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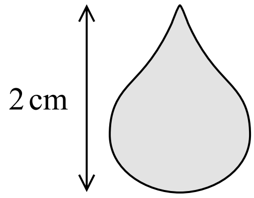

Figure 2

Figure 2 shows the image of a gold pendant which has height 2 cm. The pendant is modelled by a solid of revolution of a curve C about the y-axis. The curve C has parametric equations

$$x = \cos\theta + \frac{1}{2}\sin 2\theta, \quad y = -(1 + \sin\theta) \quad 0 \leqslant \theta \leqslant 2\pi$$

(a) Show that a Cartesian equation of the curve C is

$$x^2 = -(y^4 + 2y^3)$$

(4)

(b) Hence, using the model, find, in cm^3, the volume of the pendant.

(4)

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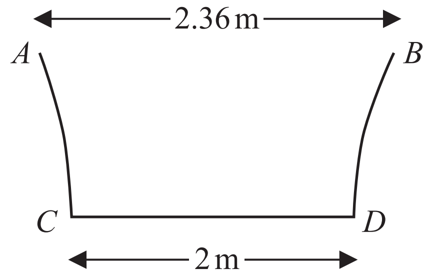

Figure 1

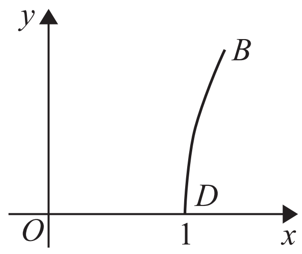

Figure 2

Figure 1 shows the central vertical cross section $ABCD$ of a paddling pool that has a circular horizontal cross section. Measurements of the diameters of the top and bottom of the paddling pool have been taken in order to estimate the volume of water that the paddling pool can contain.

Using these measurements, the curve BD is modelled by the equation

$$y = \ln(3.6x - k) \qquad 1 \leqslant x \leqslant 1.18$$

as shown in Figure 2.

(a) Find the value of k.

(1)

(b) Find the depth of the paddling pool according to this model.

(2)

The pool is being filled with water from a tap.

(c) Find, in terms of h, the volume of water in the pool when the pool is filled to a depth of h m.

(5)

Given that the pool is being filled at a constant rate of 15 litres every minute,

(d) find, in cm h^{-1}, the rate at which the water level is rising in the pool when the depth of the water is 0.2 m.

(3)

7.

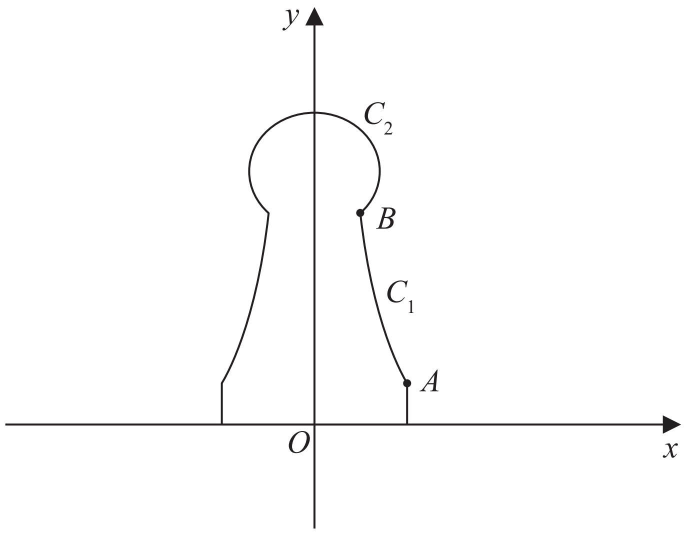

Figure 1

A student wants to make plastic chess pieces using a 3D printer. Figure 1 shows the central vertical cross-section of the student's design for one chess piece. The plastic chess piece is formed by rotating the region bounded by the y-axis, the x-axis, the line with equation $x = 1$, the curve C_1 and the curve C_2 through 360° about the y-axis.

The point A has coordinates (1, 0.5) and the point B has coordinates (0.5, 2.5) where the units are centimetres.

The curve C_1 is modelled by the equation

$$x = \frac{a}{y + b} \qquad 0.5 \leqslant y \leqslant 2.5$$

(a) Determine the value of a and the value of b according to the model.

(2)

The curve C_2 is modelled to be an arc of the circle with centre (0, 3).

(b) Use calculus to determine the volume of plastic required to make the chess piece according to the model.

