x = \frac{\pi}{3}$ or even $\theta = \frac{\pi}{3}$. Need not see working here. May be implied by $\theta = \frac{\pi}{9}$ in final answer (allow $(3\theta) = 1.05$ or $\theta = 0.349$ as decimals or $(3\theta) = 60$ or $\theta = 20$ as degrees for this mark)

Do not allow $\tan 3\theta = -\sqrt{3}$ nor $\tan 3\theta = \pm\frac{1}{\sqrt{3}}$

M1: Adding π or 2π to a previous value however obtained. It is not dependent on the previous mark.

(May be implied by final answer of $\theta = \frac{4\pi}{9}$ **or** $\frac{7\pi}{9}$). This mark may also be given for answers as decimals [4.19 or 7.33], or degrees (240 or 420).

A1: Need all three correct answers in terms of π and **no extras in range**.

Three correct answers implies M1M1A1

NB : $\theta = 20°, 80°, 140°$ earns M1M1A0 and 0.349, 1.40 and 2.44 earns M1M1A0

(ii) (a) **M1:** Applies $\sin^2 x = 1 - \cos^2 x$ (allow even if brackets are missing e.g. $4 \times 1 - \cos^2 x$).

This must be awarded in (ii) (a) for an expression with k not after k = 3 is substituted.

dM1: Uses formula or completion of square to obtain cos$x$ = expression in k

(Factorisation attempt is M0) **A1:** cao - award for their final simplified expression

(b) **M1**: **Either** attempts to substitute k = 3 into their answer to obtain two values for cos$x$

Or restarts with k = 3 to find two values for cos$x$ (They cannot earn marks in ii(a) for this)

In both cases they need to have applied $\sin^2 x = 1 - \cos^2 x$ (brackets may be missing) **and** correct method for solving their quadratic (usual rules – see notes) The values for cos$x$ may be >1 or < -1

dM1: Obtains **two correct** values for x

A1: Obtains **all three correct values** in degrees (allow awrt 139 and 221) including 0. Ignore excess answers outside range (including 360 degrees) Lose this mark for excess answers in the range or radian answers.

Question Number	Scheme	Marks
9. (a)	Either: (Cost of polishing top and bottom (two circles) is) $3\times 2\pi r^2$ **or (**Cost of polishing curved surface area is) $2\times 2\pi rh$ or both - just need to see at least one of these products	B1
	Uses volume to give $(h=)\dfrac{75\pi}{\pi r^2}$ or $(h=)\dfrac{75}{r^2}$ (simplified) (if V is misread – see below)	B1ft
	$(C)=6\pi r^2+4\pi r\left(\dfrac{75}{r^2}\right)$ Substitutes expression for h into area or cost expression of form Ar^2+Brh	M1
	$C=6\pi r^2+\dfrac{300\pi}{r}$ *	A1* (4)
(b)	$\left\{\dfrac{\mathrm{d}C}{\mathrm{d}r}=\right\}\ 12\pi r-\dfrac{300\pi}{r^2}$ or $12\pi r-300\pi r^{-2}$ (then isw)	M1 A1 ft
	$12\pi r-\dfrac{300\pi}{r^2}=0$ **so** r^k = value where $k=\pm2,\pm3,\pm4$	dM1
	Use **cube** root to obtain $r=\left(their\ \frac{300}{12}\right)^{\frac{1}{3}}$ $(=2.92)$ - allow r = 3, and thus C =	ddM1
	Then C = awrt 483 or 484	A1cao (5)
(c)	$\left\{\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}=\right\}\ 12\pi+\dfrac{600\pi}{r^3}>0$ **so minimum**	B1ft (1) **[10]**

Notes

(a) **B1:** States $3\times 2\pi r^2$ or states $2\times 2\pi rh$

B1ft: Obtains a **correct** expression for h in terms of r **(**ft only follows misread of V)

M1: Substitutes their expression for h into **area or cost** expression of form Ar^2+Brh

A1*: Had correct expression for C and achieves **given** answer in part (a) including "C =" or "Cost=" and **no errors seen** such as C = area expression without multiples of (£)3 and (£)2 at any point. Cost and area must be perfectly distinguished at all stages for this A mark.