(9)

7. **Solutions based entirely on graphical or numerical methods are not acceptable.**

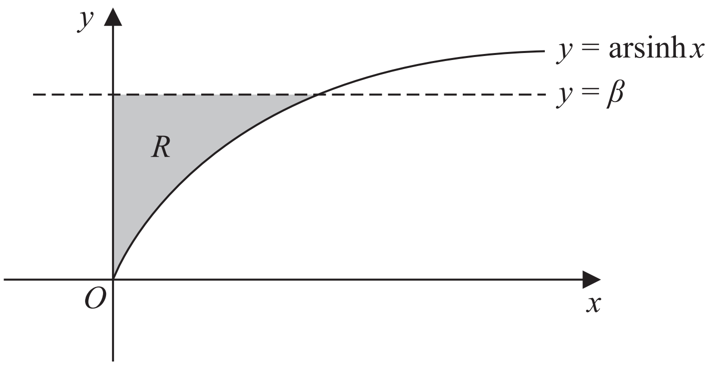

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \text{arsinh}\,x \qquad x \geqslant 0$$

and the straight line with equation $y = \beta$

The line and the curve intersect at the point with coordinates (α, β)

Given that $\beta = \frac{1}{2}\ln 3$

(a) show that $\alpha = \frac{1}{\sqrt{3}}$

(3)

The finite region R, shown shaded in Figure 1, is bounded by the curve with equation $y = \text{arsinh}\,x$, the y-axis and the line with equation $y = \beta$

The region R is rotated through 2π radians about the y-axis.

(b) Use calculus to find the exact value of the volume of the solid generated.

(6)

8. (a) Given

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad n \in \mathbb{N}$$

show that

$$32\cos^6\theta \equiv \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$$

(5)

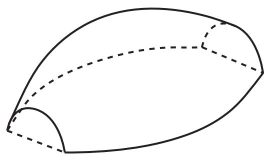

Figure 1

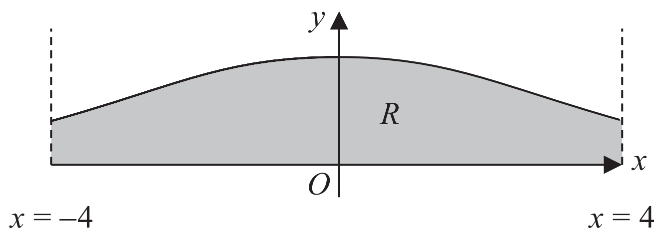

Figure 2

Figure 1 shows a solid paperweight with a flat base.

Figure 2 shows the curve with equation

$$y = H\cos^3\left(\frac{x}{4}\right) \qquad -4 \leqslant x \leqslant 4$$

where H is a positive constant and x is in radians.

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = -4$, the line with equation $x = 4$ and the x-axis.

The paperweight is modelled by the solid of revolution formed when R is rotated **180°** about the x-axis.

Given that the maximum height of the paperweight is 2 cm,

(b) write down the value of H.

(1)

(c) Using algebraic integration and the result in part (a), determine, in cm^3, the volume of the paperweight, according to the model. Give your answer to 2 decimal places.

[*Solutions based entirely on calculator technology are not acceptable*.]

(5)

(d) State a limitation of the model.

(1)

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

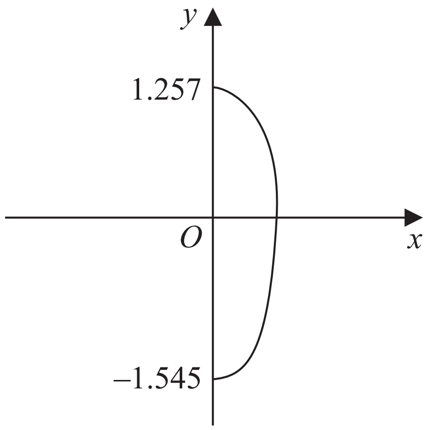

Figure 2

John picked 100 berries from a plant.

The largest berry picked was approximately 2.8 cm long.

The shape of this berry is modelled by rotating the curve with equation

$$16x^2 + 3y^2 - y\cos\left(\frac{5}{2}y\right) = 6 \qquad x \geqslant 0$$

shown in Figure 2, about the y-axis through 2π radians, where the units are cm.

Given that the y intercepts of the curve are –1.545 and 1.257 to four significant figures,

(a) use algebraic integration to determine, according to the model, the volume of this berry.

(6)

Given that the 100 berries John picked were then squeezed for juice,

(b) use your answer to part (a) to decide whether, in reality, there is likely to be enough juice to fill a 200 cm^3 cup, giving a reason for your answer.

(2)

9.

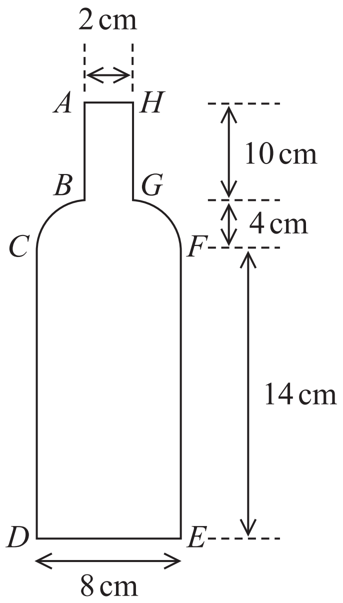

Figure 1

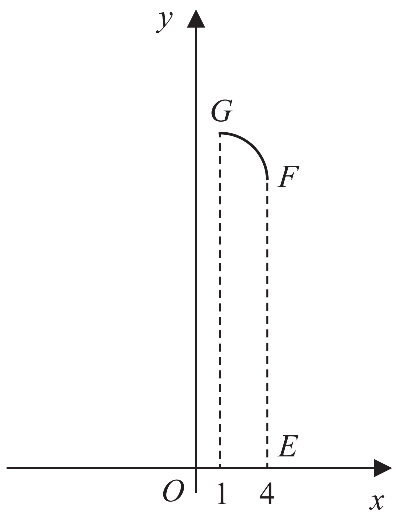

Figure 2

A mathematics student is modelling the profile of a glass bottle of water. Figure 1 shows a sketch of a central vertical cross-section *ABCDEFGHA* of the bottle with the measurements taken by the student.

The horizontal cross-section between *CF* and *DE* is a circle of diameter 8 cm and the horizontal cross-section between *BG* and *AH* is a circle of diameter 2 cm.

The student thinks that the curve *GF* could be modelled as a curve with equation

$$y = ax^2 + b \qquad 1 \leqslant x \leqslant 4$$

where a and b are constants and O is the fixed origin, as shown in Figure 2.

(a) Find the value of a and the value of b according to the model. **(2)**

(b) Use the model to find the volume of water that the bottle can contain. **(7)**

(c) State a limitation of the model. **(1)**

The label on the bottle states that the bottle holds approximately 750 cm^3 of water.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning. **(1)**

9. $$f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \qquad x > 0$$

The finite region bounded by the curve $y = f(x)$, the line $x = \frac{1}{8}$, the x-axis and the line $x = 8$ is rotated through θ radians about the x-axis to form a solid of revolution.

Given that the volume of the solid formed is $\frac{461}{2}$ units cubed, use algebraic integration to find the angle θ through which the region is rotated.

(8)

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3.

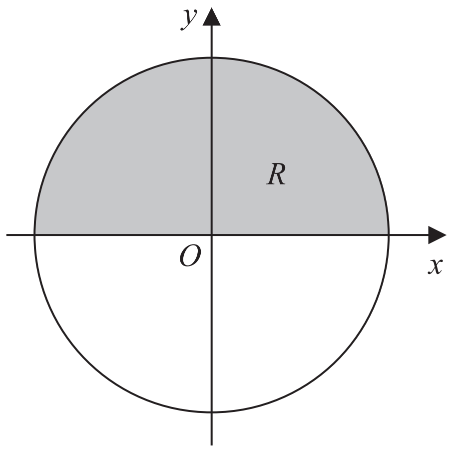

Figure 1

Figure 1 shows a circle with radius r and centre at the origin.
The region R, shown shaded in Figure 1, is bounded by the x-axis and the part of the circle for which $y > 0$
The region R is rotated through 360° about the x-axis to create a sphere with volume V

Use integration to show that $V = \dfrac{4}{3}\pi r^3$

(5)

P 6 2 6 8 5 A 0 6 2 8

9.

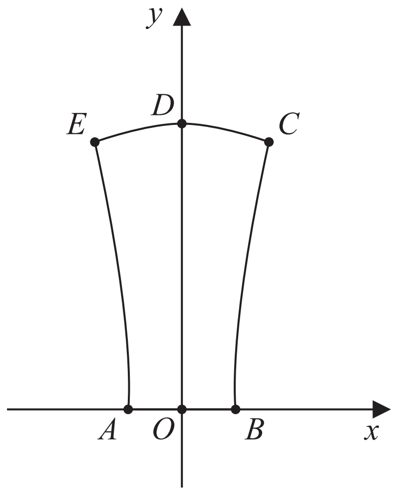

Figure 2

Figure 2 shows the vertical cross-section, *AOBCDE*, through the centre of a wax candle.

In a model, the candle is formed by rotating the region bounded by the y-axis, the line *OB*, the curve *BC*, and the curve *CD* through 360° about the y-axis.

The point *B* has coordinates (3, 0) and the point *C* has coordinates (5, 15).

The units are in centimetres.

The curve *BC* is represented by the equation

$$y = \frac{\sqrt{225x^2 - 2025}}{a} \qquad 3 \leqslant x < 5$$

where a is a constant.

(a) Determine the value of a according to this model.