N.B. Candidates using Curved Surface Area = $\dfrac{2V}{r}$ - please send to review

(b) **M1:** Attempts to differentiate as evidenced by at least one term differentiated correctly

A1ft: Correct derivative – allow $12\pi r-300\pi r^{-2}$ then isw if the power is misinterpreted (ft only for misread)

dM1: Sets their $\dfrac{\mathrm{d}C}{\mathrm{d}r}$ to 0, and obtains r^k = value where k = 2, 3 or 4 (needs correct collection of powers of r from their original derivative expression – allow errors dividing by 12π)

ddM1: Uses **cube** root to find r **or** see r = awrt 3 as evidence of cube root and substitutes into correct expression for C to obtain value for C

A1: Accept awrt 483 or 484

(c) **B1ft: Finds** correct expression for $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}$ and deduces value of $\dfrac{\mathrm{d}^2C}{\mathrm{d}r^2}$ > 0 so minimum (r may have been wrong)

OR checks gradient to left and right of 2.92 and shows gradient goes from negative to zero to positive so minimum

OR checks value of C to left and right of 2.92 and shows that C > 483 so deduces minimum (i.e. uses shape of graph) Only ft on misread of V for each ft mark (see below)

N..B. Some candidates have **misread** the volume as 75 instead of 75π. PTO for marking instruction.

Following this misread candidates cannot legitimately obtain the printed answer in part (a). Either they obtain $C = 6\pi r^2 + \dfrac{300}{r}$ or they "fudge" their working to appear to give the printed answer.

The policy for a misread is **to subtract 2 marks from A or B marks**. In this case the A mark is to be subtracted from part (a) and the final A mark is to be subtracted from part (b)
The maximum mark for part (a) following this misread is 3 marks. The award is B1 B1 M1 A0 as a maximum.

(a) B1: as before

B1: Uses volume to give $(h =)\dfrac{75}{\pi r^2}$

M1: $(C) = 6\pi r^2 + 4\pi r\left(\dfrac{75}{\pi r^2}\right)$

A0: Printed answer is not obtained without error

Most Candidates may then adopt the printed answer and gain up to full marks for the rest of the question so 9 of the 10 marks maximum in all.

Any candidate who proceeds with **their** answer $C = 6\pi r^2 + \dfrac{300}{r}$ may be awarded up to 4 marks in part (b). These are M1A1dM1ddM1A0 and then the candidate may also be awarded the B1 mark in part (c). So 8 of the 10 marks maximum in all.

(b) M1 A1: $\left\{\dfrac{dC}{dr} =\right\} 12\pi r - \dfrac{300}{r^2}$ or $12\pi r - 300r^{-2}$ (then isw)

dM1: $12\pi r - \dfrac{300}{r^2} = 0$ **so** r^k = value where k = 2, 3 or 4 or $12\pi r - \dfrac{300}{r^2} = 0$ **so** r^k = value

ddM1: Use **cube** root to obtain r = $\left(their\ \frac{300}{12\pi}\right)^{\frac{1}{3}}$ $(=1.996)$ - allow r = 2, and thus C = … must use $C = 6\pi r^2 + \dfrac{300}{r}$

A0: Cannot obtain C = 483 or 484

(c) B1: $\left\{\dfrac{d^2C}{dr^2} =\right\} 12\pi + \dfrac{600}{r^3} > 0$ **so minimum** OR checks gradient to left and right of 1.966 and shows gradient goes from negative to zero to positive so minimum
OR checks value of C to left and right of 1.966 and shows that C > 225.4 so deduces minimum (i.e. uses shape of graph)
There is an example in Practice of this misread.

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