(2)

The curve *CD* is represented by the equation

$$y = 16 - 0.04x^2 \qquad 0 \leqslant x < 5$$

(b) Using algebraic integration, determine, according to the model, the exact volume of wax that would be required to make the candle.

(9)

(c) State a limitation of the model.

(1)

When the candle was manufactured, $700\,\text{cm}^3$ of wax were required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

8.

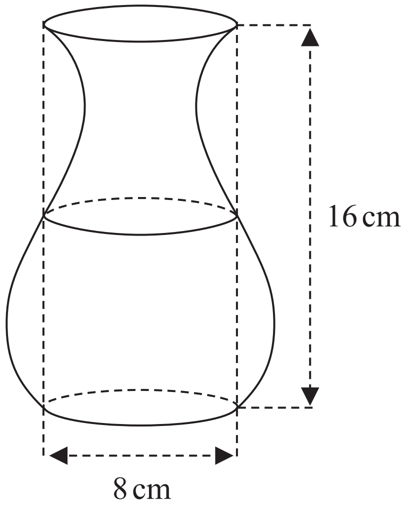

Figure 1

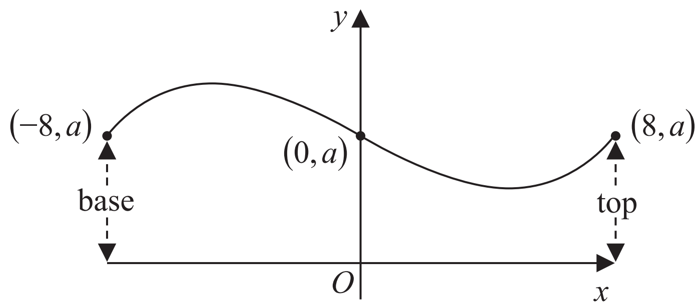

Figure 2

Figure 1 shows a sketch of a 16 cm tall vase which has a flat circular base with diameter 8 cm and a circular opening of diameter 8 cm at the top.

A student measures the circular cross-section halfway up the vase to be 8 cm in diameter.

The student models the shape of the vase by rotating a curve, shown in Figure 2, through 360° about the x-axis.

(a) State the value of a that should be used when setting up the model.

(1)

Two possible equations are suggested for the curve in the model.

Model A $\qquad y = a - 2\sin\left(\frac{45}{2}x\right)^{\circ}$

Model B $\qquad y = a + \frac{x(x-8)(x+8)}{100}$

For each model,

(b) (i) find the distance from the base at which the widest part of the vase occurs,

(ii) find the diameter of the vase at this widest point.

(7)

The widest part of the vase has diameter 12 cm and is just over 3 cm from the base.

(c) Using this information and making your reasoning clear, suggest which model is more appropriate.

(1)

(d) Using algebraic integration, find the volume for the vase predicted by Model B. You must make your method clear.

(5)

The student pours water from a full one litre jug into the vase and finds that there is 100 ml left in the jug when the vase is full.

(e) Comment on the suitability of Model B in light of this information.

(1)

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

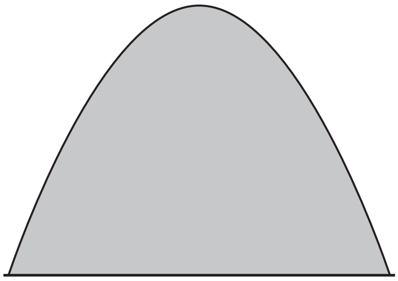

Figure 1

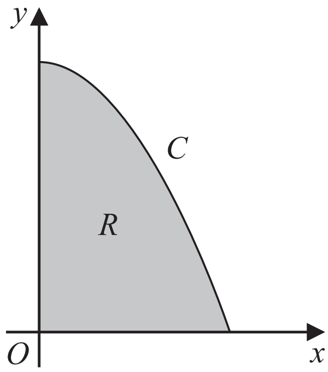

Figure 2

A large pile of concrete waste is created on a building site.

Figure 1 shows a central vertical cross-section of the concrete waste.

The curve C, shown in Figure 2, has equation

$$y + x^2 = 2 \qquad 0 \leqslant x \leqslant \sqrt{2}$$

The region R, shown shaded in Figure 2, is bounded by the y-axis, the x-axis and the curve C.

The volume of concrete waste is modelled by the volume of revolution formed when R is rotated through 360° about the y-axis. The units are metres.

The density of the concrete waste is $900\,\text{kg}\,\text{m}^{-3}$

(a) Use the model to estimate the mass of the concrete waste. Give your answer to 2 significant figures.

(6)

(b) Give a limitation of the model.

(1)

The mass of the concrete waste is approximately 5500 kg.

(c) Use this information and your answer to part (a) to evaluate the model, giving a reason for your answer.

(1)

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